Exercise 1 – LTI Basics 1

Jan Heiland

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We consider the linear time invariant system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned}$$

with $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,p}$, and $C \in \mathbb{R}^{q,n}$ and let the Kalman controllability matrix

$$K = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}$$

have rank $r \leq n$.

1 Controllability and Hautusz Test

1. Show that there exists an invertible matrix $T \in \mathbb{R}^{n,n}$ such that

   $$T^{-1}AT = \begin{bmatrix} A_1 & A_2 \\ 0 & A_3 \end{bmatrix} \quad \text{and} \quad T^{-1}B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},$$

   with (A_1, B_1) is a controllable matrix pair.
2. Show that (A, B) is controllable if, and only if,

   $$\begin{bmatrix} \lambda I - A & B \end{bmatrix}$$

   has full row rank for any $\lambda \in \mathbb{C}$.

2 Invariance of Controllability under Transformations

Show that controllability is invariant under

1. State transformations $x(t) \leftarrow \tilde{x}(t) := T^{-1}x(t)$, with $T \in \mathbb{R}^{n,n}$ invertible,
2. Input scalings $u(t) \leftarrow \tilde{u}(t) := R^{-1}u(t)$, with $R \in \mathbb{R}^{m,m}$ invertible.
3. Linear state feedback transformations $u(t) = -Fx(t) + v(t)$, with $F \in \mathbb{R}^{m,n}$.