

Exercise 2 – LTI Basics 2

Jan Heiland

June 24, 2021

We consider the linear time invariant system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t)\end{aligned}$$

with $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,p}$, and $C \in \mathbb{R}^{q,n}$.

Properties of a Given System

Let the system matrices be given as

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -4 & -2 \\ 0 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.$$

1. Check this system for stability.
2. Check this system for controllability.
3. Compute the transfer function

$$G(s) = C(sI - A)^{-1}B$$

and comment on *minimality* of the realization.

4. For this transfer function, compute the \mathcal{H}_∞ norm

$$\|G\|_\infty = \sup_{\omega \in \mathbb{R}} |G(i\omega)|$$

where i is the imaginary unit.