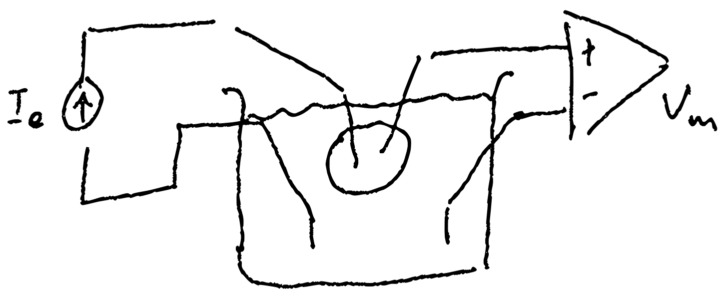
Lecture 2 RC neuron model 22 November

Alan Hodgkin & Andrew Huxley, 1952

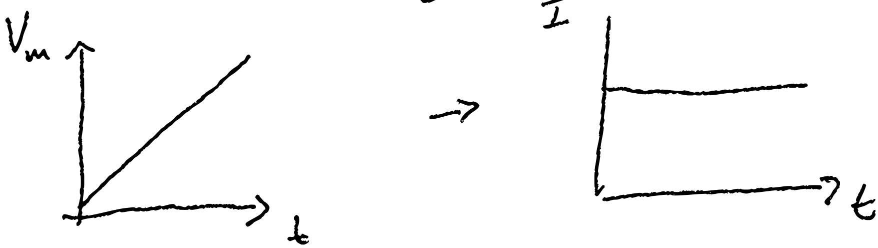
Q^{n}s

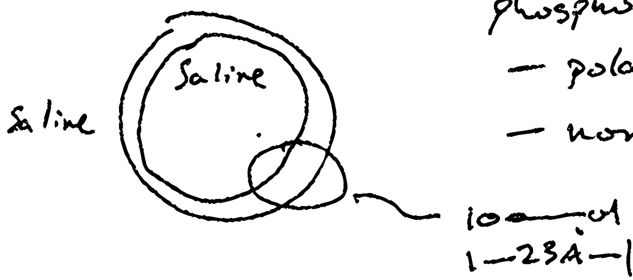
- How do neurons respond to current?
- Membrane cap^y and resistance
- "batteries" of a neuron



- Every aspect of computation and signaling in a neuron is controlled by voltage. The control is mediated by the voltage sensitivity of ion channels.
- All infor^n a neuron received is by current from other cells. How does current IN leads to volts out.

$$\text{voltage}(t) = \int_0^t \text{current}(t)\, d\tau$$



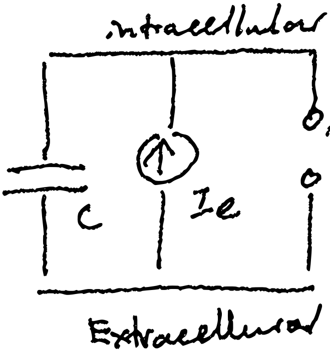


phospholipid bilayer:

- polar head
- non-polar tail

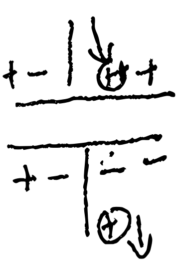
Å = 10^{-8} cm

This is a capacitor



$V_m = V_{in} - V_{out}$

membrane potential.



positive charge produces a capacitance charge.

There is a charge imbalance ΔQ thus there is an electric field $\vec{E}$ and a voltage ΔV.

The capacitance is

$$\boxed{\Delta Q = C \Delta V}$$

Capacitance Current

$$\rightarrow I_c(t) = \frac{dQ}{dt} = C \frac{dV_m}{dt},$$

Kirchoff's law:

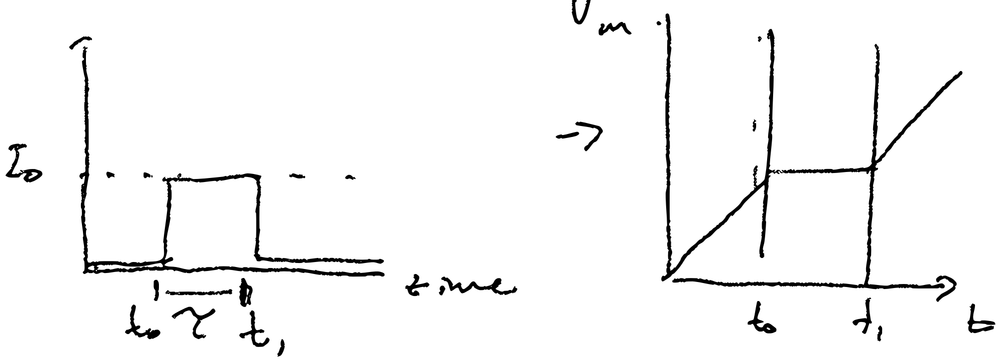
$$-I_c + I_e = 0$$

$$\Rightarrow \boxed{I_e(t) = C \frac{dV_m}{dt}}$$

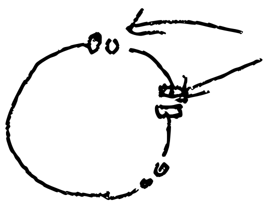
$$V_m(t) = V_0 + \frac{1}{C} \underbrace{\int_0^t I_e(t)\, dt}_{\Delta Q}$$

Thus the total change in voltage is just

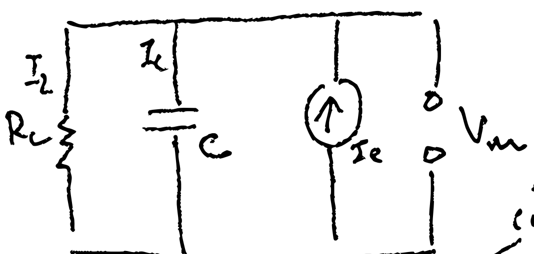
$$\boxed{\Delta V = \frac{1}{C} \Delta Q}$$



Leaky capacitor



Ion channels contribute to membrane current.



$$I_e = I_c + I_2$$

$$I_e = I_L + c\frac{dV}{dt}$$

electrode current (into the cell is +ve)

ionic current (+ve if leaving the cell)

$$\left(I_e = \frac{V_m}{R_2} + c\frac{dV}{dt} \right) \times R_L$$

$$\rightarrow V_m + R_2 c \frac{dV}{dt} = I_e R_L$$

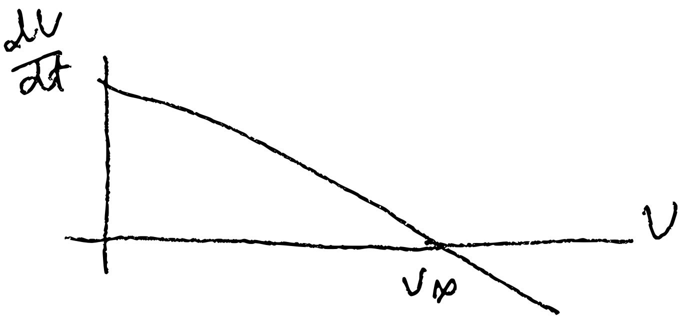
$$V_m + R_2 c \frac{dV_m}{dt} = I_e R_L$$

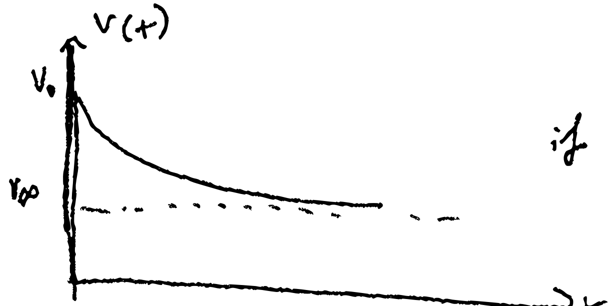
steady state $\Rightarrow$

$$V_m = I_e R_L \rightarrow V_\infty = I_e R_L$$

$$\rightarrow \quad V_m + \tau \frac{dV_m}{dt} = V_\infty \qquad \tau = R_L C$$

$$\rightarrow \quad \frac{dV}{dt} = -\frac{1}{\tau}(V - V_\infty)$$





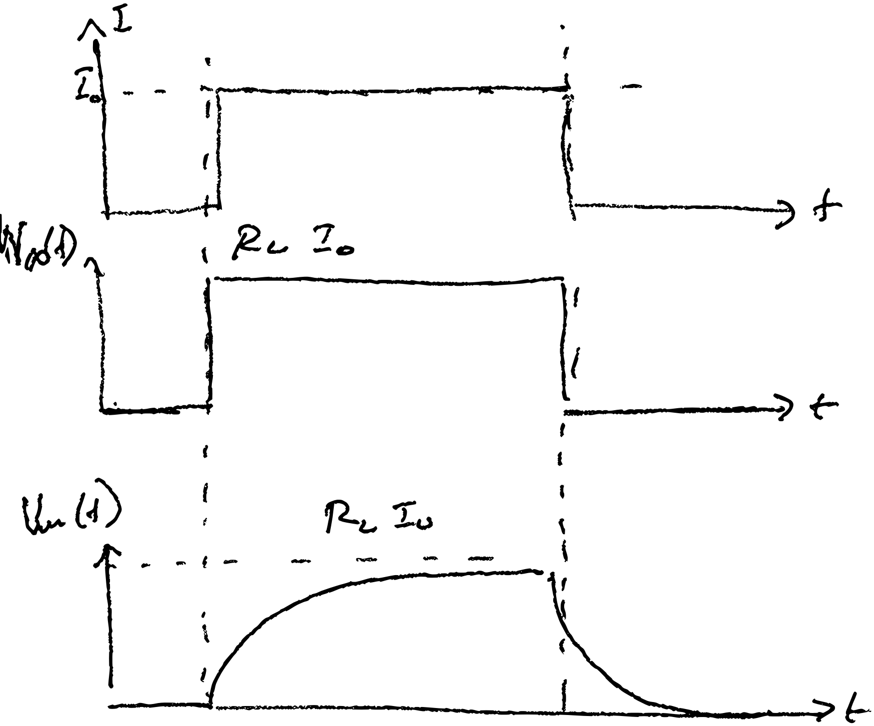
if τ is small $\frac{dV}{dt}$ is large so faster convergence.

The general solution:

$$\boxed{V(t) - V_\infty = (V_0 - V_\infty)\, e^{-t/\tau}}$$

Example

If we have an injection:



It is a low pass filter, only passes low frequencies.

Time scale

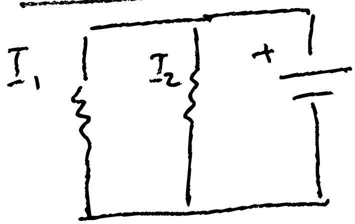
$$R = 10\ M\Omega \qquad C \approx 10^{-10}\ F$$

$\rightarrow \quad \tau = \sim 10 - 100\ ms,$

using conductance $G_L = \frac{1}{R_L}$ or

$$\boxed{I_L = G_L V}$$

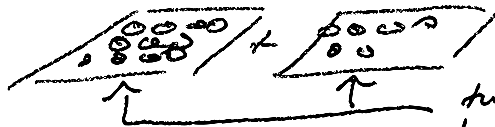
Two Conductance,



$$I_{tot} = I_1 + I_2$$

$$I_{tot} = (G_1 + G_2)V$$

$$G_{tot} = G_1 + G_2$$



twice the area

twice the holes.

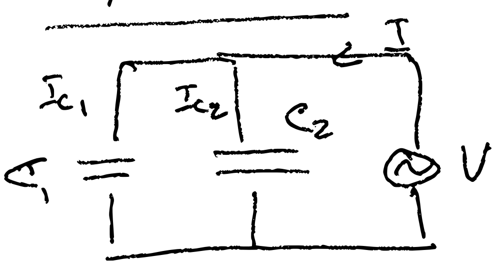
$$I_2 = G_2 V_m$$

$$= A g_2 V_m$$

specific leak conductance

membrane area

Capacitance



$$I_{ctot} = I_{c1} + I_{c2}$$

$$= C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt}$$

$$= (C_1 + C_2)\frac{dV}{dt}$$

$$\rightarrow \quad C_{tot} = C_1 + C_2$$

$$C = C_m A$$

specific capacitance (10 nF/mm²)

$$A = 4\pi r^2$$

membrane area

Time constant

$$\tau_m = R_L C = \frac{C}{G_L} = \frac{c_m A}{g_L A} = \frac{c_m}{g_L}$$

only a constance of the membrane properties. not of cell geometry.

Let's add a battery

* Some ion channels push the membrane potential positive
* others push it negatively.
* Together these channels gives the neural machinery flexible control of the voltage.

Where do batteries come from?

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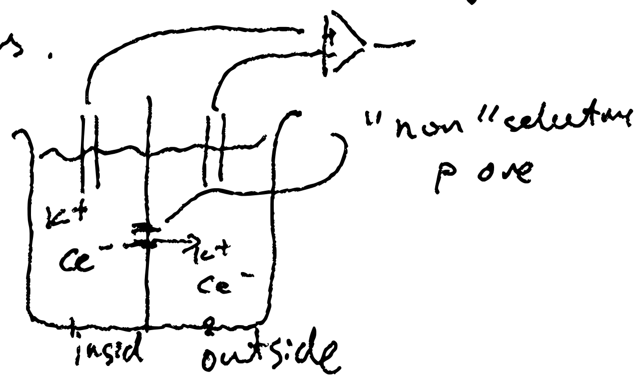
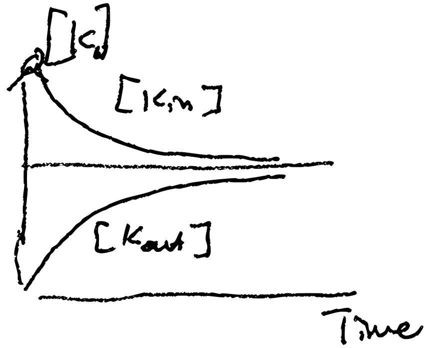
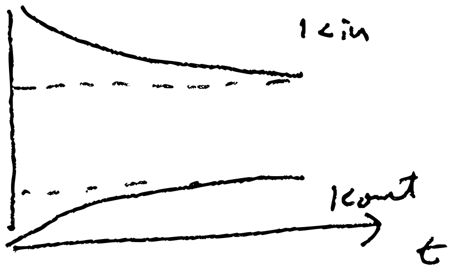
1) Ion concentration gradients
2) Ion selective permeability of ion channels.





Now if we have an "ion selective" pore passes only K^+ ion.



$\Delta V = V_1 - V_2$

once we poked a hole there is a voltage difference.

↑ equilibrium potential.