Finite Element Analysis of Electromagnetic Waves in Anisotropic Media

Syed Khalid (21BEE1288) Niraj VITian (21BEE1289)
Aditya Suresh (21BEE1290)

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Introduction to Electromagnetic Waves

- Electromagnetic waves are solutions to Maxwell's equations.
- They propagate in free space and through various media.
- **Anisotropic media** exhibit direction-dependent properties.

Maxwell's equations in the differential form:

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Anisotropic Media

- Anisotropic media are materials whose properties differ based on direction.
- Example: Crystals, certain composites.
- They affect how electromagnetic waves propagate, reflecting different velocities in different directions.

Constitutive relations in anisotropic media:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad \text{(Electric displacement)}$$
$$\mathbf{B} = \mu \mathbf{H} \quad \text{(Magnetic flux density)}$$

where the permittivity ε and permeability μ are tensors:

$$\varepsilon_{ij}, \mu_{ij}$$

Finite Element Method (FEM) Overview

- The Finite Element Method (FEM) is a numerical method for solving partial differential equations (PDEs).
- FEM discretizes the computational domain into small elements.
- The weak form of Maxwell's equations is solved over these elements.

Weak form of Maxwell's equations in anisotropic media:

$$\int_{\Omega} \varepsilon_{ij} \mathbf{E}_i \mathbf{E}_j d\Omega = \int_{\Omega} \mathbf{J} \mathbf{E} d\Omega$$

where **E** is the electric field, and **J** is the source current density.

Simulation with FEniCS

- We used FEniCS, a Python-based open-source finite element package.
- FEniCS allows for automatic code generation of finite element models.

Basic workflow in FEniCS:

- Define the mesh and function spaces.
- Formulate the problem (weak form).
- Solve using FEM.

Results and Analysis

- The simulation shows the propagation of electromagnetic waves in the anisotropic medium.
- Results show the directional dependence of wave propagation.

Figure: Wave propagation in anisotropic media