Exercise 2 – LTI Basics 2

Jan Heiland

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We consider the linear time invariant system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

with $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,p}$, and $C \in \mathbb{R}^{q,n}$.

Properties of a Given System

Let the system matrices be given as

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -4 & -2 \\ 0 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.$$

1. Check this system for stability.
2. Check this system for controllability.
3. Compute the transfer function

   $$G(s) = C(sI - A)^{-1}B$$

   and comment on *minimality* of the realization.
4. For this transfer function, compute the $\mathcal{H}_\infty$ norm

   $$\|G\|_\infty = \sup_{\omega \in \mathbb{R}} |G(i\omega)|$$

   where i is the imaginary unit.