Quantum Mechanics Griffiths Notes

Syed Khalid

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1 Ch1 Wave Function

1.1 Schrodinger Equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

Probability of finding particle between a and b at time $t = \int\limits_a^b |\Psi(x,t)|^2 \, \mathrm{d}x$

1.2 Probability

For a Discrete Variable j :

$$\langle j\rangle = \frac{\sum jN(j)}{N_{total}} = \sum_{j=0}^{\infty} jP(j)$$

$$\langle f(j)\rangle = \sum_{j=0}^{\infty} f(j)P(j)$$

$$\sigma = \sqrt{\langle(\langle j\rangle - j)^2\rangle} = \sqrt{\langle j^2\rangle - \langle j\rangle^2}$$

For a Continuous Variable x:

$$P(c \in [x, x+dx]) = \rho(x)dx$$

$$P(x \in [a,b]) = \int\limits_a^b \rho(x)\,\mathrm{d}x$$

$$\int\limits_{-\infty}^{\infty} \rho(x)\,\mathrm{d}x = 1$$

$$\langle x\rangle = \int\limits_{-\infty}^{\infty} x\rho(x)\,\mathrm{d}x$$

$$\langle f(x)\rangle = \int\limits_{-\infty}^{\infty} f(x)\rho(x)\,\mathrm{d}x$$

1.3 Nomralization

From the Born's Statistical Interpretation of Ψ:

$$\int\limits_{-\infty}^{\infty} |\Psi(x,t)|^2\,\mathrm{d}x = 1$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{-\infty}^{\infty} |\Psi(x,t)|^2\,\mathrm{d}x = 0$$

Hence once the wave function is normalized at any t its normalized for all t.

1.4 Momentum

$$\langle x \rangle = \int\limits_D \Psi^*[x]\Psi \, \mathrm{d}x$$

$$\langle p \rangle = \int\limits_D \Psi^*[-i\hbar\frac{\partial}{\partial x}]\Psi \, \mathrm{d}x$$

1.5 Uncertainty Principle

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

2 Time Independependent Schrodinger Equations

2.1 Stationary States

Let $V(x)$ be independependent of time and $\Psi(x,t) = \psi(x)\varphi(t)$. From Schrodinger Equations we get the TISE:

$$\boxed{-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V\psi = E\psi}, \quad \frac{\mathrm{d}\varphi}{\mathrm{d}t} = -\frac{iE}{\hbar}\varphi \implies \varphi(t) = e^{-iEt/\hbar}$$

Where $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ are stationary states as the probability density and every expectation value is independependent of time.

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V(x), \quad \boxed{\hat{H}\psi = E\psi}, \quad \langle H \rangle = E$$

TISE yields an infinite number of solutions each associated with an allowed energy. Any wave function can be written as a linear combination of these infinite stationary states:

$$\Psi(x,t) = \sum_{i=0}^{\infty} c_i \Psi_i(x,t) = \sum_{i=1}^{\infty} c_i \psi_i(x) e^{-iEt/\hbar}$$

Where $|c_i|^2$ represent te probability of the measurement of energy returning E_i. Thus:

$$\sum_{i=1}^{\infty} |c_i|^2 = 1, \quad \langle H \rangle = \sum_{i=1}^{\infty} |c_i|^2 E_i$$

2.2 Infinite Square Well

We definte $V(x) = 0$ when $x \in (0,a)$ else, $V(x) = \infty$. TISE becomes:

$$\frac{\partial^2 \psi(x)}{\partial x^2} = -(\frac{\sqrt{2mE}}{\hbar})^2 \psi(x)$$

Resembling the Simple Harmonic Oscillator($\frac{\partial^2 f}{\partial x^2} = -k^2 f$). Applying boundary conditions $\psi(0) = 0, \;\; \psi(a) = 0$ and solving we get:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\Big(\frac{n\pi}{a}x\Big), \quad n = 1, 2, 3\ldots$$

$$E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}$$

$\psi_n(x)$ are alternatively even and odd w.r.t to $x = a$. $\psi_i(x)$ has $i - 1$ nodes. Also they are orthonormal:

$$\int\limits_{-\infty}^{\infty} \psi_m(x)^* \psi_n(x) \, \mathrm{d}x = \delta_{mn}$$

They are also complete:

$$\Psi(x,t) = \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n sin(\frac{n\pi}{a}x) e^{-it(n^2\pi^2\hbar/2ma^2)}, \quad c_n = \sqrt{\frac{2}{a}} \int\limits_0^a sin(\frac{n\pi}{a}x)\Psi(x,0) \, \mathrm{d}x$$

This is nothing but the Fourier series.

2.3 Harmonic Oscillator

Defined by the potential- $V(x) = \frac{1}{2}m\omega^2x^2$. Most arbitary potentials can be expressed in this form;

$$V(x) = V(x_0) + V'(x_0)(x - x_0) + \frac{1}{2}V''(x_0)(x - x_0)^2 + \ldots$$

where x_0 is a minima. As

2.3.1 Algebraic Method

2.3.2 Analytic Method

2.4 Free Particle

2.5 Delta-Function Potential

2.5.1 Bound States and Scattering States

2.5.2 Delta-Function Well

2.6 Finite Square Well

$$\textbf{4.} \quad V_1 = \frac{R_1}{R_1 + R_2} \times V = \frac{10k}{10k + 20k}5V = 1.67V$$

$$V_2 = \frac{R_2}{R_1 + R_2} \times V = \frac{20k}{10k + 20k}5V = 3.33V$$

$$I_1 = \frac{R_2}{R_1 + R_2} \times I = \frac{2k}{1k + 2k} \times 15mA = 5mA$$

$$I_2 = \frac{R_1}{R_1 + R_2} \times I = \frac{1k}{1k + 2k} \times 15mA = 2.5mA$$

$$C_{12} = \frac{C_1 \times C_2}{C_1 + C_2} \implies \frac{20\text{mF} \times 30\text{mF}}{20\text{mF} + 30\text{mF}} = 12\text{mF}$$

$$C_{34} = C_3 + C_4 \implies 40\text{mF} + 20\text{mF} = 60\text{mF}$$

$$C_{total} = \frac{C_{12} \times C_{34}}{C_{12} + C_{34}} \implies \frac{12\text{mF} \times 60\text{mF}}{12\text{mF} + 60\text{mF}} = 10\text{mF}$$

$$Q = C_{total} \times V_{total} \implies 10\text{mF} \times 30\text{V} = 300\text{mC}$$

$$V_1 = \frac{Q}{C_1} \implies \frac{300\text{mC}}{20\text{mF}} = 15\text{V}$$

$$V_2 = \frac{Q}{C_2} \implies \frac{300\text{mC}}{30\text{mF}} = 10\text{V}$$

$$V_3 = \frac{Q}{C_{34}} \implies \frac{300\text{mC}}{60\text{mF}} = 5\text{V}$$

$$R_{total} = R_1 + R_2 + R_3 \implies 10k\Omega + 20k\Omega + 30k\Omega = 60k\Omega$$

$$C_{total} = C_1 + \frac{C_2 \times C_3}{C_2 + C_3} \implies 10\text{nF} + \frac{20\text{nF} \times 20\text{nF}}{20\text{nF} + 20\text{nF}} = 20\text{nF}$$

$$L_{total} = \frac{L_1 \times (L_2 + L_3)}{L_1 + L_2 + L_3} \implies \frac{10\text{mH} \times (4\text{mH} + 8\text{mH})}{10\text{mH} + 4\text{mH} + 8\text{mH}} = 5.45\text{mH}$$