ARTICLE TEMPLATE

An Application of Spatiotemporal Modeling to Finite Population Abundance Prediction

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ARTICLE HISTORY
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ABSTRACT
This template is for authors who are preparing a manuscript for a Taylor & Francis journal using the LaTeX document preparation system and the 'interact} class file, which is available via selected journals' home pages on the Taylor & Francis website.

KEYWORDS
Sections; lists; figures; tables; mathematics; fonts; references; appendices

1. Introduction

1.1. *Motivation*

Moose surveys in Alaska and western Canada are often performed annually in many regions. The primary goal of these surveys is to predict moose abundance, the total number of moose, in the region. Because of time and money constraints, only some areas (sites) in the region of interest are selected to be in the survey. Biologists fly to these selected sites, count the number of moose, and can then use a spatial statistical model to find a prediction for the finite abundance for that year (Ver Hoef 2008).

Though these surveys are annual, each survey is analysed completely independently of surveys from previous years (cite examples). For example, a model for the survey conducted in the year 2019 only uses counts on sites that were sampled in that year. However, using counts from previous years in a model that incorporates both spatial and temporal correlation (spatiotemporal) could result in a prediction that is more precise than predictions from a spatial model using only counts from the most recent survey year.

Though the framework of the motivation is given with an example on moose surveys, this type of analysis could be useful for many examples involving prediction in a finite region with sites that are surveyed regularly.

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1.2. *Background*

Paragraph about background of spatiotemporal models

Paragraph about background of finite populations

Small paragraph combining and summarizing the previous two paragraphs

Small paragraph that gives the outline for the rest of the paper

2. Methods

We now give details on the development of the predictor for abundance. We first detail the spatiotemporal model, and we then use the spatiotemporal model with a finite population correction factor to give a Best-Linear-Unbiased-Predictor (BLUP) and its variance for total abundance in a given year.

2.1. *Spatiotemporal Model*

Let $Y(\mathbf{s}_i, t_j)$, $i = 1, 2, \ldots, n_{sp}$ and $j = 1, 2, \ldots, n_t$ be a random variable, where the vector $\mathbf{s}_i$ contains the coordinates for the i^{th} spatial site location and where t_j is the j^{th} time point. With each spatial location at each time point, the total number of data points (observed and unobserved) is $n_{sp} \cdot n_t \equiv N$. Then, a model for $mathbfy(\mathbf{s}_i, t_j)$, a vector of the $Y(\mathbf{s}_i, t_j)$ is

$$\mathbf{y}(\mathbf{s}_i, t_j) = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}(\mathbf{s}_i, t_j), \tag{1}$$

where $\mathbf{X}$ is a design matrix for fixed effects and $\boldsymbol{\beta}$ is a parameter vector of fixed effects. The error $\boldsymbol{\epsilon}(\mathbf{s}_i, t_j)$ can be decomposed into spatial and temporal components, as in Dumelle et al. (2021). In particular, a sum-with-error linear mixed model for response vector $\mathbf{y}(\mathbf{s}_i, t_j)$ is

$$\mathbf{y}(\mathbf{s}_i, t_j) = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}_{sp}\boldsymbol{\delta} + \mathbf{Z}_{sp}\gamma + \mathbf{Z}_t\boldsymbol{\tau} + \mathbf{Z}_t\boldsymbol{\eta} + \boldsymbol{\nu}. \tag{2}$$

$\mathbf{Z}_{sp}$ is an $N \times n_{sp}$ matrix where the values in a row corresponding to an observation in location $\mathbf{s}_i$ are a 1 in the i^{th} column and 0's in all other columns. Similarly, $\mathbf{Z}_t$ is an $N \times n_t$ matrix where the values in a row corresponding to an observation at time point $\mathbf{t}_j$ are a 1 in the j^{th} column and 0's in all other columns. We assume that $\boldsymbol{\delta}$, $\boldsymbol{\gamma}$, $\boldsymbol{\tau}$, $\boldsymbol{eta}$, and $\boldsymbol{\nu}$ all are all mean $\mathbf{0}$ vectors of length n_{sp}, n_{sp}, n_t, n_t, and N, respectively, with the covariances

$$\begin{aligned}
cov(\boldsymbol{\delta}) &= \sigma_\delta^2 \mathbf{R}_{sp} \\
cov(\boldsymbol{\gamma}) &= \sigma_\gamma^2 \mathbf{I}_{sp} \\
cov(\boldsymbol{\tau}) &= \sigma_\tau^2 \mathbf{R}_t \\
cov(\boldsymbol{\eta}) &= \sigma_\eta^2 \mathbf{I}_t \\
cov(\boldsymbol{\nu}) &= \sigma_\nu^2 \mathbf{I}_N,
\end{aligned}$$

where $\mathbf{R}_{sp}$ is a spatial correlation matrix and $\mathbf{R}_t$ is a temporal correlation matrix. σ_δ^2 and σ_γ^2 are the spatial partial sill and spatial nugget, σ_τ^2 and σ_η^2 are the temporal partial sill and temporal nugget, and σ^2_ν is spatiotemporal independent error variance parameter.

If we assume that $\boldsymbol{\delta}$, $\boldsymbol{\gamma}$, $\boldsymbol{\tau}$, $\boldsymbol{\eta}$, and $\boldsymbol{\nu}$ are mutually independent of each other, then

$$var(\mathbf{y}) \equiv \boldsymbol{\Sigma} = \sigma^2_\delta \mathbf{Z}_{sp}\mathbf{R}_{sp}\mathbf{Z}'_{sp} + \sigma^2_\gamma \mathbf{Z}_{sp}\mathbf{I}_{sp}\mathbf{Z}'_{sp} + \sigma^2_\tau \mathbf{Z}_t\mathbf{R}_t\mathbf{Z}'_t + \sigma^2_\eta \mathbf{Z}_t\mathbf{I}_t\mathbf{Z}'_t + \sigma^2_\nu \mathbf{I}_N. \quad (3)$$

One common form of $\mathbf{R}_{sp}$ is exponential. For observations at locations i and i' at $h_{ii'}$ distance apart, the i^{th} row and i'^{th} column of $\mathbf{R}_{sp}$ is

$$r_{sp,ii'} = \exp(-h_{ii'}/\phi), \quad (4)$$

where ϕ is the range parameter.

One common form of $\mathbf{R}_t$ is also exponential, more commonly known in time series as AR(1). For observations at time points j and j' that are $m_{jj'}$ units apart, the j^{th} row and j'^{th} column of $\mathbf{R}_t$ is

$$r_{t,jj'} = \exp(-m_{jj'}/\rho), \quad (5)$$

where ρ is the autocorrelation parameter.

If we assume that $\mathbf{y}$ is multivariate normal with mean $\mathbf{X}\boldsymbol{\beta} \equiv \boldsymbol{\mu}$ and variance $\boldsymbol{\Sigma}$, then all parameters can be estimated with Maximum Likelihood or Restricted Maximum Likelihood.

2.2. *Finite Population Kriging*

The model in equation 2 is used for all N observations at n_{sp} sites and n_t time points. But, we typically do not have the resources to sample every spatial site in every year. Additionally, for many wildlife management decisions, we are most interested in prediction of the abundance in the most current year of the survey.

Let the subscript s denote observations that were sampled (both past and present), and let the subscript u denote observations that were unsampled. Then, we can reorder the response vector so that

$$\mathbf{y} = [\mathbf{y}'_u, \mathbf{y}'_s]'. \quad (6)$$

Let $\tilde{\mathbf{y}} = [\tilde{\mathbf{y}}'_u, \tilde{\mathbf{y}}'_s]'$ denote the fixed, realized values of the response variable for one data-generating process. Our primary goal is to use the model developed in the previous section to predict values for $\tilde{\mathbf{y}}_u$ from the observed data in $\tilde{\mathbf{y}}_s$. That is, we want to find optimal weights $\mathbf{a}'$ to apply to the sampled data $\mathbf{a}'\tilde{\mathbf{y}}_s$, such that $\mathbf{a}'\mathbf{y}_s$ is the Best Linear Unbiased Predictor (BLUP) for $\mathbf{b}'_a\mathbf{y}_a$. If we are interested in the total abundance across all years, then $\mathbf{b}_a$ is a column vector of 1's. so that we are adding up all values of the response for our predictor of total abundance.

Unbiasedness implies that $E(\mathbf{a}'\mathbf{y}_s) = E(\mathbf{b}'_a\mathbf{y}_a)$ for all $\boldsymbol{\beta}$. So, denoting $\mathbf{X}_s$ as the design matrix for sampled sites, $\mathbf{a}'\mathbf{X}_s\boldsymbol{\beta} = \mathbf{b}'\mathbf{X}\boldsymbol{\beta}$ for every $\boldsymbol{\beta}$, implying that $\mathbf{a}'\mathbf{X}_s = \mathbf{b}'{}_a\mathbf{X}$.

The kriging weights are then found by finding $\boldsymbol{\lambda}$ such that]

$$E\{(\mathbf{a}'\mathbf{y}_s - \mathbf{b}'{}_a\mathbf{y}_a)(\mathbf{a}'\mathbf{y}_s - \mathbf{b}'{}_a\mathbf{z}_a)\} - E\{(\boldsymbol{\lambda}'\mathbf{y}_s - \mathbf{b}'{}_a\mathbf{z}_a)(\boldsymbol{\lambda}'\mathbf{y}_s - \mathbf{b}'{}_a\mathbf{z}_a)\} \quad (7)$$

is greater than 0 for all $\mathbf{a}'$. The prediction equations are

$$\begin{pmatrix} \mathbf{\Sigma}_{s,s} & \mathbf{X}_s \\ \mathbf{X}_s' & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\lambda} \\ m \end{pmatrix} = \begin{pmatrix} \mathbf{\Sigma}_{s,s} & \mathbf{\Sigma}_{s,u} \\ \mathbf{X}_s' & \mathbf{X}_u' \end{pmatrix} \begin{pmatrix} \mathbf{b}_s \\ \mathbf{b}_u \end{pmatrix}, \tag{8}$$

where again the subscripts s and u denote sampled and unsampled observations. For example, letting n denote the number of sampled observations, $\mathbf{\Sigma}_{s,s}$ denotes the $n \times n$ submatrix of $\mathbf{\Sigma}$ corresponding only to rows and columns of sampled observations and $\mathbf{\Sigma}_{u,s}$ denotes the $(N-n) \times n$ submatrix of $\mathbf{\Sigma}$ corresponding to rows of observations that were not sampled and columns of observations that were sampled. Then, $\boldsymbol{\lambda}$ is an $n \times 1$ vector.

Then, we can solve for the prediction weights as

$$\boldsymbol{\lambda}_s = \mathbf{b}_s' + \mathbf{b}_u'(\mathbf{\Sigma}_{u,s}\mathbf{\Sigma}_{s,s}^{-1}) - \mathbf{b}_u'(\mathbf{\Sigma}_{u,s}\mathbf{\Sigma}_{s,s}^{-1})\mathbf{X}_s(\mathbf{X}_s'\mathbf{\Sigma}_{s,s}^{-1}\mathbf{X}_s)^{-1}\mathbf{X}_s'\mathbf{\Sigma}_{s,s}^{-1} + \mathbf{b}_u'\mathbf{X}_u'(\mathbf{X}_s'\mathbf{\Sigma}_{s,s}^{-1}\mathbf{X}_s)^{-1}\mathbf{X}_s\mathbf{\Sigma}_{s,s}^{-1}. \tag{9}$$

Our prediction for the total abundance across all years of the survey is then

$$\boldsymbol{\lambda}_s'\tilde{\mathbf{y}}_s, \tag{10}$$

with a prediction variance of

$$E((\boldsymbol{\lambda}_s'\mathbf{y}_s - \mathbf{b}_a'\mathbf{y}_a)(\boldsymbol{\lambda}_s'\mathbf{y}_s - \mathbf{b}_a'\mathbf{y}_a)) = \boldsymbol{\lambda}_s'\mathbf{\Sigma}_{s,s}\boldsymbol{\lambda}_s - 2\mathbf{b}_a'\mathbf{\Sigma}_{a,s}\boldsymbol{\lambda}_s + \mathbf{b}_a'\mathbf{\Sigma}\mathbf{b}_a. \tag{11}$$

However, we often are not interested in predicting total abundance across multiple years and instead would want a prediction of the total abundance in the most recent year of the survey. Therefore, for this goal, $\mathbf{b}_a$ should not be a vector of 1's and should instead take a value of 1 if the observation is in the current year of the survey and a 0 if the observation is not in the current year of the survey:

$$\mathbf{b}_a = [\mathbf{b}_{up}', \mathbf{b}_{uc}', \mathbf{b}_{sp}', , \mathbf{b}_{sc}']' = [\mathbf{0}', \mathbf{1}', \mathbf{0}', \mathbf{1}']', \tag{12}$$

where the subscripts up, uc, sp, and sc denote unsampled sites in past years, unsampled sites in current years, sampled sites in past years, and sampled sites in current years, respectively.

$\boldsymbol{\lambda}_s$ can then be rewritten as

$$\boldsymbol{\lambda}_s = \mathbf{b}_s' + \mathbf{b}_{uc}'(\mathbf{\Sigma}_{uc,s}\mathbf{\Sigma}_{s,s}^{-1}) - \mathbf{b}_{uc}'(\mathbf{\Sigma}_{uc,s}\mathbf{\Sigma}_{s,s}^{-1})\mathbf{X}_s(\mathbf{X}_s'\mathbf{\Sigma}_{s,s}^{-1}\mathbf{X}_s)^{-1}\mathbf{X}_s'\mathbf{\Sigma}_{s,s}^{-1} + \mathbf{b}_{uc}'\mathbf{X}_{uc}'(\mathbf{X}_s'\mathbf{\Sigma}_{s,s}^{-1}\mathbf{X}_s)^{-1}\mathbf{X}_s\mathbf{\Sigma}_{s,s}^{-1}. \tag{13}$$

The prediction variance can then be rewritten as

$$\boldsymbol{\lambda}_s'\mathbf{\Sigma}_{s,s}\boldsymbol{\lambda}_s - 2\mathbf{b}_c'\mathbf{\Sigma}_{c,s}\boldsymbol{\lambda}_s + \mathbf{b}_c'\mathbf{\Sigma}_{\mathbf{c},\mathbf{c}}\mathbf{b}_c, \tag{14}$$

where c denotes observations in the current year.

Table 1. Example of a table showing that its caption is as wide as the table itself and justified.

	Type					
Class	One	Two	Three	Four	Five	Six
Alpha[a]	A1	A2	A3	A4	A5	A6
Beta	B2	B2	B3	B4	B5	B6
Gamma	C2	C2	C3	C4	C5	C6

[a]This footnote shows how to include footnotes to a table if required.

3. Application

4. Simulation

5. Discussion

5.1. *Tables*

The `interact` class file will deal with positioning your tables in the same way as standard LaTeX. It should not normally be necessary to use the optional `[htb]` location specifiers of the `table` environment in your manuscript; you may, however, find the `[p]` placement option or the `endfloat` package useful if a journal insists on the need to separate tables from the text.

The `tabular` environment can be used as shown to create tables with single horizontal rules at the head, foot and elsewhere as appropriate. The captions appear above the tables in the Interact style, therefore the `\tbl` command should be used before the body of the table. For example, Table˜1 is produced using the following commands:

```
\begin{table}
\tbl{Example of a table showing that its caption is as wide as
 the table itself and justified.}
{\begin{tabular}{lcccccc} \toprule
 & \multicolumn{2}{l}{Type} \\ \cmidrule{2-7}
 Class & One & Two & Three & Four & Five & Six \\ \midrule
 Alpha\textsuperscript{a} & A1 & A2 & A3 & A4 & A5 & A6 \\
 Beta & B2 & B2 & B3 & B4 & B5 & B6 \\
 Gamma & C2 & C2 & C3 & C4 & C5 & C6 \\ \bottomrule
\end{tabular}}
\tabnote{\textsuperscript{a}This footnote shows how to include
 footnotes to a table if required.}
\label{sample-table}
\end{table}
```

To ensure that tables are correctly numbered automatically, the `\label` command should be included just before `\end{table}`.

The `\toprule`, `\midrule`, `\bottomrule` and `\cmidrule` commands are those used by `booktabs.sty`, which is called by the `interact` class file and included in the Interact LaTeX bundle for convenience. Tables produced using the standard commands of the `tabular` environment are also compatible with the `interact` class file.

5.2. *The list of references*

References should be listed at the end of the main text in alphabetical order by authors' surnames, then chronologically (earliest first). If references have the same author(s), editor(s), etc., arrange by year of publication, with undated works at the end. A single-author entry precedes a multi-author entry that begins with the same name. If the reference list contains two or more items by the same author(s) in the same year, add a, b, etc. and list them alphabetically by title. Successive entries by two or more authors when only the first author is the same are alphabetized by co-authors' surnames. If a reference has more than ten named authors, list only the first seven, followed by 'et al.'. If a reference has no author or editor, order by title; if a date of publication is impossible to find, use 'n.d.' in its place.

The following list shows some sample references prepared in the Taylor & Francis Chicago author-date style.

(Adelman 2009; Albiston 2005)

References

Adelman, Rachel. 2009. "'Such Stuff as Dreams Are Made On': God's Footstool in the Aramaic Targumim and Midrashic Tradition." Paper presented at the annual meeting for the Society of Biblical Literature, New Orleans, Louisiana, Nov. 21—24.

Albiston, Catherine R. 2005. "Bargaining in the Shadow of Social Institutions: Competing Discourses and Social Change in the Workplace Mobilization of Civil Rights." *Law and Society Review* 39 (1): 11–47.

Dumelle, Michael, Jay M Ver Hoef, Claudio Fuentes, and Alix Gitelman. 2021. "A linear mixed model formulation for spatio-temporal random processes with computational advances for the product, sum, and product–sum covariance functions." *Spatial Statistics* 43: 100510.

Ver Hoef, Jay M. 2008. "Spatial methods for plot-based sampling of wildlife populations." *Environmental and Ecological Statistics* 15 (1): 3–13.

6. Appendices

Any appendices should be placed after the list of references, beginning with the command `\appendix` followed by the command `\section` for each appendix title, e.g.

```
\appendix
\section{This is the title of the first appendix}
\section{This is the title of the second appendix}
```

produces:

Appendix A. This is the title of the first appendix

Appendix B. This is the title of the second appendix

Subsections, equations, figures, tables, etc. within appendices will then be automatically numbered as appropriate. Some theorem-like environments may need to have their counters reset manually (e.g. if they are not numbered within sections in the main text). You can achieve this by using `\numberwithin{remark}{section}` (for example) just after the `\appendix` command.

Please note that if the `endfloat` package is used on a document containing appendices, the `\processdelayedfloats` command must be included immediately before the `\appendix` command in order to ensure that the floats in the main body of the text are numbered as such.

Appendix A. Troubleshooting

Authors may occasionally encounter problems with the preparation of a manuscript using LaTeX. The appropriate action to take will depend on the nature of the problem:

(i) If the problem is with LaTeX itself, rather than with the actual macros, please consult an appropriate LaTeX 2_ε manual for initial advice. If the solution cannot be found, or if you suspect that the problem does lie with the macros, then please contact Taylor & Francis for assistance (`latex.helpdesk@tandf.co.uk`).

(ii) Problems with page make-up (e.g. occasional overlong lines of text; figures or tables appearing out of order): please do not try to fix these using 'hard' page make-up commands – the typesetter will deal with such problems. (You may, if you wish, draw attention to particular problems when submitting the final version of your manuscript.)

(iii) If a required font is not available on your system, allow TeX to substitute the font and specify which font is required in a covering letter accompanying your files.

Appendix B. Obtaining the template and class file

B.1. *Via the Taylor & Francis website*

This article template and the `interact` class file may be obtained via the 'Instructions for Authors' pages of selected Taylor & Francis journals.

Please note that the class file calls up the open-source LaTeX packages booktabs.sty, epsfig.sty and rotating.sty, which will, for convenience, unpack with the downloaded template and class file. The template calls for natbib.sty and subfigure.sty, which are also supplied for convenience.

B.2. *Via e-mail*

This article template, the `interact` class file and the associated open-source LaTeX packages are also available via e-mail. Requests should be addressed to `latex.helpdesk@tandf.co.uk`, clearly stating for which journal you require the template and class file.