BMAT102L Notes

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1 Laplace Transnforms

1.1 Theory

<u>Definition:-</u> Let $f(t)$ be a function of t defined for $0 \leq t \leq \infty$ then the Laplace transform of f is defined as :

$$\mathcal{L}\left\{f(t)\right\} = \int_0^\infty e^{-st} f(t) dt = F(s).$$

1.2 Formulae

1. $\mathcal{L}\left\{1\right\} = \frac{1}{s}$
2. $\mathcal{L}\left\{t^n\right\} = \frac{\Gamma(n+1)}{s^{n+1}}$
3. $\mathcal{L}\left\{e^{at}\right\} = \frac{1}{s-a}$
4. $\mathcal{L}\left\{\sin at\right\} = \frac{a}{s^2+a^2}$
5. $\mathcal{L}\left\{\cos at\right\} = \frac{s}{s^2+a^2}$
6. $\mathcal{L}\left\{\sinh at\right\} = \frac{a}{s^2-a^2}$
7. $\mathcal{L}\left\{\cosh at\right\} = \frac{s}{s^2-a^2}$

1.3 First Shifting Theorem

For Laplace Transforms :- If $\mathcal{L}\left\{f(t)\right\} = f(s)$ then $\mathcal{L}\left\{e^{at} f(t)\right\} = f(s-a)$

eg :- $\mathcal{L}\left\{e^{2t} \cdot \sin 3t\right\} = \frac{3}{(s-2)^2+9}$
$\mathcal{L}\left\{e^{-7t} t^4\right\} = \frac{\Gamma(5)}{(s+7)^5}$
For Inverse Laplace Transform-
If $\mathcal{L}^{-1}\left\{f(s)\right\} = f(t)$ then, $\mathcal{L}^{-1}\left\{f(s-a)\right\} = e^{at} f(t) = e^{at} \mathcal{L}^{-1}\left\{f(s)\right\}$

eg:- $\mathcal{L}^{-1}\left\{\frac{s}{s^2-4s+13}\right\} = \mathcal{L}^{-1}\left\{\frac{s-2+2}{(s-2)^2+9}\right\} \implies \mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2+9}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2+9}\right\} = e^{2t}\cos 3t + \frac{2}{3} e^{2t} \sin 3t$

1.4 Second Shifting Theorem

Unit Step (Heaviside) function :- $H(t-a) = \begin{cases} 1 & t \geq a \\ 0 & t < a \end{cases}$

For Laplace Transforms :- If $\mathcal{L}\left\{f(t)\right\} = f(s)$ then $\mathcal{L}\left\{f(t-a)H(t-a)\right\} = e^{-as} f(s)$

eg:- $q(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right) & t \geq \frac{\pi}{3} \\ 0 & t < \frac{\pi}{3} \end{cases} = \sin\left(t - \frac{\pi}{3}\right) \begin{cases} 1 & t \geq \frac{\pi}{3} \\ 0 & t < \frac{\pi}{3} \end{cases}$
$\mathcal{L}\left\{q(t)\right\} = \mathcal{L}\left\{\sin\left(t - \frac{\pi}{3}\right) H\left(t - \frac{\pi}{3}\right)\right\} \implies e^{\frac{\pi}{3}s} \cdot \frac{1}{s^2+1}$
For Inverse Laplace Transform :- If $\mathcal{L}^{-1}\left\{f(s)\right\} = f(t)$ then $\mathcal{L}^{-1}\left\{e^{-as} f(s)\right\} = f(t-a)H(t-a)$
$\mathcal{L}^{-1}\left\{e^{-as} \frac{1}{s-1}\right\} = e^{t-a} H(t-a)$

2 Multiplication Property

If $\mathcal{L}\left\{f(t)\right\} = f(s)$ then $\boxed{\mathcal{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n} f(s)}$
$\mathcal{L}\{t \sin 2t\} = -\frac{d}{ds}(\frac{2}{s^2+4}) = \frac{4s}{(s^2+4)^2}$ Now, $\int_0^\infty e^{-3t} t \sin 2t dt = \frac{4 \cdot 3}{(3^2+4)^2}$
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