PHS597 Assignment 6
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The true parameters are set to be (0.1, 1.3, 0.8, -1) and 1000 bootstrap samples are used for all scenarios.

True parameter	N=10			N=15			N=50			N=100		
	2.5%	97.5%	width	2.5%	97.5%	width	2.5%	97.5%	width	2.5%	97.5%	width
0.1	-1.25	1.87	3.12	-0.30	2.31	2.61	-0.47	0.69	1.15	0.066	0.56	0.50
1.3	-1.56	2.78	4.33	0.39	1.98	1.59	0.90	1.67	0.77	1.06	1.45	0.39
0.8	-0.39	1.53	1.92	-0.37	1.33	1.70	0.29	0.96	0.67	0.57	0.92	0.35
-1	-2.38	0.39	2.77	-1.39	0.80	2.19	-1.28	-0.67	0.61	-1.11	-0.78	0.33

Table 1. Non-parametric Bootstrap 2.5^{th} and 97.5^{th} percentile for estimates

True parameter	N=10			N=15			N=50			N=100		
	2.5%	97.5%	width	2.5%	97.5%	width	2.5%	97.5%	width	2.5%	97.5%	width
0.1	-0.51	1.32	1.83	-0.36	1.34	1.71	-0.40	0.69	1.08	0.034	0.61	0.57
1.3	-0.55	2.22	2.77	0.60	1.36	0.76	0.85	1.61	0.76	1.07	1.48	0.40
0.8	0.035	1.12	1.09	0.004	1.16	1.16	0.25	0.98	0.72	0.54	0.93	0.39
-1	-1.77	-0.14	1.63	-1.55	-0.11	1.44	-1.24	-0.62	0.62	-1.12	-0.77	0.35

Table 1. Parametric Bootstrap 2.5^{th} and 97.5^{th} percentile for estimates

Comparing the Parametric Bootstrap to non-parametric:

1. Parametric bootstrap allows the simulation of a distribution, which not limit to use original data.
2. Compared to the empirical distribution used in non-parametric bootstrap, the parametric generation process will lead to a smoother sampling distribution. This allows parametric bootstrap works better when the sample size is small. From the above tables (in this case, we use the true model), we can see when the sample size is small, the confidence interval is narrower in parametric case than non-parametric case. And as the sample size increases, the two procedures lead to roughly same results. The sampling distribution in parametric bootstrap is a better approximation of the true distribution (where original sample came from), compared to the empirical distribution in non-parametric bootstrap, when sample size is small.