# ASCON

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**Abstract.** In this paper we explore ASCON - its design and hardware implementation as well as its cryptanalysis automated analysis implementation. With regards to design, we focus through the construction of the Mode - Authenticated Encryption and the Permutation Components. In cryptanalysis, we present distinguishers from Zero-Sum, Linear and Differential cryptanalysis - truncated, impossible differentials. The Hardware Implementation of the Permutation is presented in Verilog along with Area and Component Analysis. Automated Analysis using basic MILP as well as including Logical Conditional Modelling is covered in this paper.

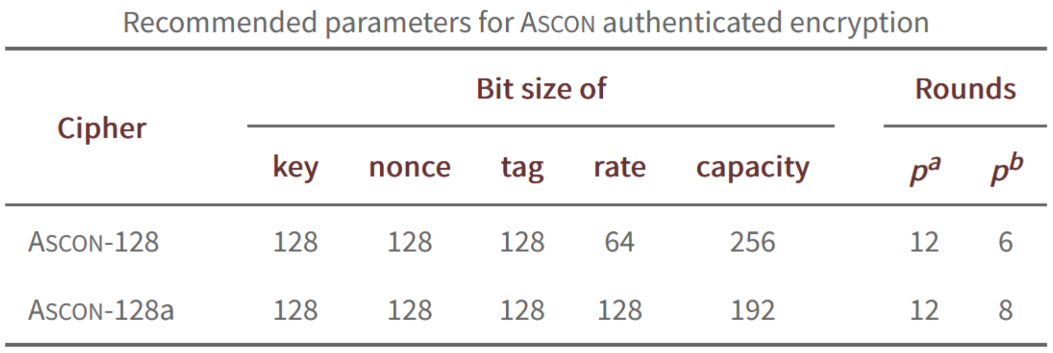
**Keywords:** ASCON · AEAD · Cryptanalysis · Verilog Implementation · Automated Analysis

## 1 Cipher Construction

ASCON cipher suite consists of authenticated encryption with associated data (AEAD) and hashing functionality. Authenticated encryption consists of the authenticated ciphers Ascon-128 and Ascon-128a, which vary on the number of rounds in the core permutation as well as in the rate and capacity. Apart from that ASCON Cipher suite also provides the hash functions Ascon-Hash and Ascon-Hasha, as well as the Extendable Output Functions (XOFs) Ascon-Xof and Ascon-Xofa. We will however focus mainly on the Authenticated Encryption Mode but more specifically the permutation of the ASCON. All the mode schemes have 128-bit security and the permutation is run over a 320 bit state. Authentication Encryption in specific is also designed to work through single pass - not needing to run twice to get the Authentication Tag as well as online - not needing to know the entire Plaintext or Associated Data to start running the algorithm.

The Permutation is design to have several features and complement over the balance of high security, performance and robustness. Ascon permutation operations are all run over 64 bit words making it fast for 64-bit platforms, while bitsliced implementation of s-box can easily be parallelized and run faster in 32-bit, 16-bit, 8-bit platforms. Ascon uses S-box with low algebraic degree and other less complex factors which allows for a small area usage.

## 1.1 Cipher Specifications



Recommended parameters for ASCON authenticated encryption

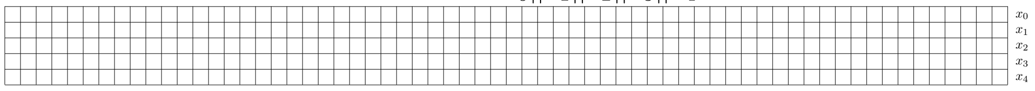
| Cipher | Bit size of | | | | | Rounds | |
|---|---|---|---|---|---|---|---|
| | key | nonce | tag | rate | capacity | $p^a$ | $p^b$ |
| ASCON-128 | 128 | 128 | 128 | 64 | 256 | 12 | 6 |
| ASCON-128a | 128 | 128 | 128 | 128 | 192 | 12 | 8 |

Figure 1: Authentication Encryption Specifications

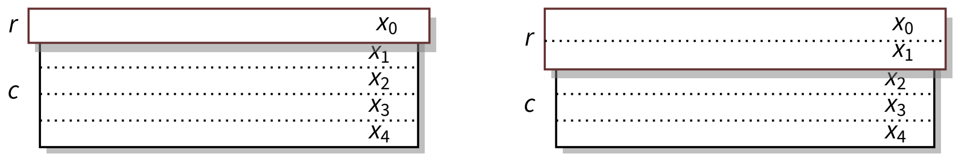
## 1.2 State Design

ASCON uses a 320-Bit State which can be divided into 5 - 64-Bit words - $x_0, x_1, x_2, x_3, x_4$ namely. The Least Significant Bit is the Right most Bit of $x_4$ and the Most Significant is the Left most bit of $x_0$. It follows a reverse row indexing of the bits.

Table 1: $S = x_0||x_1||x_2||x_3||x_4$



Apart from that the state can also be divided into Rate Part $S_r$ and the capacity part $S_c$ the following way



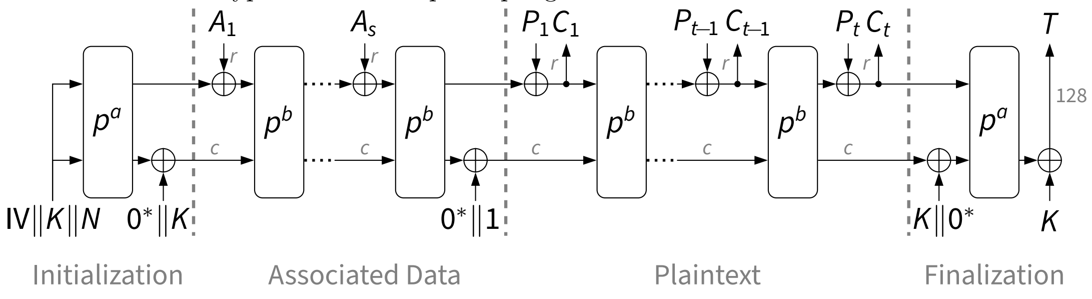
(a) 64-Bit Rate

(b) 128-Bit Rate

Figure 2: $S = S_r||S_c$

## 1.3 Authenticated Encryption

Authenticated Encryption uses a Duplex-Sponge Construction



### 1.3.1 Initialization

Initialization vectors is of 64 bits and it depends on the key size ($k$), rate size ($r$), end permutation rounds ($a$), and the core permutation rounds ($b$) - each filling up 8 bits and the rest being padded the following way

$$IV_{k,r,a,b} \leftarrow k||r||a||b||0^{160-k}$$
$$\text{ASCON-128} = -\texttt{80400c0600000000}$$
$$\text{ASCON-128a} - \texttt{80800c0800000000}$$

The State is then filled with the IV in $x_0$, Key next in $x_1, x_2$ and the Nonce in the remaining $x_3, x_4$

$$S \leftarrow IV||K||V$$

The State is then run through the end permutation $p^a$ after being added with a padded key the following way

$$S \leftarrow p^a(S) \oplus \left(0^{320-k}||K\right)$$

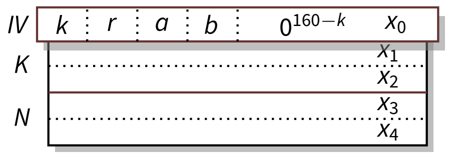


Figure 3: Initialization State

### 1.3.2 Associated Data Processing

After initialization we process the Associated Data by first padding the Associated Data $A$ and splitting it into $s$ $r$-Bit Blocks the following way

$$A_1, \ldots, A_s \leftarrow A||1||0^{r-1-(|A| \bmod r)}$$

Then each Associated Data Block $A_i$ is digested by injecting it into the core permutation

$$S \leftarrow p^b\left((S_r \oplus A_i)||S_c\right),\ 1 \leq i \leq s$$

After all the Blocks are digested we add a 1-bit Domain Separation Constant to add security over attacks that swap Associated Data and Plaintext

$$S \leftarrow S \oplus \left(0^{319}||1\right)$$

### 1.3.3 Encryption

After Associated Data Processing the Plaintext $P$ is processed by first and padding it splitting into $t$ $r$-Bit Blocks the following way

$$P_1, \ldots, P_t \leftarrow P||1||0^{r-1-(|P| \bmod r)}$$

Then Plaintext Block $P_i$ is injected into the core permutation after which the corresponding Ciphertext Block $C_i$ is extracted from the rate part.

$$C_i \leftarrow S_r \oplus P_i \quad 1 \leq i \leq t$$
$$S \leftarrow \begin{cases} p^b\left(C_i||S_c\right) & \text{if } 1 \leq i < t \\ C_i||S_c & \text{if } i = t \end{cases}$$

The last Ciphertext $C_t$ is unpadded to have the same size as the last Plaintext Block $P_t$ before padding so that the whole Plaintext and Ciphertext have same size

$$C'_t \leftarrow \lfloor C_t \rfloor_{|P| \bmod r}$$

### 1.3.4 Finalization

In the finalization phase we first add a Padded Key to State

$$S \leftarrow S \oplus \left(0^r || K || 0^{c-k}\right)$$

It is then run through the End Permutation

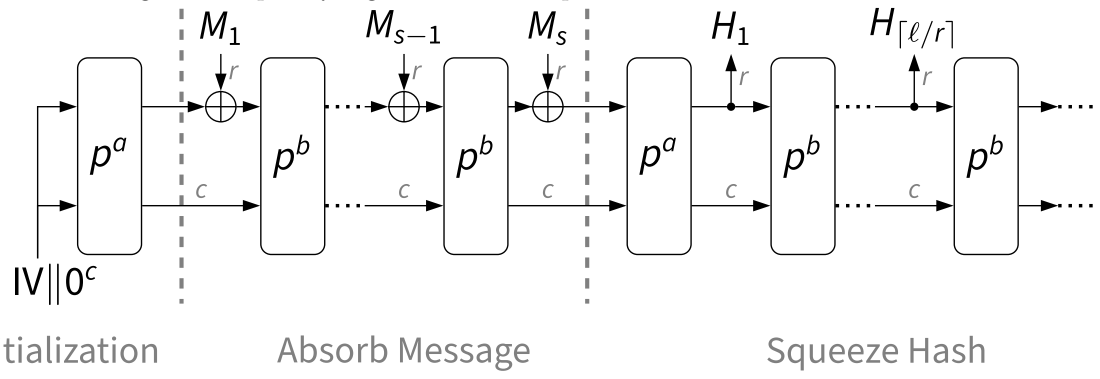
$$S \leftarrow p^a(S)$$

The Tag $T$ is extracted from the Least Significant 128 Bits after adding Key

$$T \leftarrow \lceil S \rceil^{128} \oplus \lceil K \rceil^{128}$$

## 1.4 Hash

The hash construction is similar to AEAD except it doesn't use Duplex-Sponge Construction. The Messages is completely digested and the squeeze is done in the end.



## 1.5 Permutation

Permutation consists of three components run one after the other - Round Constant $p_C$, S-Box $p_S$, Linear Diffusion Layer $p_L$

$$p = p_L \circ p_S \circ p_C$$

### 1.5.1 Round Constant ($p_C$)

The Round Constant is essentially adding a 64-bit round constant register $c_r$ on $x_2$. There are 12 registers and the register to be applied at a specific round $r$ depends on the following formula

$$p^a \rightarrow c_r \text{ and } p^b \rightarrow c_{a-b+r}$$

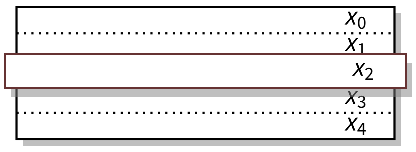


Figure 4: $c_r$ on $x_2$

Table 2: Round Constant Registers

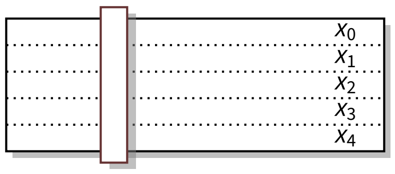
| $p^{12}$ | $p^8$ | $p^6$ | Constant | $p^{12}$ | $p^8$ | $p^6$ | Constant |
|---|---|---|---|---|---|---|---|
| 0 | | | 000000000000000000f0 | 6 | 2 | 0 | 00000000000000000096 |
| 1 | | | 000000000000000000e1 | 7 | 3 | 1 | 00000000000000000087 |
| 2 | | | 000000000000000000d2 | 8 | 4 | 2 | 00000000000000000078 |
| 3 | | | 000000000000000000c3 | 9 | 5 | 3 | 00000000000000000069 |
| 4 | 0 | | 000000000000000000b4 | 10 | 6 | 4 | 0000000000000000005a |
| 5 | 1 | | 000000000000000000a5 | 11 | 7 | 5 | 0000000000000000004b |

### 1.5.2 S-Box ($p_S$)

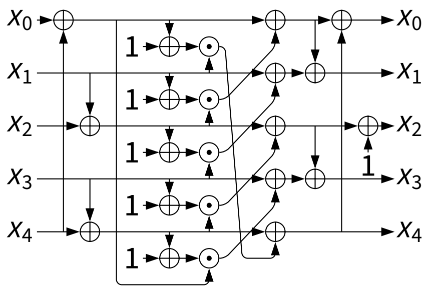
The below 5-Bit S-box is applied in a Bit Sliced Manner across $(x_0, x_1, x_2x_3, x_4)$ with $x_0$ being the Most Significant and $x_4$ being the Least Significant. 64 parallel applications can hence be done over Bit-Sliced S-box

Table 3: 5-Bit ASCON S-Box

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 1a | 1b | 1c | 1d | 1e | 1f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $S(x)$ | 4 | b | 1f | 14 | 1a | 15 | 9 | 2 | 1b | 5 | 8 | 12 | 1d | 3 | 6 | 1c | 1e | 13 | 7 | e | 0 | d | 11 | 18 | 10 | c | 1 | 19 | 16 | a | f | 17 |



(a) Bit-Sliced Application



(b) ANF Form

### 1.5.3 Linear Layer

Linear Layer is applied within each word $w_i$ with different Linear Functions $\Sigma_i(x_i)$ with essentially linearly combines each word with their right circular rotations the following way.

$$x_i \leftarrow \Sigma_i(x_i) \quad 0 \leq i \leq 4$$

$$\begin{aligned}
x_0 &\leftarrow \Sigma_0(x_0) = x_0 \oplus (x_0 \ggg 19) \oplus (x_0 \ggg 28) \\
x_1 &\leftarrow \Sigma_1(x_1) = x_1 \oplus (x_1 \ggg 61) \oplus (x_1 \ggg 39) \\
x_2 &\leftarrow \Sigma_2(x_2) = x_2 \oplus (x_2 \ggg 1) \oplus (x_2 \ggg 6) \\
x_3 &\leftarrow \Sigma_3(x_3) = x_3 \oplus (x_3 \ggg 10) \oplus (x_3 \ggg 17) \\
x_4 &\leftarrow \Sigma_4(x_4) = x_4 \oplus (x_4 \ggg 7) \oplus (x_4 \ggg 41)
\end{aligned}$$

# 2 Cryptanalysis

## 2.1 S-Box Analysis

### 2.1.1 DDT

The Differential Branch Number is 3 and the Differential Uniformity of ASCON S-box is 8. Hence the maximum Differential Probability of the S-Box is $2^{-2}$

Table 4: $\mathrm{DDT}[\Delta_i, \Delta_o] = |\{x : S(x \oplus \Delta_i) \oplus S(x) = \Delta_o\}|$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 1a | 1b | 1c | 1d | 1e | 1f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 32 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · |
| 1 | · | · | · | · | · | · | · | · | · | 4 | · | 4 | · | 4 | · | 4 | · | · | · | · | · | · | · | · | 4 | · | 4 | · | 4 | · | 4 | · |
| 2 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | 4 | · | 4 | · | 4 | · | 4 | · | 4 | · | 4 | · | 4 | · | 4 |
| 3 | · | 4 | · | · | · | 4 | · | · | · | 4 | · | · | · | 4 | · | · | 4 | · | · | · | 4 | · | · | · | 4 | · | · | · | 4 | · | · | · |
| 4 | · | · | · | · | · | · | 8 | · | · | · | · | · | · | · | 8 | · | · | · | · | · | · | · | 8 | · | · | · | · | · | · | · | 8 | · |
| 5 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | 4 | · | 4 | 4 | · | 4 | · | 4 | · | 4 | · | · | 4 | · | 4 |
| 6 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 |
| 7 | · | · | 4 | 4 | · | · | 4 | 4 | · | · | 4 | 4 | · | · | 4 | 4 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · |
| 8 | · | · | · | · | · | · | 4 | 4 | · | · | · | · | · | · | 4 | 4 | · | · | · | · | · | · | 4 | 4 | · | · | · | · | · | · | 4 | 4 |
| 9 | · | 2 | · | 2 | 2 | · | 2 | · | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 | · | 2 | · | 2 | · | 2 | 2 | · | 2 | · |
| 10 | · | 2 | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 |
| 11 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 |
| 12 | · | 8 | · | · | · | · | · | · | 8 | · | · | · | · | · | · | · | 8 | · | · | · | · | · | · | · | · | 8 | · | · | · | · | · | · |
| 13 | · | 2 | · | 2 | · | 2 | · | 2 | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | · | 2 | · | 2 | · | 2 | · | 2 |
| 14 | · | 4 | 4 | · | 4 | · | · | 4 | · | · | · | · | · | · | · | · | · | 4 | 4 | · | 4 | · | · | 4 | · | · | · | · | · | · | · | · |
| 15 | · | · | · | · | · | · | · | · | 4 | 4 | · | · | 4 | 4 | · | · | · | · | · | · | · | · | · | · | 4 | 4 | · | · | 4 | 4 | · | · |
| 16 | · | · | · | · | · | · | · | · | · | 8 | · | 8 | · | · | · | · | · | · | · | · | · | · | · | · | 8 | · | 8 | · | · | · | · | · |
| 17 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | 8 | · | 8 | · | 8 | · | 8 | · | · | · | · | · | · | · | · |
| 18 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · |
| 19 | · | · | 8 | · | 8 | · | · | · | · | · | 8 | · | 8 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · |
| 20 | · | · | · | · | 4 | 4 | 4 | 4 | · | · | · | · | 4 | 4 | 4 | 4 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · |
| 21 | · | · | · | · | · | 4 | · | 4 | · | 4 | · | 4 | · | · | · | · | · | 4 | · | 4 | · | · | · | · | · | · | · | · | · | 4 | · | 4 |
| 22 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 23 | · | · | 4 | · | 4 | · | · | · | · | · | 4 | · | 4 | · | · | · | · | · | 4 | · | 4 | · | · | · | · | · | 4 | · | 4 | · | · | · |
| 24 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 |
| 25 | · | · | · | 4 | · | · | 4 | · | 4 | · | · | · | · | 4 | · | · | 4 | · | · | · | · | 4 | · | · | · | · | · | 4 | · | · | 4 | · |
| 26 | · | 2 | 2 | · | · | 2 | 2 | · | 2 | · | · | 2 | 2 | · | · | 2 | · | 2 | 2 | · | · | 2 | 2 | · | 2 | · | · | 2 | 2 | · | · | 2 |
| 27 | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · |
| 28 | · | 4 | · | 4 | · | · | · | · | 4 | · | 4 | · | · | · | · | · | 4 | · | 4 | · | · | · | · | · | · | 4 | · | 4 | · | · | · | · |
| 29 | · | · | · | 4 | · | 4 | · | · | 4 | · | · | · | · | · | 4 | · | 4 | · | · | · | · | · | 4 | · | · | · | · | 4 | · | 4 | · | · |
| 30 | · | · | · | · | · | · | · | · | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | · | · | · | · | · | · | · | · | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 31 | · | · | 4 | 4 | 4 | 4 | · | · | · | · | · | · | · | · | · | · | · | · | 4 | 4 | 4 | 4 | · | · | · | · | · | · | · | · | · | · |

Table 5: DDT Frequency Analysis

| $f(\Delta_i, \Delta_o)$ | $S$ |
|---|---|
| 0 | 707 |
| 2 | 176 |
| 4 | 120 |
| 6 | 0 |
| 8 | 20 |

### 2.1.2 LAT

Linear Branch Number is 3 and Maximum Absolute Linear Bias of the ASCON S-Box is 8. Hence the The Maximum Linear bias of the S-box is $2^{-2}$

Table 6: $\text{LAT}[\alpha, \beta] = |\{x : \alpha^T \cdot x = \beta^T \cdot S(x)\}| - 16$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 1a | 1b | 1c | 1d | 1e | 1f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 16 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · |
| 1 | · | · | · | · | · | · | 8 | · | · | 4 | 4 | · | · | -4 | 4 | · | · | · | 4 | 4 | · | · | 4 | -4 | 4 | · | -4 | · | -4 | · | -4 | · |
| 2 | · | · | · | · | · | · | -8 | 8 | · | · | 4 | 4 | · | · | 4 | 4 | · | · | 4 | 4 | · | · | -4 | -4 | · | · | · | · | · | · | · | · |
| 3 | · | 8 | · | · | · | · | · | · | · | 4 | · | 4 | · | 4 | · | -4 | -8 | · | · | · | · | · | · | · | 4 | · | 4 | · | 4 | · | -4 | · |
| 4 | · | · | · | 4 | · | -4 | · | · | · | · | 4 | · | · | 4 | -4 | -4 | · | · | 4 | · | -4 | · | · | · | · | -8 | · | -4 | -4 | · | 4 | -4 |
| 5 | · | · | · | 4 | · | 4 | · | · | · | -4 | · | · | · | · | · | -4 | · | · | · | -4 | 4 | · | -4 | -4 | 4 | · | -4 | 4 | · | -8 | · | -4 |
| 6 | · | · | · | 4 | · | -4 | · | · | · | · | · | -4 | · | 4 | · | · | · | · | · | -4 | -4 | · | -4 | -4 | · | 8 | · | -4 | -4 | · | -4 | 4 |
| 7 | · | · | · | -4 | · | -4 | · | · | · | 4 | 4 | 4 | · | · | -4 | · | · | · | -4 | · | -4 | · | · | · | -4 | · | -4 | 4 | · | -8 | · | 4 |
| 8 | · | · | · | · | · | · | · | · | · | · | 4 | 4 | · | · | -4 | -4 | · | · | · | · | · | · | · | · | · | 8 | -4 | 4 | · | 8 | 4 | -4 |
| 9 | · | · | · | · | · | · | · | -8 | · | -4 | · | 4 | · | 4 | · | 4 | · | · | 4 | 4 | · | · | -4 | 4 | 4 | · | · | 4 | -4 | · | · | 4 |
| 10 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | 4 | 4 | · | · | 4 | 4 | · | 8 | 4 | -4 | · | -8 | 4 | -4 |
| 11 | · | 8 | · | · | · | · | · | · | · | -4 | 4 | · | · | -4 | -4 | · | 8 | · | · | · | · | · | · | · | 4 | · | · | -4 | 4 | · | · | 4 |
| 12 | · | · | -8 | 4 | -8 | -4 | · | · | · | · | · | 4 | · | -4 | · | · | · | · | -4 | · | 4 | · | · | · | · | · | 4 | · | -4 | · | · | · |
| 13 | · | · | · | -4 | -8 | 4 | · | · | · | 4 | -4 | -4 | · | · | -4 | · | · | · | · | 4 | -4 | · | -4 | -4 | 4 | · | · | · | · | · | 4 | · |
| 14 | · | · | · | -4 | 8 | -4 | · | · | · | · | -4 | · | · | -4 | -4 | -4 | · | · | · | 4 | 4 | · | -4 | -4 | · | · | 4 | · | -4 | · | · | · |
| 15 | · | · | 8 | -4 | -8 | -4 | · | · | · | -4 | · | · | · | · | · | -4 | · | · | 4 | · | 4 | · | · | · | -4 | · | · | · | · | · | -4 | · |
| 16 | · | · | · | · | · | · | -8 | · | · | 4 | · | -4 | -4 | · | -4 | · | · | · | · | · | 4 | -4 | 4 | 4 | 4 | · | -4 | · | -4 | · | -4 | · |
| 17 | · | · | · | · | · | · | · | · | -8 | · | -4 | 4 | -4 | -4 | · | · | · | 8 | 4 | -4 | -4 | -4 | · | · | · | · | · | · | · | · | · | · |
| 18 | · | -8 | · | · | · | · | · | · | · | -4 | 4 | · | -4 | · | · | -4 | · | · | -4 | 4 | -4 | -4 | · | · | 4 | · | 4 | · | 4 | · | -4 | · |
| 19 | · | · | · | · | · | · | -8 | -8 | · | · | · | · | 4 | -4 | 4 | -4 | · | · | · | · | -4 | 4 | 4 | -4 | · | · | · | · | · | · | · | · |
| 20 | · | · | · | 4 | · | 4 | · | · | · | 4 | 4 | -4 | -4 | -4 | · | -4 | · | · | 4 | · | · | 4 | -4 | 4 | -4 | · | 4 | 4 | · | · | · | 4 |
| 21 | · | · | · | 4 | · | -4 | · | · | · | · | · | -4 | 4 | · | -4 | 4 | · | 8 | · | 4 | · | 4 | · | · | · | · | · | 4 | 4 | · | -4 | -4 |
| 22 | · | · | · | -4 | · | -4 | · | · | · | 4 | · | · | -4 | 4 | 4 | · | 8 | · | · | -4 | · | 4 | · | · | 4 | · | 4 | 4 | · | · | · | -4 |
| 23 | · | · | · | 4 | · | -4 | · | · | 8 | · | -4 | · | -4 | · | · | · | · | · | 4 | · | · | -4 | 4 | -4 | · | · | · | 4 | 4 | · | 4 | 4 |
| 24 | · | · | · | · | · | · | · | -8 | · | 4 | 4 | · | -4 | · | · | 4 | · | · | · | · | 4 | -4 | -4 | -4 | -4 | · | · | -4 | 4 | · | · | -4 |
| 25 | · | · | · | · | · | · | · | · | · | · | · | · | 4 | -4 | -4 | 4 | · | -8 | 4 | -4 | -4 | -4 | · | · | · | · | 4 | 4 | · | · | -4 | -4 |
| 26 | · | 8 | · | · | · | · | · | · | · | -4 | · | -4 | -4 | · | 4 | · | · | · | -4 | 4 | -4 | -4 | · | · | -4 | · | · | 4 | -4 | · | · | -4 |
| 27 | · | · | · | · | · | · | · | · | 8 | · | -4 | 4 | -4 | -4 | · | · | · | · | · | · | -4 | 4 | -4 | 4 | · | · | -4 | -4 | · | · | -4 | -4 |
| 28 | · | · | 8 | 4 | · | -4 | · | · | · | 4 | · | · | 4 | -4 | 4 | · | · | · | -4 | · | · | -4 | -4 | 4 | 4 | · | · | · | · | · | 4 | · |
| 29 | · | · | · | -4 | · | 4 | · | · | 8 | · | 4 | · | 4 | · | · | · | · | 8 | · | -4 | · | -4 | · | · | · | · | 4 | · | -4 | · | · | · |
| 30 | · | · | · | 4 | · | 4 | · | · | · | 4 | -4 | 4 | 4 | 4 | · | -4 | 8 | · | · | 4 | · | -4 | · | · | -4 | · | · | · | · | · | -4 | · |
| 31 | · | · | 8 | 4 | · | 4 | · | · | · | · | · | 4 | -4 | · | -4 | 4 | · | · | -4 | · | · | 4 | 4 | -4 | · | · | 4 | · | -4 | · | · | · |

Table 7: LAT Frequency Analysis

| $\epsilon$ | $S$ |
|---|---|
| -8 | 18 |
| -4 | 174 |
| 0 | 647 |
| 4 | 162 |
| 8 | 22 |

### 2.1.3 BCT

Boomerang Uniformity is 16

Table 8: $\text{BCT}[\Delta_i, \nabla_o] = |\{S^{-1}(S(x) \oplus \nabla_o) \oplus S^{-1}(S(x \oplus \nabla_o) \oplus \Delta_i) = \Delta_i\}|$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | a | b | c | d | e | f | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 1a | 1b | 1c | 1d | 1e | 1f |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 | 32 |
| 1 | 32 | · | · | · | · | · | 8 | · | · | 4 | · | 4 | · | 4 | 8 | 4 | · | 4 | · | 4 | 4 | · | 12 | · | 8 | · | 8 | · | 4 | 4 | 12 | 4 |
| 2 | 32 | 12 | 4 | · | 4 | · | · | 4 | 8 | · | · | · | · | · | · | · | 8 | 8 | 4 | 4 | 4 | 4 | · | 8 | · | 12 | · | 4 | · | 4 | · | 4 |
| 3 | 32 | 4 | 8 | · | 8 | 4 | · | · | · | 12 | 8 | 8 | 8 | 4 | · | · | 4 | · | · | · | 4 | · | · | · | 12 | · | 8 | · | 4 | · | · | · |
| 4 | 32 | · | · | · | 4 | 8 | 12 | 8 | · | 16 | · | 16 | 4 | 8 | 12 | 8 | · | 16 | · | 16 | 4 | 8 | 12 | 8 | 16 | · | 16 | · | 4 | 8 | 12 | 8 |
| 5 | 32 | · | · | · | · | · | 8 | · | · | 4 | · | 4 | · | 4 | 8 | 4 | · | 4 | · | 4 | 4 | · | 12 | · | 8 | · | 8 | · | 4 | 4 | 12 | 4 |
| 6 | 32 | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 |
| 7 | 32 | · | 8 | 4 | 4 | · | 4 | 4 | · | 8 | 8 | 12 | 4 | · | 4 | 4 | · | · | 4 | · | 4 | · | · | · | 8 | · | 12 | · | 4 | · | · | · |
| 8 | 32 | · | · | 4 | · | · | 8 | 4 | 4 | · | · | · | · | 4 | 4 | 4 | 4 | 8 | · | 8 | · | 12 | 4 | 12 | · | · | · | 4 | · | · | 8 | 4 |
| 9 | 32 | 2 | · | 2 | 2 | · | 2 | · | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 | · | 2 | · | 2 | · | 2 | 2 | · | 2 | · |
| 10 | 32 | 2 | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 | · | 2 | 2 | · | 2 | · | · | 2 |
| 11 | 32 | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 | · | · | 2 | 2 |
| 12 | 32 | 12 | 16 | 8 | 16 | 8 | · | 4 | 12 | · | 8 | · | 8 | · | 4 | · | 12 | 16 | 8 | 16 | 8 | 16 | 4 | 16 | · | 12 | · | 8 | · | 8 | · | 4 |
| 13 | 32 | 2 | · | 2 | · | 2 | · | 2 | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | · | 2 | · | 2 | · | 2 | · | 2 |
| 14 | 32 | 12 | 4 | · | 4 | · | · | 4 | 8 | · | · | · | · | · | · | · | 8 | 8 | 4 | 4 | 4 | 4 | · | 8 | · | 12 | · | 4 | · | 4 | · | 4 |
| 15 | 32 | 4 | 8 | 4 | 8 | · | · | · | 8 | 4 | 12 | · | 12 | 4 | · | · | 4 | · | 4 | · | · | · | · | · | 4 | 8 | · | 4 | 4 | 4 | · | · |
| 16 | 32 | 4 | 16 | 4 | 16 | 8 | 16 | 8 | · | 12 | 16 | 12 | 16 | 8 | 16 | 8 | 4 | · | 4 | · | 8 | · | 8 | · | 12 | · | 12 | · | 8 | · | 8 | · |
| 17 | 32 | 8 | · | 8 | · | 8 | 16 | 8 | 16 | 4 | · | 4 | · | 4 | 16 | 4 | 16 | 12 | · | 12 | · | 12 | 16 | 12 | · | 8 | · | 8 | · | 8 | 16 | 8 |
| 18 | 32 | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · | 2 | · |
| 19 | 32 | 16 | 12 | 8 | 12 | 8 | · | · | 16 | 16 | 12 | 8 | 12 | 8 | · | · | 16 | · | 8 | 4 | 8 | 4 | · | · | 16 | 16 | 8 | 4 | 8 | 4 | · | · |
| 20 | 32 | · | · | · | 4 | 4 | 12 | 4 | · | 8 | · | 8 | 4 | 4 | 12 | 4 | · | · | · | · | · | · | 8 | · | 8 | · | 8 | · | · | · | 8 | · |
| 21 | 32 | · | · | · | · | 4 | 8 | 4 | · | 4 | · | 4 | · | · | 8 | · | · | 12 | · | 12 | · | 8 | 8 | 8 | · | · | · | · | · | 4 | 8 | 4 |
| 22 | 32 | · | · | · | · | · | · | · | · | · | · | · | · | · | · | · | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 23 | 32 | · | 8 | 4 | 4 | · | 4 | 4 | · | 8 | 8 | 12 | 4 | · | 4 | 4 | · | · | 4 | · | 4 | · | · | · | 8 | · | 12 | · | 4 | · | · | · |
| 24 | 32 | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 |
| 25 | 32 | · | · | 4 | · | · | 8 | 4 | 4 | · | · | · | · | 4 | 4 | 4 | 4 | 8 | · | 8 | · | 12 | 4 | 12 | · | · | · | 4 | · | · | 8 | 4 |
| 26 | 32 | 2 | 2 | · | · | 2 | 2 | · | 2 | · | · | 2 | 2 | · | · | 2 | · | 2 | 2 | · | · | 2 | 2 | · | 2 | · | · | 2 | 2 | · | · | 2 |
| 27 | 32 | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · | · | · | 2 | 2 | 2 | 2 | · | · |
| 28 | 32 | 4 | 8 | 4 | 8 | · | · | · | 8 | 4 | 12 | · | 12 | 4 | · | · | 4 | · | 4 | · | · | · | · | · | 4 | 8 | · | 4 | 4 | 4 | · | · |
| 29 | 32 | 8 | · | 4 | · | 4 | · | · | 12 | · | · | · | · | · | 4 | · | 12 | 8 | · | 8 | · | 8 | 4 | 8 | · | 8 | · | 4 | · | 4 | · | · |
| 30 | 32 | · | · | · | · | · | · | · | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | · | · | · | · | · | · | · | · | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 31 | 32 | 8 | 12 | 4 | 12 | 4 | · | · | 8 | · | 8 | · | 8 | · | · | · | 8 | · | 4 | 4 | 4 | 4 | · | · | · | 8 | · | · | · | · | · | · |

Table 9: BCT Frequency Analysis

| $f(\Delta_i, \nabla_o)$ | $S$ |
|---|---|
| 0 | 445 |
| 2 | 176 |
| 4 | 150 |
| 8 | 110 |
| 12 | 50 |
| 16 | 30 |

### 2.1.4 BDT

Boomerang Differential Table (BDT) combines DDT and BCT to find converging points that adhere to both Differential and Boomerang Conditions.

$$\text{DDT}[\Delta_i, \Delta_o, \nabla_o] = |\{x : S(x \oplus \Delta_i) \oplus S(x) = \Delta_o \& S^{-1}(S(x) \oplus \nabla_o) \oplus S^{-1}(S(x \oplus \nabla_o) \oplus \Delta_i) = \Delta_i\}|$$

Table 10: BDT Frequency Analysis

| $f(\Delta_i, \Delta_o, \nabla_o)$ | $S$ |
|---|---|
| 0 | 31624 |
| 2 | 352 |
| 4 | 720 |
| 8 | 40 |
| 32 | 32 |

## 2.2 Differential and Linear Cryptanalysis

The Differential and Linear Branch Numbers of the Linear Diffusion Layer $\Sigma_i$ is 4. The number of active S-box after 3 rounds in both Differential and Linear Trails are 15 and 13 respectively. Using heuristics, one can figure out the minimum number of active S-boxes for both differential and linear trails are calculated for upto 5 rounds and the results are the following

Table 11: Minimum Number of Active Sboxes

| Rounds ($R$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Minimum # Active Sboxes (Differential) | 1 | 4 | 15 | $\leq 44$ | $\leq 78$ |
| Minimum # Active Sboxes (Linear) | 1 | 4 | 13 | $\leq 43$ | $\leq 67$ |

We can add additional constraints on the differences, for instance we can constrain the differences - both input and output - to be restricted only to the rate part of the state ($S_r$). This might be useful in forgery attacks in the Authenticated Encryption Scheme. However, it might not be that fruitful as the Differential Properties of Permutations are quite secure.

$$\Delta_i^r \xrightarrow{p^b} \Delta_o^r$$

## 2.3 Truncated Differentials

For a specific input difference $\Delta_i$ of an S-box, if some bits of the output difference $\Delta_o^*$ remain invariant, then we call such bits **undisturbed**.

$$Pr\left[\Delta_i \xrightarrow{S} \Delta_o^*\right] = 1$$

For ASCON S-Box, due to its low algebraic degree $(d(S) = 2)$ has found to have around 23 such undisturbed bits which are shown below as follows. The Inverse S-box however has found to have only 2 undisturbed bits as it algebraic degree being higher $\left(d\left(S^{-1}\right) = 3\right)$

Table 12: Undisturbed Bits

| $\Delta_i$ | $\Delta_o^*$ |
|---|---|
| 00001 | *1*** |
| 00010 | 1***1 |
| 00011 | ***0* |
| 00100 | **110 |
| 00101 | 1**** |
| 01111 | *1*0* |
| 10000 | *10** |
| 10001 | 10**1 |
| 10011 | 0***0 |
| 10100 | 0*1** |
| 10101 | ****1 |
| 10110 | 1**** |
| 00110 | ****1 |
| 00111 | 0**1* |
| 01000 | **11* |
| 01011 | ***1* |
| 01100 | **00* |
| 01110 | *0*** |
| 10111 | ****0 |
| 11000 | **1** |
| 11100 | **0** |
| 11110 | *1*** |
| 11111 | *0*** |

Since there are so many available undisturbed bits with probability 1, we exhaust all of them to find the longest possible Truncated Differential. One such undisturbed bits $\left[\texttt{10011} \xrightarrow{S} \texttt{0***0}\right]$ has found to generate a 3.5 Round Truncated Differential as follows

Table 13: 3.5 Round Truncated Differential Distinguisher

| | |
|---|---|
| $I$ | 1000000000000000000000000000000000000000000000000000000000000000 |
| | 0000000000000000000000000000000000000000000000000000000000000000 |
| | 0000000000000000000000000000000000000000000000000000000000000000 |
| | 1000000000000000000000000000000000000000000000000000000000000000 |
| | 1000000000000000000000000000000000000000000000000000000000000000 |
| $S_1$ | 0000000000000000000000000000000000000000000000000000000000000000 |
| | *000000000000000000000000000000000000000000000000000000000000000 |
| | *000000000000000000000000000000000000000000000000000000000000000 |
| | *000000000000000000000000000000000000000000000000000000000000000 |
| | 0000000000000000000000000000000000000000000000000000000000000000 |
| $P_1$ | 0000000000000000000000000000000000000000000000000000000000000000 |
| | *00000000000000000000000000000000000000*000000000000000000000*00 |
| | **0000*000000000000000000000000000000000000000000000000000000000 |
| | *000000000*000000*0000000000000000000000000000000000000000000000 |
| | 0000000000000000000000000000000000000000000000000000000000000000 |
| $S_2$ | **0000*000*000000*000000000000000000000*000000000000000000000*00 |
| | **0000*000*000000*000000000000000000000*000000000000000000000*00 |
| | **0000*000*000000*000000000000000000000*000000000000000000000*00 |
| | **0000*000*000000*000000000000000000000*000000000000000000000*00 |
| | *000000000*000000*000000000000000000000*000000000000000000000*00 |
| $P_2$ | **0*00*000*00000**0**0000*00**0000*0*0**00000*000000000000*00*00 |
| | **0*00**00*000*00*000000000000000000*00**0000*000*000000*0*00**0 |
| | ****00**00***000***0000*000000000000000**0000*000000000000000**0 |
| | **0000**00**00*0***0*00*000*000000*0000*000000000*000000*0000*00 |
| | *000*00*00*00000**000000*0000000000000**0*0000*0000*000000*00*00 |
| $S_3$ | *****0**00***0*0*****00***0***0000*0*0****000**00*0*0000*0*00**0 |
| | *****0**00***0*0*****00***0***0000*0*0****000**00*0*0000*0*00**0 |
| | *****0**00***0*0***0*00**00*000000*0*0****000**00*0*0000*0*00**0 |
| | *****0**00***0*0*****00***0***0000*0*0****000**00*0*0000*0*00**0 |
| | **0**0**00**00*0*****00***0***0000*0*0****000**00*0*0000*0*00**0 |
| $P_3$ | ********0*****************************************0***0********0 |
| | *****0************************0*0***********0**0****0*0*****0*** |
| | *******************************00**********0****0****00*****0*** |
| | *****0**************************0**********0***0****0*0*****0*** |
| | *********0**0************************0************0***0********* |
| $S_4$ | ******************************************************0********* |
| | ******************************************************0********* |
| | ******************************************************0********* |
| | ******************************************************0********* |
| | ******************************************************0********* |

## 2.4 Impossible Differentials

Using automation tools, 5 Round Impossible Trails of the ASCON Permutation are found. One of which is as follows

Table 14: 5 Round Impossible Trail

| | | | |
|---|---|---|---|
| $x_0$ | 0000000000000000 | | 0100000000100002 |
| $x_1$ | 0000000000000000 | | 0000000000000000 |
| $x_2$ | 0000000000000000 | $\not\to$ | 0000000000000000 |
| $x_3$ | 0000000000000000 | | 0000000000000000 |
| $x_4$ | 8000000000000000 | | 0000000000000000 |

However, for a random permutation the above 5 round impossible differential would require a probability of $p = 2^{320}$. Hence to use it as a distinguisher, a whole codebook is required. It also cannot be used in key recovery or attacks like forgery as the Output Differences are fully specified.

A combination of 3.5 Truncated Distinguisher in the forward direction and a new Impossible Trail can be used in the backward direction however to generate a Truncated Impossible Differential in what is known as **miss-in-the-middle-technique**. To get the Round Differential, we will need to use Inverse S-box and Inverse Linear Diffusion. Using (*Rivest, 2011*) Theorem of finding Inverse of Rotations, we can find inverses of the ASCON linear functions $\Sigma_i$ as they have odd number of terms ($k = 3$) in the linear layer. Hence using that theorem, here are the terms in both the linear functions $\Sigma_i$ and their inverses $\Sigma_i^{-1}$

Table 15: Linear Function and Inverse Terms

| Function | Rotations | # Terms |
|---|---|---|
| $\Sigma_0$ | 0 19 28 | 3 |
| $\Sigma_0^{-1}$ | 0 3 6 9 11 12 14 15 17 18 19 21 22 24 25 27 30 33 36 38 39 41 42 44 45 47 50 53 57 60 63 | 31 |
| $\Sigma_1$ | 0 61 39 | 3 |
| $\Sigma_1^{-1}$ | 0 1 2 3 4 8 11 13 14 16 19 21 23 24 25 27 28 29 30 35 39 43 44 45 47 48 51 53 54 55 57 60 61 | 33 |
| $\Sigma_2$ | 0 1 6 | 3 |
| $\Sigma_2^{-1}$ | 0 2 4 6 7 10 11 13 14 15 17 18 20 23 26 27 28 32 34 35 36 37 40 42 46 47 52 58 59 60 61 62 63 | 33 |
| $\Sigma_3$ | 0 10 17 | 3 |
| $\Sigma_3^{-1}$ | 1 2 4 6 7 9 12 17 18 21 22 23 24 26 27 28 29 31 32 33 35 36 37 40 42 44 47 48 49 53 58 61 63 | 31 |
| $\Sigma_4$ | 0 7 41 | 3 |
| $\Sigma_4^{-1}$ | 0 1 2 3 4 5 9 10 11 13 16 20 21 22 24 25 28 29 30 31 35 36 40 41 44 45 46 47 48 50 53 55 60 61 63 | 35 |

Using the Inverse we now built an Impossible trail such that it should not match at atleast one bit in $S_4$ of 3.5 Round Truncated Differential. However since the Inverse S-box has only 2 undisturbed bits, the generated Differential is of length 1.5 Round only. Here is the generated combined 5-Round Differential (3.5 Forward + 1.5 Backward) which only differs in 1-bit of $S_4$

Table 16: 5 Round Impossible Truncated Distinguisher

| | |
|---|---|
| 3.5 Truncated Differential Distinguisher | |
| $S_4$ | `******************************************************0*********` |
| | `******************************************************0*********` |
| | `******************************************************0*********` |
| | `******************************************************0*********` |
| | `******************************************************0*********` |
| 1.5 Round Impossible Backward Differential | |
| $S_4$ | `****************************************************************` |
| | `****************************************************************` |
| | `****************************************************************` |
| | `1110000101100010110011111011111101010100101000110100011010100101` |
| | `****************************************************************` |
| $P_4$ | `0**0*0**0*00*0000**00****0****0***0***00*0*0*00***000*0000*00*0*` |
| | `0**0*0**0*00*0000**00****0****0***0***00*0*0*00***000*0000*00*0*` |
| | `0**0*0**0*00*0000**00****0****0***0***00*0*0*00***000*0000*00*0*` |
| | `0110101101001000011001111011110111011100101010011100010000100101` |
| | `0**0*0**0*00*0000**00****0****0***0***00*0*0*00***000*0000*00*0*` |
| $S_5$ | `0000000000000000000000000000000000000000000000000000000000000000` |
| | `0000000000000000000000000000000000000000000000000000000000000000` |
| | `0000000000000000000000000000000000000000000000000000000000000000` |
| | `0110101101001000011001111011110111011100101010011100010000100101` |
| | `0000000000000000000000000000000000000000000000000000000000000000` |
| $P_5$ | `0000000000000000000000000000000000000000000000000000000000000000` |
| | `0000000000000000000000000000000000000000000000000000000000000000` |
| | `0000000000000000000000000000000000000000000000000000000000000000` |
| | `1000000000000000000000000000000000000000000000000000000000000000` |
| | `0000000000000000000000000000000000000000000000000000000000000000` |

## 2.5 Zero-Sum Distinguisher

Zero-sum distinguishers over round-reduced versions for the Keccak permutation have been analysis across multiple research analyses. Since ASCON's Sbox is based on Keccak S-box $\chi$, we will be using similar methods used in Keccak Distinguisher especially that of Boura et al. Using similar techniques, we are able to construct 12 round distinguishers (essentially the whole end permutation).

Keccak S-box $\chi$ covers most of the ASCON S-box Search Criteria, however it has low Differential and Linear branch numbers of 2 whereas ASCON requires 3, a Fix Point at 0 whereas ASCON requires no Fix Points, and all Outputs depend on just 3 Inputs whereas ASCON requires atleast 4 Inputs. Hence basing on $\chi$, ASCON S-box is employs few lightweight affine transformations to the input and output of  to check through the search criteria. However, since its just affine transformation, they algebraic properties like algebraic degree of ASCON stays the same as that of $\chi$ which is 2. Hence for the permutation the algebraic degree is atmost 2. The inverse S-box, however, has alegraic degree 3 making reverse permutations degree at most 3. We will be exploiting that property of Ascon S-box to yield a Zero-Sum Distinguisher.

$$d(S) = 2 \rightarrow d(p) \leq 2 \rightarrow d(p^r) \leq 2^r$$

$$d(S^{-1}) = 3 \rightarrow d(p^{-1}) \leq 3 \rightarrow d\left((p^{-1})^r\right) = 3^r$$

**Basic 12 Round Distinguisher**

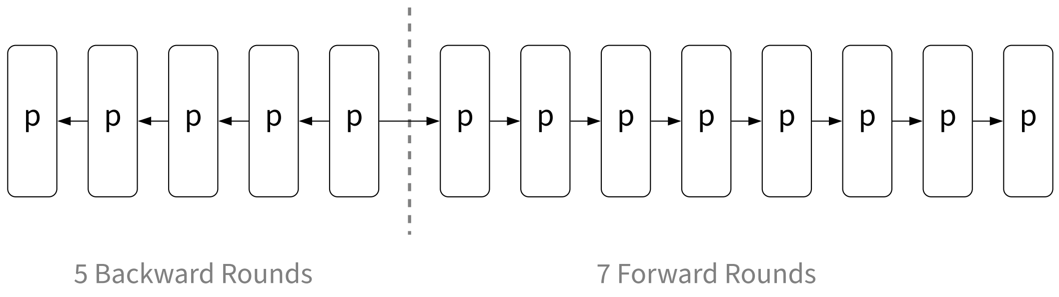


Figure 6: Basic 12 Round Distinguisher

In this we start at an intermediate state after Round 5. Hence there will be 5 backward (inverse) rounds and 7 forward rounds. For the forward 7-rounds, the upper bound degree will be $d\left(p^7\right) = 2^7 = 128$. For the backward 5 rounds of inverse permutation the upper bound will be $d\left((p^{-1})^5\right) = 3^5 = 243$. Hence with $d = \max\{2^7, 3^5\} + 1 = 244$, we start out with $320 - 244 = 76$ of constant bits and 244 variable bits in the intermediate state to create a set of $2^244$ input intermediate states. For each of these input intermediate states, we calculate 7 rounds forward and 5 rounds backward. The sum of all the resulting input and output states should be equal to 0.

**Free Middle Round Distinguisher**

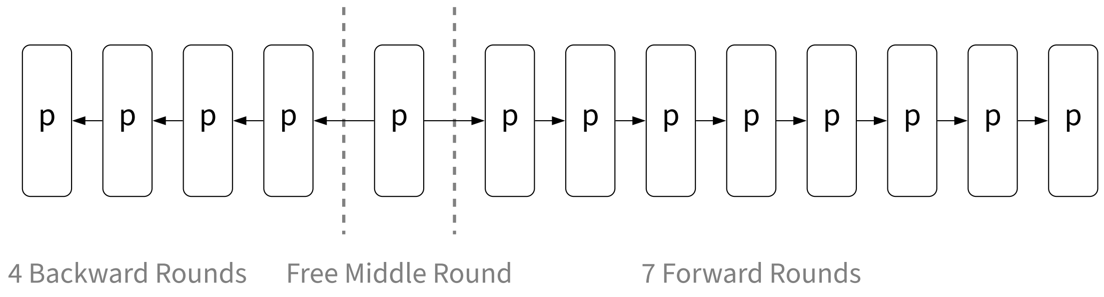


Figure 7: Free Middle Round Distinguisher

In this we start similarly as above at an intermediate state after Round 5. However this time we start out with a degree that is a multiple of 5-bit Sbox and we fill the $d$ variable bits such that Any S-box that is filled should be completely filled. Hence, the remaining constant bits fill out S-boxes completely as well. After application of S-box as well the variable bit and constant bit positions and number will be same. The starting Point of backward trail can be start of S-box of Round 5 whereas the starting point of Forward Rounds can be the Output of S-box of Round 5. The Linear Layer won't have much of an affect hence we get an entire free round without increase in algebraic degree. Hence there will be 4 backward (inverse) rounds and 7 forward rounds. For the forward 7-rounds, the upper bound degree will be $d\left(p^7\right) = 2^7 = 128$. For the backward 4 rounds of inverse permutation the upper bound will be $d\left((p^{-1})^4\right) = 3^4 = 81$. Hence with $d = \max\{2^7, 3^4\} + 1 = 129$, Since we need $d$ to be multiple of 5 our degree can be 130. we start out with $320 - 130 = 190$ of constant bits and 130 variable bits in the intermediate state to create a set of $2^130$ input intermediate states. For each of these input intermediate states, we calculate 7 rounds forward and 4 rounds backward. The sum of all the resulting input and output states should be equal to 0.

# 3 Verilog Implementation

## 3.1 S-Box

The S-box is implemented is 3 ways - using a Look-Up Table (LuT), Algebraic Normal Form (ANF) and Karnaugh Map (K-Map). The ASIC Area Analysis using Genus and the FPGA Area Analysis using Vivado can be found below. It is to be noted that in ASIC K-Maps have lower areas as they are essentially run on AND and OR Gates only which are cheaper compared to XOR. However as the algebraic degree of ASCON is 2, even ANF has lower area compared to LuT.

Table 17: ASIC Area Analysis - Genus

| LuT | ANF | K-MAP |
|---|---|---|
| 43.920 | 42.480 | 39.240 |

Table 18: FPGA Area Analysis - Vivado

| Component | Slice LuT (Logic) | OBUF | IBUF | LUT5 | LUT4 |
|---|---|---|---|---|---|
| Number Used | 3 | 5 | 5 | 3 | 2 |

### 3.1.1 Schematics

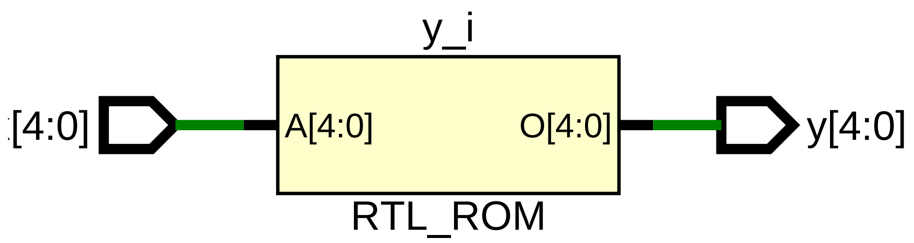


Figure 8: LuT

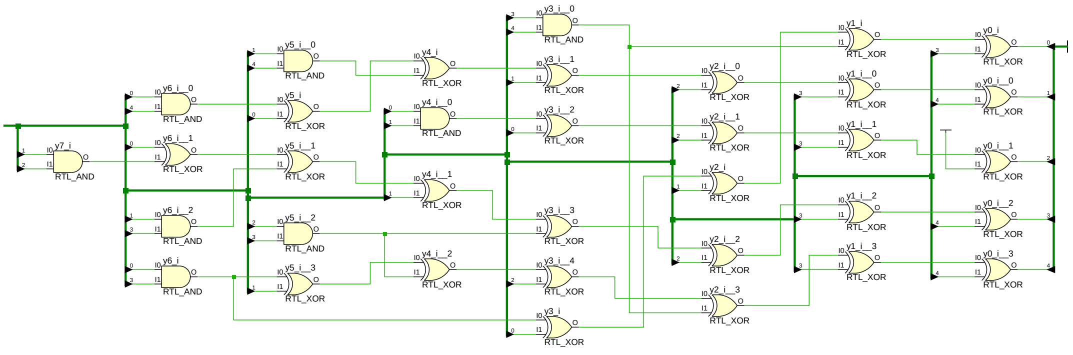


Figure 9: ANF

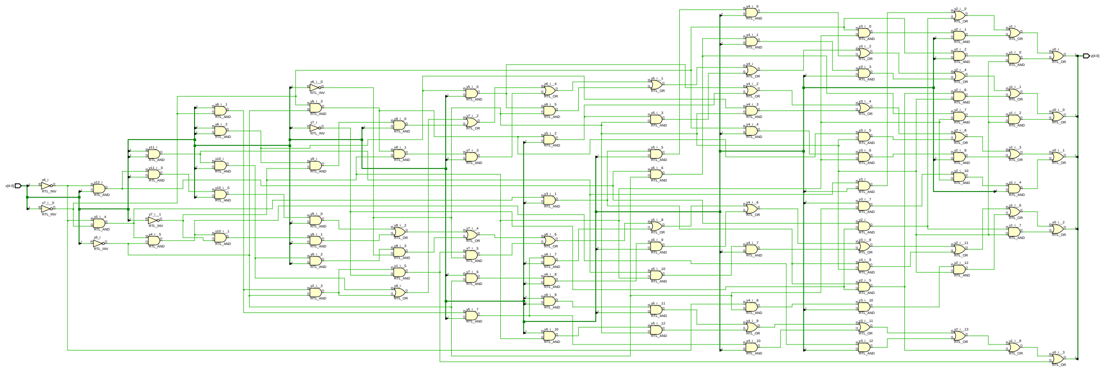


Figure 10: K-Map

## 3.2 Linear Layer

The Linear Layer takes in 320 Bits Input State and returns back 320 Bit Output State after applying the 5 Linear Functions $\Sigma_i$ on each word $x_i$. Its takes in 1958.400 GE on ASIC and here are the primitives used in FPGA

Table 19: Linear Layer FPGA Area Analysis - Vivado

| Components | Slice LuT (Logic) | OBUF | LUT3 | IBUF |
|---|---|---|---|---|
| Number Used | 169 | 320 | 320 | 320 |

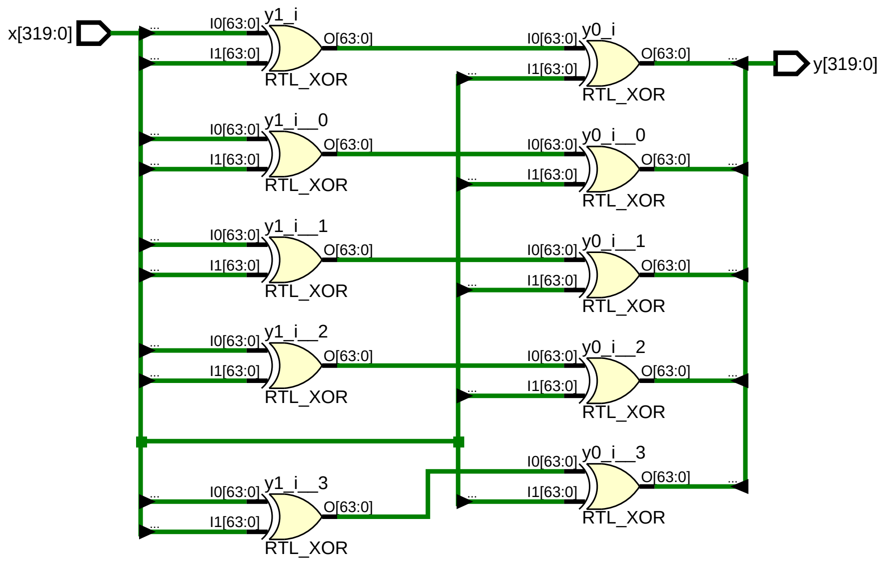


Figure 11: Linear Layer Schematic

## 3.3 Round Constant

Round Constant Module takes in a clock, start signal and a register containing number of round - could be 12 for end permutation ($p^a$) or 6, 8 for core permutation ($p^b$), It returns the 1 Byte of Round Constant Register after every clock cycle depending on the number of rounds as per discussed in the cipher design. After all the rounds are complete, it returns 00 byte for one clock cycle and then repeats round iteration. It takes in 321.840 GE in ASIC and here are the primitives used in FPGA

Table 20: Round Constant FPGA Analysis - Vivado

| Components | Slice LuT (Logic) | Slice Register (Flip Flop) | FDRE | FDRE | OBUF | LUT1 | LUT6 | IBUF | LUT4 | LUT2 | LUT3 | FDSE | LUT5 | BUFG (Clock) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number Used | 29 | 14 | 12 | 10 | 9 | 7 | 7 | 7 | 6 | 5 | 3 | 2 | 1 | 1 |

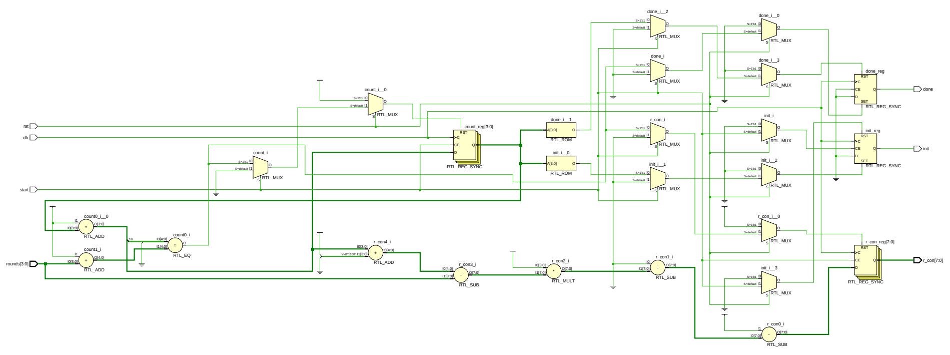


Figure 12: Round Constant Schematic

## 3.4 Permutation

The Permutation takes in a clock, start signal, number of rounds and 320 Bit Input. It returns 320 Bit Output after the rounds are completed for one clock cycle and then repeats the permutation from start again in a loop. To acheive an iterated design, D-Flip Flops are used to store intermediate round outputs and Multiplexers are used to select start and done signals. As we use 3 Different S-boxes, here are the Area Analysis for the same

Table 21: LuT Sbox - 9148.320 GE

| Components | Slice LuT (Logic) | Slice Registers (Flip Flop) | LUT3 | FDRE | IBUF | OBUF | LUT5 | LUT4 | LUT6 | LUT | 1LUT2 | BUFG (Clock) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number Used | 740 | 334 | 862 | 334 | 326 | 320 | 196 | 110 | 99 | 9 | 5 | 1 |

Table 22: KMAP Sbox - 8848.800 GE

| Components | Slice LuT (Logic) | Slice Registers (Flip Flop) | LUT3 | FDRE | IBUF | OBUF | LUT5 | LUT6 | LUT4 | LUT1 | LUT2 | BUFG (Clock) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number Used | 777 | 334 | 955 | 334 | 326 | 320 | 196 | 134 | 13 | 9 | 5 | 1 |

Table 23: ANF Sbox - 9056.160 GE

| Components | Slice LuT (Logic) | Slice Registers (Flip Flop) | LUT3 | FDRE | IBUF | OBUF | LUT6 | LUT5 | LUT4 | LUT1 | LUT2 | BUFG (Clock) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number Used | 883 | 334 | 955 | 334 | 326 | 320 | 246 | 140 | 13 | 9 | 5 | 1 |

As expected K-Maps do the best in ASIC whereas LuT does better in FPGA as they are better programmed to run LuTs.

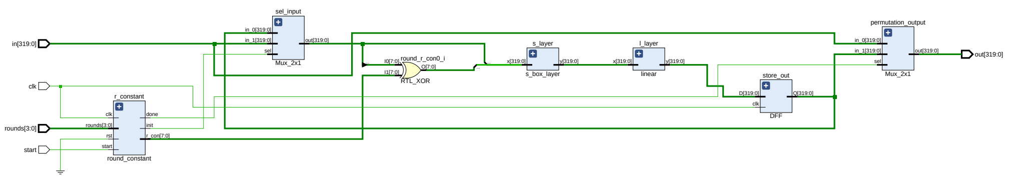
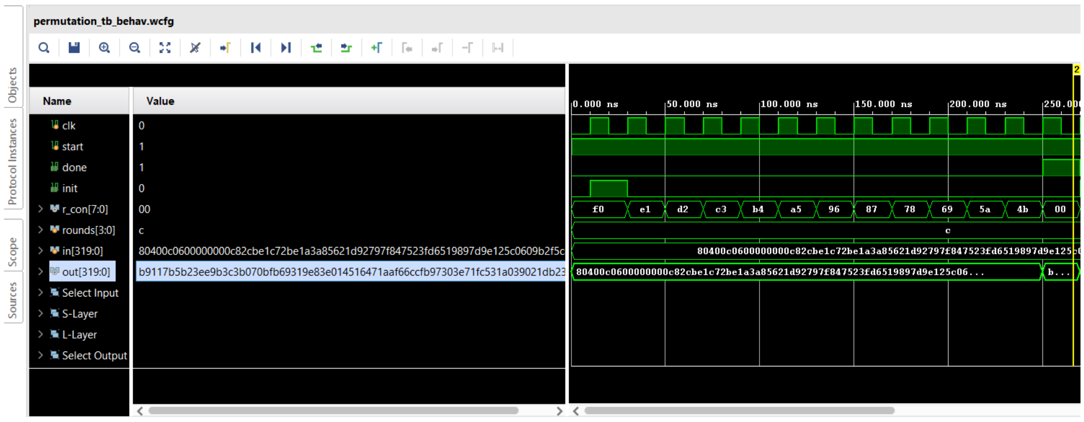


Figure 13: Permutaion Schematic

Test Bench for the Permutation function is also implemented for 12, 8 and 6 rounds. For all the 3 round options we take the following input and we get the following outputs. The input is taken from ASCON Python Implementation referenced in the Official Website.

Table 24: Test Vectors

| Word | Input | 12 Rounds Output | 8 Rounds Output | 6 Rounds Output |
|---|---|---|---|---|
| $x_0$ | 80400c0600000000 | b9117b5b23ee9b3c | 7e23b301941a4a62 | 25c593c800b08021 |
| $x_1$ | c82cbe1c72be1a3a | 3b070bfb69319e83 | 9b907c66bad8b7ed | 3cb7732ae5f81096 |
| $x_2$ | 85621d92797f8475 | e014516471aaf66c | 332e8d3ba929ce17 | e8edf67b79fb4c3e |
| $x_3$ | 23fd6519897d9e12 | cfb97303e71fc531 | efc6c3a1c24a7b4e | 2d1d39a69214abf8 |
| $x_4$ | 5c0609b2f5ca3aaa | a039021db23a484c | 45a7f92724354d6a | 9e9ce6a15408bc5d |


permutation_tb_behav.wcfg
Objects
Protocol Instances
Scope
Sources
0.000 ns
50.000 ns
100.000 ns
150.000 ns
200.000 ns
250.000
f0
e1
d2
c3
b4
a5
96
87
78
69
5a
4b
00
c
80400c0600000000c82cbe1c72be1a3a85621d92797f847523fd6519897d9e125c0
80400c0600000000c82cbe1c72be1a3a85621d92797f847523fd6519897d9e125c06...
b...

| Name | Value |
|---|---|
| clk | 0 |
| start | 1 |
| done | 1 |
| init | 0 |
| r_con[7:0] | 00 |
| rounds[3:0] | c |
| in[319:0] | 80400c0600000000c82cbe1c72be1a3a85621d92797f847523fd6519897d9e125c0609b2f5c |
| out[319:0] | b9117b5b23ee9b3c3b070bfb69319e83e014516471aaf66ccfb97303e71fc531a039021db23 |
| Select Input | |
| S-Layer | |
| L-Layer | |
| Select Output | |

Figure 14: 12 Round Test Bench Wave Configuration

## 4 Automated Analysis

### 4.1 Basic MILP Model

A Basic MILP Model is designed which takes in basic properties of S-Box like Branch Number and Linear Layer Model to get the minimum number of active S-boxes for an $r$ round differential. To create that model, we will need to define the following variables.

- $x_{r,w,b} \in \{0,1\}$ - S-box input Bit $b$ $(0 \leq b \leq 63)$ of word $X_w$ $(0 \leq w \leq 4)$ of round $r$ active or not.

- $y_{r,w,b} \in \{0,1\}$ - S-box output Bit $b$ $(0 \leq b \leq 63)$ of word $X_w$ $(0 \leq w \leq 4)$ of round $r$ active or not.

- $d_{r,b} \in \{0,1\}$ - $b^{th}$ $(0 \leq b \leq 63)$ S-box of round $r$ is active

- $u_{r,w,b} \in \{0,1\}$ - Linear layer model in word $x_w$ $(0 \leq w \leq 4)$ of round $r$.

Our Objective Function is to Minimize the Number of Active S-Boxes across the Differential Trail

$$\min \sum_{r=1}^{R} \sum_{b=0}^{63} d_{r,b}$$

For Constraints, for starters we should add **non-triviality condition** - that atleast one input bit should be active at start.

$$\sum_{w=0}^{4} \sum_{b=0}^{63} x_{0,w,b} \geq 1$$

Then we can model in the Basic Properties of S-box like Branch Number

$$\sum_{w=0}^{4} (x_{r,w,b} + y_{r,w,b}) \geq 3d_{r,b}$$

If the S-box is Active, at least one input and output bits and at max 5 input and output bits should be active

$$d_{r,b} \leq \sum_{w=0}^{4} x_{r,w,b} \leq 5d_{r,b}$$

$$d_{r,b} \leq \sum_{w=0}^{4} y_{r,w,b} \leq 5d_{r,b}$$

We can also add in that, if there is an active input or output bit, S-Box should be active

$$d_{r,b} \geq x_{r,w,b} \quad 0 \leq w \leq 4$$
$$d_{r,b} \geq y_{r,w,b} \quad 0 \leq w \leq 4$$

The Linear Layer can be modelled in using Mouha et al.'s XOR Framework

$$\begin{aligned}
y_{r,0,b} + y_{r,0,b-19} + y_{r,0,b-28} + x_{r+1,0,b} &= 2u_{r,0,b} \\
y_{r,1,b} + y_{r,1,b-61} + y_{r,1,b-39} + x_{r+1,1,b} &= 2u_{r,1,b} \\
y_{r,2,b} + y_{r,2,b-1} + y_{r,2,b-6} + x_{r+1,2,b} &= 2u_{r,2,b} \\
y_{r,3,b} + y_{r,3,b-10} + y_{r,3,b-17} + x_{r+1,3,b} &= 2u_{r,3,b} \\
y_{r,4,b} + y_{r,4,b-7} + y_{r,4,b-41} + x_{r+1,4,b} &= 2u_{r,4,b}
\end{aligned}$$

$$\begin{array}{c|c|c|c}
2u_{r,0,b} \geq y_{r,0,b} & 2u_{r,0,b} \geq y_{r,0,b-19} & 2u_{r,0,b} \geq y_{r,0,b-28} & 2u_{r,0,b} \geq x_{r+1,0,b} \\
2u_{r,1,b} \geq y_{r,1,b} & 2u_{r,1,b} \geq y_{r,1,b-61} & 2u_{r,1,b} \geq y_{r,1,b-39} & 2u_{r,1,b} \geq x_{r+1,1,b} \\
2u_{r,2,b} \geq y_{r,2,b} & 2u_{r,2,b} \geq y_{r,2,b-1} & 2u_{r,2,b} \geq y_{r,2,b-6} & 2u_{r,2,b} \geq x_{r+1,2,b} \\
2u_{r,3,b} \geq y_{r,3,b} & 2u_{r,3,b} \geq y_{r,3,b-10} & 2u_{r,3,b} \geq y_{r,3,b-17} & 2u_{r,3,b} \geq x_{r+1,3,b} \\
2u_{r,4,b} \geq y_{r,4,b} & 2u_{r,4,b} \geq y_{r,4,b-7} & 2u_{r,4,b} \geq y_{r,4,b-41} & 2u_{r,4,b} \geq x_{r+1,4,b}
\end{array}$$

The results after modelling it in the automation tool gurobi and running for 3 rounds are the following

Table 25: Basic MILP Model

| Rounds | Active S-Boxes | Variables (Real) | Variables (Binary) | Inequalities | Time Taken |
|---|---|---|---|---|---|
| 1 | 1 | 960 | 384 | 2561 | 0.03 s |
| 2 | 4 | 1920 | 448 | 5121 | 0.51 s |
| 3 | 12 | 2880 | 512 | 7681 | 2 min 52.47 s |

## 4.2 Logical Conditonal Modelling

In the above basic MILP Model we have not modeled the intricacies of the S-box. To add in few elements of ASCON S-box, we can use the Undisturbed Bits to create Logical Conditional Modelling the following way.

$$x_{r,0,b}, x_{r,1,b}, x_{r,2,b}, x_{r,3,b}, x_{r,4,b}) = (\delta_0, \delta_1, \delta_2, \delta_3, \delta_4) \Rightarrow y_{r,w,b} = \delta$$

$$\sum_{w'=0}^{4} (-1)^{\delta_i} x_{r,w',b} + (-1)^{\delta+1} y_{r,w,b} - \delta + \sum_{w'=0}^{4} \delta_i \geq 0$$

There are 13 such Undisturbed bits such that $\Delta_o^r$ has one bit invariant.

Table 26: Undisturbed Bits

| $\Delta_i$ | $\Delta_o^*$ |
|---|---|
| 00001 | *1*** |
| 00011 | ***0* |
| 00101 | 1**** |
| 11111 | *0*** |

| $\Delta_i$ | $\Delta_o^*$ |
|---|---|
| 10101 | ****1 |
| 10110 | 1**** |
| 11110 | *1*** |
| 01110 | *0*** |
| 01011 | ***1* |

| $\Delta_i$ | $\Delta_o^*$ |
|---|---|
| 10111 | ****0 |
| 11000 | **1** |
| 11100 | **0** |
| 00110 | ****1 |

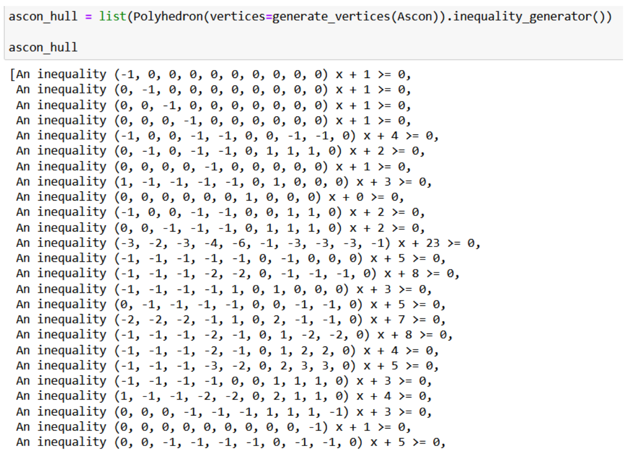
After Modelling these inequalities for every S-box in every round and implementing it on the automation tool for 3 rounds, we get the following results

Table 27: Logical Conditional Modelling

| Rounds | Active S-Boxes | Variables (Real) | Variables (Binary) | Inequalities | Time Taken |
|---|---|---|---|---|---|
| 1 | 1 | 965 | 384 | 3393 | 0.02 s |
| 2 | 4 | 1925 | 448 | 6785 | 1.32 s |
| 3 | 12 | 2885 | 512 | 10177 | 3 min 28.54 s |

## 4.3 Convex Hull

Convex Hull can exhaustively represent every element of ASCON S-box, however it would also lead to huge overhead as per S-box 2415 inequalities will be added in whereas in the above just 13 per S-box. Hence while we found the inequalities as shown below, we have not integrated it into the automation tool.



```
ascon_hull = list(Polyhedron(vertices=generate_vertices(Ascon)).inequality_generator())

ascon_hull
```

```
[An inequality (-1, 0, 0, 0, 0, 0, 0, 0, 0, 0) x + 1 >= 0,
 An inequality (0, -1, 0, 0, 0, 0, 0, 0, 0, 0) x + 1 >= 0,
 An inequality (0, 0, -1, 0, 0, 0, 0, 0, 0, 0) x + 1 >= 0,
 An inequality (0, 0, 0, -1, 0, 0, 0, 0, 0, 0) x + 1 >= 0,
 An inequality (-1, 0, 0, -1, -1, 0, 0, -1, -1, 0) x + 4 >= 0,
 An inequality (0, -1, 0, -1, -1, 0, 1, 1, 1, 0) x + 2 >= 0,
 An inequality (0, 0, 0, 0, -1, 0, 0, 0, 0, 0) x + 1 >= 0,
 An inequality (1, -1, -1, -1, -1, 0, 1, 0, 0, 0) x + 3 >= 0,
 An inequality (0, 0, 0, 0, 0, 0, 1, 0, 0, 0) x + 0 >= 0,
 An inequality (-1, 0, 0, -1, -1, 0, 0, 1, 1, 0) x + 2 >= 0,
 An inequality (0, 0, -1, -1, -1, 0, 1, 1, 1, 0) x + 2 >= 0,
 An inequality (-3, -2, -3, -4, -6, -1, -3, -3, -3, -1) x + 23 >= 0,
 An inequality (-1, -1, -1, -1, -1, 0, -1, 0, 0, 0) x + 5 >= 0,
 An inequality (-1, -1, -1, -2, -2, 0, -1, -1, -1, 0) x + 8 >= 0,
 An inequality (-1, -1, -1, -1, 1, 0, 1, 0, 0, 0) x + 3 >= 0,
 An inequality (0, -1, -1, -1, -1, 0, 0, -1, -1, 0) x + 5 >= 0,
 An inequality (-2, -2, -2, -1, 1, 0, 2, -1, -1, 0) x + 7 >= 0,
 An inequality (-1, -1, -1, -2, -1, 0, 1, -2, -2, 0) x + 8 >= 0,
 An inequality (-1, -1, -1, -2, -1, 0, 1, 2, 2, 0) x + 4 >= 0,
 An inequality (-1, -1, -1, -3, -2, 0, 2, 3, 3, 0) x + 5 >= 0,
 An inequality (-1, -1, -1, -1, 0, 0, 1, 1, 1, 0) x + 3 >= 0,
 An inequality (1, -1, -1, -2, -2, 0, 2, 1, 1, 0) x + 4 >= 0,
 An inequality (0, 0, 0, -1, -1, -1, 1, 1, 1, -1) x + 3 >= 0,
 An inequality (0, 0, 0, 0, 0, 0, 0, 0, 0, -1) x + 1 >= 0,
 An inequality (0, 0, -1, -1, -1, -1, 0, -1, -1, 0) x + 5 >= 0,
```

Figure 15: Convex-Hull - 2415 Inequalities

# 5 Conclusion

In conclusion we have covered all four aspects of the cipher - Construction, Cryptanalysis, Hardware Implementation, Automated Analysis extensively. We have covered the design of the Authenticated Encryption as well as the permutation components. We have analyses the cipher through various cryptanalysis techniques like Differential, Linear, Zero-Sum. We have then implemented the permutation in verilog and have done the Area Analysis in both ASIC and FPGA. In the end, we have implemented Basic as well as Logical Condtional Modelling in MILP to find out the minimum number of active S-boxes in a Differential.

# 6 References

- https://ascon.iaik.tugraz.at
- [**DEMS15**] *Cryptanalysis of Ascon* - Christoph Dobraunig, Maria Eichlseder, Florian Mendel, and Martin Schläffer. **IACR** - 2015/030 (pp. 28, 31–33, 35).
- [**Tez16**] *Truncated, Impossible, and Improbable Differential Analysis of Ascon* - Cihangir Tezcan. **IACR** - 2016/490 (pp. 28, 33).