

Ignic Current

18th Nov 2020

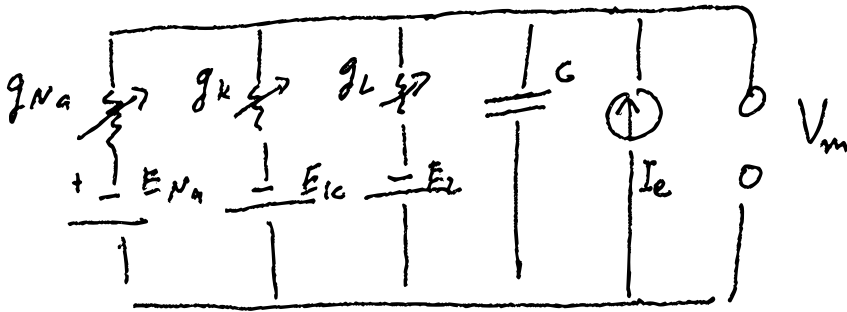
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* Silicon neural pixels can record brain activities.

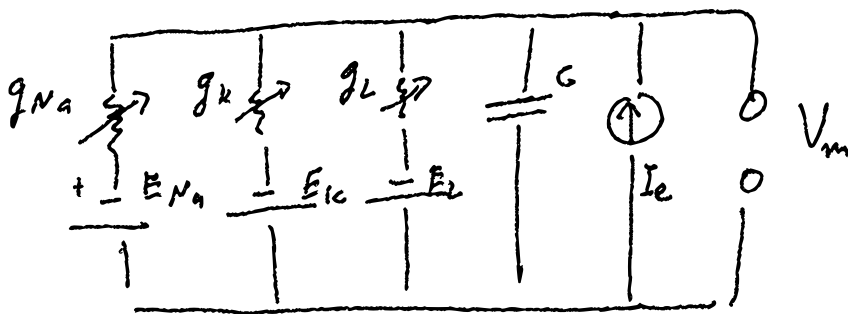
* How to find correlation in large datasets.

Song birds listen and imitate their parents.
So they can memories and practice.

Math of neuron



- Neurons are very complicated
- Different neurons are defined by the genes that are expressed.
- Different neurons also have different morphology, the spatial properties also shape how a neuron behaves.



Ions flow by two methods :

- Diffusion
- Drift in an electric field

Thermal energy:

* Every way a particle come to equilibrium that is proportional to the temperature.

The constant is Boltzmann constant

$$kT = 4 \times 10^{-21} \text{ Joules at } 300 \text{ K}$$

kinetic energy: $\left(\frac{1}{2} m v_x^2 \right) = \frac{1}{2} kT$

The mass of a sodium ion is $3.8 \times 10^{-26} \text{ kg}$

$$\Rightarrow \bar{v}_x = 3.2 \times 10^2 \text{ m/s}$$

In a solution the particles are crashing into each other. A particle in a solution collides with a water molecule 10^{13} times per second.

* an ion can diffuse across a cell body $10\mu\text{m}$ in 50ms

* to diffuse down a dendrite (1mm) takes around 10mins .

* for 1m long motor neuron takes 10 years !

Random walk 1D

Consider a particle moving \leftarrow or \rightarrow at V_x for a time τ before a collision.

If we have N particles, the i^{th} particle position is $x_i(n)$. Let $\delta = \pm V_x \tau$

$$n = t / \tau$$

$$x_i(n) = x_i(n-1) \pm \delta$$

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$$\begin{aligned}\langle x_i(n) \rangle &= \frac{1}{N} \sum_i x_i(n) \quad \text{average} \\ &= \frac{1}{N} \sum_i x_i(n-1) + \frac{1}{N} \sum_i (\pm \delta)\end{aligned}$$

$$\langle x_i(n) \rangle = \langle x_i(n-1) \rangle,$$

or center of the distribution is the same.

$$\begin{aligned}\langle |x(n)| \rangle &= \sqrt{\langle x^2(n) \rangle} \\ &= \frac{1}{N} \sum_i x_i^2(n) \\ &= \frac{1}{N} \sum_i x_i^2(n-1) \pm 2\delta x_i(n-1) + \delta^2 \\ &= \langle x^2(n-1) \rangle + \cancel{\langle \pm 2\delta x(n-1) \rangle} + \langle \delta^2 \rangle \\ &= \langle x^2(n-1) \rangle + \delta^2\end{aligned}$$

At each t the variance grows at δ^2 .

$$\langle x^2(0) \rangle = 0, \quad \langle x^2(1) \rangle = \delta^2, \quad \langle x^2(2) \rangle = 2\delta^2$$

$$\rightarrow \langle x^2(n) \rangle = \cancel{t\delta^2} n \delta^2$$

$$\langle x^2(t) \rangle = \frac{t}{N} \delta^2$$

$$\langle x^2 \rangle = 2Dt \quad D = \frac{\delta^2}{2\tau} \quad \text{diffusion coefficient.}$$

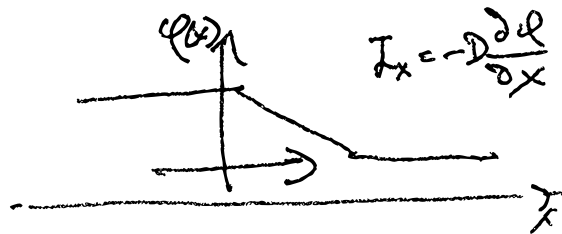
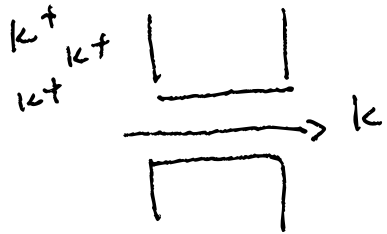
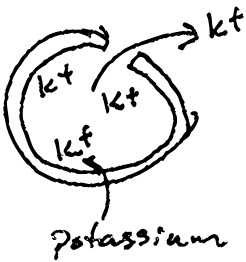
Spatial and t scales

Diffusion at short distances is fast but slow at large distances.

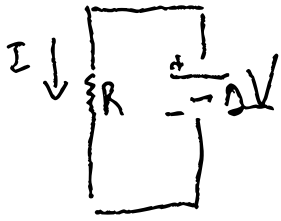
Fick's first law

• Produces a net flow from area of high concentration to lower.

$$J_x = -D \frac{\partial C}{\partial x} \quad \text{concentration}$$

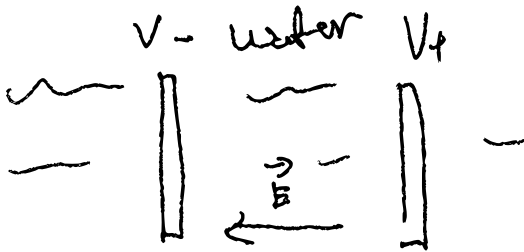


In a circuit current is \propto to voltage diff².



$$\Delta V = IR$$

$$\rightarrow I = \frac{\Delta V}{R}$$



$$E = \frac{\Delta V}{L}$$

- $\vec{F} = q\vec{E}$ is going to drag the particle through the liquid.

- $\vec{F} = f\vec{v}_d$ frictional coefficient
 $f = k \frac{\eta}{D}$

The drift velocity $\vec{v}_d = \frac{D}{kT} \vec{F} = \frac{D}{\mu T} \vec{E}$

- $I \propto v_d A$ and $I \propto EA = \frac{\Delta V}{L} A$

$$\rightarrow I \propto \left(\frac{1}{\rho}\right) \frac{\Delta V}{L} A \quad \rho = \text{resistivity } (\Omega \text{m}).$$

$$I = \left(\frac{A}{\rho L} \right) \Delta V \rightarrow R = \frac{\rho L}{A}$$

$\rho = 1.6 \mu\Omega \text{ cm}$ copper

$\rho = 60 \Omega \text{ cm}$ for mammalian saline.

→ Need huge voltage to diffuse a small current. That's why the brain ~~can~~ created the axon.