Optimization models for day-ahead scheduling

1. Introduction

This draft contains an optimization problem for day-ahead scheduling. This first problem is the optimal power flow (OPF), a common problem in power system planning and operation, which still a challenge to know the global solution for this problem, in large instances. The main characteristics for our toy problem is:

- 3 buses and 3 thermal generators; All generators are the same, with limits on active power = 400 MW and range for reactive power is [-300, 300] MVAr. Each bus have one generator.
- 1 load located at bus 3, with demand P + jQ = 180 + j30 MVA.
- All buses are conected by a single line, with r + jx = 0,025 + j0,1 pu of impedance.
- The unitary cost (CVU) from termal generators are: [10, 12, 20], respectively.

Considering these characteristics, we present in the rest of this draft some formulations of the OPF problem.

2. Problem Formulation – DC OPF

The DC OPF formulation to this problem is (assuming power in pu) :

$$
\begin{aligned}
&\min 1000Pg_1 + 1200Pg_2 + 2000Pg_3 \\
&\text{s.t.:} \\
&Pg_1 = \frac{\theta_1 - \theta_2}{0,1} + \frac{\theta_1 - \theta_3}{0,1} \qquad -1 \le \frac{\theta_1 - \theta_2}{0,1} \le 1 \\
&Pg_2 = \frac{\theta_2 - \theta_3}{0,1} + \frac{\theta_2 - \theta_1}{0,1} \qquad -1 \le \frac{\theta_1 - \theta_3}{0,1} \le 1 \\
&Pg_3 - 1.8 = \frac{\theta_3 - \theta_1}{0,1} + \frac{\theta_3 - \theta_2}{0,1} \quad -1 \le \frac{\theta_2 - \theta_3}{0,1} \le 1 \\
&0 \le Pg_1 \le 4;\ 0 \le Pg_2 \le 4;\ 0 \le Pg_3 \le 4 \\
&-\pi \le \theta_1 \le \pi;\ -\pi \le \theta_2 \le \pi;\ -\pi \le \theta_3 \le \pi \\
&\theta_0 = 0
\end{aligned}
$$

The following results are obtained by solving this problem with gurobi:

Pg = [1.2; 0.6; 0.0]; Theta = [0.0; -0.02; -0.1]; objective cost = 1920 $/hour.

Solved in 2 iterations (linear programming) and 0.08 seconds.

3. Problem Formulation – AC OPF with no approximations

The standart AC OPF formulation to this problem is (in polar coordinates, assuming power in pu):

$$\min 1000Pg_1 + 1200Pg_2 + 2000Pg_3$$

s.t.:

$$Pg_1 = V_1 \begin{bmatrix} V_1 \cdot 4.70588235 + V_2 \cdot \left(-2.35294118\cos\left(\theta_1 - \theta_2\right) + 9.41176471\sin\left(\theta_1 - \theta_2\right)\right) + \\ V_3 \cdot \left(-2.35294118\cos\left(\theta_1 - \theta_3\right) + 9.4117647\sin\left(\theta_1 - \theta_3\right)\right) \end{bmatrix}$$

$$Pg_2 = V_2 \begin{bmatrix} V_2 \cdot 4.70588235 + V_1 \cdot \left(-2.35294118\cos\left(\theta_2 - \theta_1\right) + 9.41176471\sin\left(\theta_2 - \theta_1\right)\right) + \\ V_3 \cdot \left(-2.35294118\cos\left(\theta_2 - \theta_3\right) + 9.4117647\sin\left(\theta_2 - \theta_3\right)\right) \end{bmatrix}$$

$$Pg_3 - 1.8 = V_3 \begin{bmatrix} V_3 \cdot 4.70588235 + V_1 \cdot \left(-2.35294118\cos\left(\theta_3 - \theta_1\right) + 9.41176471\sin\left(\theta_3 - \theta_1\right)\right) + \\ V_2 \cdot \left(-2.35294118\cos\left(\theta_3 - \theta_2\right) + 9.4117647\sin\left(\theta_3 - \theta_2\right)\right) \end{bmatrix}$$

$$Qg_1 = V_1 \begin{bmatrix} V_1 \cdot 18.82352941 + V_2\left(-2.35294118\sin\left(\theta_1 - \theta_2\right) - 9.41176471\cos\left(\theta_1 - \theta_2\right)\right) + \\ V_3\left(-2.35294118\sin\left(\theta_1 - \theta_3\right) - 9.41176471\cos\left(\theta_1 - \theta_3\right)\right) \end{bmatrix}$$

$$Qg_2 = V_2 \begin{bmatrix} V_2 \cdot 18.82352941 + V_1\left(-2.35294118\sin\left(\theta_2 - \theta_1\right) - 9.41176471\cos\left(\theta_2 - \theta_1\right)\right) + \\ V_3\left(-2.35294118\sin\left(\theta_2 - \theta_3\right) - 9.41176471\cos\left(\theta_2 - \theta_3\right)\right) \end{bmatrix}$$

$$Qg_3 - 0.3 = V_3 \begin{bmatrix} V_3 \cdot 18.82352941 + V_1\left(-2.35294118\sin\left(\theta_3 - \theta_1\right) - 9.41176471\cos\left(\theta_3 - \theta_1\right)\right) + \\ V_2\left(-2.35294118\sin\left(\theta_3 - \theta_2\right) - 9.41176471\cos\left(\theta_3 - \theta_2\right)\right) \end{bmatrix}$$

$$\left(2.35294118V_1^2 - 2.35294118V_1V_2\cos\left(\theta_1 - \theta_2\right) + 9.41176471V_1V_2\sin\left(\theta_1 - \theta_2\right)\right)^2 +$$
$$\left(9.41176471V_1^2 - 9.41176471V_1V_2\cos\left(\theta_1 - \theta_2\right) - 2.35294118V_1V_2\sin\left(\theta_1 - \theta_2\right)\right)^2 \le 1^2$$

$$\left(2.35294118V_1^2 - 2.35294118V_1V_3\cos\left(\theta_1 - \theta_3\right) + 9.41176471V_1V_3\sin\left(\theta_1 - \theta_3\right)\right)^2 +$$
$$\left(9.41176471V_1^2 - 9.41176471V_1V_3\cos\left(\theta_1 - \theta_3\right) - 2.35294118V_1V_3\sin\left(\theta_1 - \theta_3\right)\right)^2 \le 1^2$$

$$\left(2.35294118V_2^2 - 2.35294118V_2V_3\cos\left(\theta_2 - \theta_3\right) + 9.41176471V_2V_3\sin\left(\theta_2 - \theta_3\right)\right)^2 +$$
$$\left(9.41176471V_2^2 - 9.41176471V_2V_3\cos\left(\theta_2 - \theta_3\right) - 2.35294118V_2V_3\sin\left(\theta_2 - \theta_3\right)\right)^2 \le 1^2$$

$$\left(2.35294118V_2^2 - 2.35294118V_2V_1\cos\left(\theta_2 - \theta_1\right) + 9.41176471V_2V_1\sin\left(\theta_2 - \theta_1\right)\right)^2 +$$
$$\left(9.41176471V_2^2 - 9.41176471V_1V_2\cos\left(\theta_2 - \theta_1\right) - 2.35294118V_2V_1\sin\left(\theta_2 - \theta_1\right)\right)^2 \le 1^2$$

$$\left(2.35294118V_3^2 - 2.35294118V_3V_1\cos\left(\theta_3 - \theta_1\right) + 9.41176471V_3V_1\sin\left(\theta_3 - \theta_1\right)\right)^2 +$$
$$\left(9.41176471V_3^2 - 9.41176471V_3V_1\cos\left(\theta_3 - \theta_1\right) - 2.35294118V_3V_1\sin\left(\theta_3 - \theta_1\right)\right)^2 \le 1^2$$

$$\left(2.35294118V_3^2 - 2.35294118V_3V_2\cos\left(\theta_3 - \theta_2\right) + 9.41176471V_3V_2\sin\left(\theta_3 - \theta_2\right)\right)^2 +$$
$$\left(9.41176471V_3^2 - 9.41176471V_3V_2\cos\left(\theta_3 - \theta_2\right) - 2.35294118V_3V_2\sin\left(\theta_3 - \theta_2\right)\right)^2 \le 1^2$$

$$0 \le Pg_1 \le 4;\ 0 \le Pg_2 \le 4;\ 0 \le Pg_3 \le 4; -3 \le Qg_1 \le 3; -3 \le Qg_2 \le 3; -3 \le Qg_3 \le 3$$
$$-2\pi \le \theta_1 \le 2\pi; -2\pi \le \theta_2 \le 2\pi; -2\pi \le \theta_3 \le 2\pi$$
$$0.9 \le V_1 \le 1.1;\ 0.9 \le V_2 \le 1.1;\ 0.95 \le V_3 \le 1.05$$
$$\theta_0 = 0$$

The following results are obtained via matpower, using an Interior Point Method:

Pg = [1.1775; 0.6616; 0.0]; Qg = [-0.1959; 0.3908; 0.2614];

Theta = [0; -0.01958; -0.08913]; V = [1.069; 1.084; 1.050]

Objective cost = 1971.39 $/hour.

Converged in 3.31 seconds.

4. Problem Formulation – AC OPF in rectangular form

The OPF formulation in rectangular form is: (not finished yet)

$$\min 1000Pg_1 + 1200Pg_2 + 2000Pg_3$$

s.t.:

$$\begin{aligned}
Pg_1 = {} & V_{r1}\left(4.70588235V_{r1} + 18.82352941V_{i1} - 2.35294118V_{r2} - 9.41176471V_{i2} - 2.35294118V_{r3} - 9.41176471V_{i3}\right) + \\
& V_{i1}\left(-18.82352941V_{r1} + 4.70588235V_{i1} + 9.41176471V_{r2} - 2.35294118V_{i2} + 9.41176471V_{r3} - 2.35294118V_{i3}\right) \\
Pg_2 = {} & V_{r2}\left(4.70588235V_{r2} + 18.82352941V_{i2} - 2.35294118V_{r1} - 9.41176471V_{i1} - 2.35294118V_{r3} - 9.41176471V_{i3}\right) + \\
& V_{i2}\left(-18.82352941V_{r2} + 4.70588235V_{i2} + 9.41176471V_{r1} - 2.35294118V_{i1} + 9.41176471V_{r3} - 2.35294118V_{i3}\right) \\
Pg_3 - 1.8 = {} & V_{r3}\left(4.70588235V_{r3} + 18.82352941V_{i3} - 2.35294118V_{r1} - 9.41176471V_{i1} - 2.35294118V_{r2} - 9.41176471V_{i2}\right) + \\
& V_{i3}\left(-18.82352941V_{r3} + 4.70588235V_{i3} + 9.41176471V_{r1} - 2.35294118V_{i1} + 9.41176471V_{r2} - 2.35294118V_{i2}\right) \\
Qg_1 = {} & V_{r1}\left(18.82352941V_{r1} - 4.70588235V_{i1} - 9.41176471V_{r2} + 2.35294118V_{i2} - 9.41176471V_{r3} + 2.35294118V_{i3}\right) + \\
& V_{i1}\left(4.70588235V_{r1} + 18.82352941V_{i1} - 2.35294118V_{r2} - 9.41176471V_{i2} - 2.35294118V_{r3} - 9.41176471V_{i3}\right) \\
Qg_2 = {} & V_{r2}\left(18.82352941V_{r2} - 4.70588235V_{i2} - 9.41176471V_{r1} + 2.35294118V_{i1} - 9.41176471V_{r3} + 2.35294118V_{i3}\right) + \\
& V_{i2}\left(4.70588235V_{r2} + 18.82352941V_{i2} - 2.35294118V_{r1} - 9.41176471V_{i1} - 2.35294118V_{r3} - 9.41176471V_{i3}\right) \\
Qg_3 - 0.3 = {} & V_{r3}\left(18.82352941V_{r3} - 4.70588235V_{i3} - 9.41176471V_{r1} + 2.35294118V_{i1} - 9.41176471V_{r2} + 2.35294118V_{i2}\right) + \\
& V_{i3}\left(4.70588235V_{r3} + 18.82352941V_{i3} - 2.35294118V_{r1} - 9.41176471V_{i1} - 2.35294118V_{r2} - 9.41176471V_{i2}\right)
\end{aligned}$$

$$\begin{aligned}
&\left(2.35294118V_1^2 - 2.35294118V_1V_2\cos\left(\theta_1 - \theta_2\right) + 9.41176471V_1V_2\sin\left(\theta_1 - \theta_2\right)\right)^2 + \\
&\left(9.41176471V_1^2 - 9.41176471V_1V_2\cos\left(\theta_1 - \theta_2\right) - 2.35294118V_1V_2\sin\left(\theta_1 - \theta_2\right)\right)^2 \le 1^2 \\
&\left(2.35294118V_1^2 - 2.35294118V_1V_3\cos\left(\theta_1 - \theta_3\right) + 9.41176471V_1V_3\sin\left(\theta_1 - \theta_3\right)\right)^2 + \\
&\left(9.41176471V_1^2 - 9.41176471V_1V_3\cos\left(\theta_1 - \theta_3\right) - 2.35294118V_1V_3\sin\left(\theta_1 - \theta_3\right)\right)^2 \le 1^2 \\
&\left(2.35294118V_2^2 - 2.35294118V_2V_3\cos\left(\theta_2 - \theta_3\right) + 9.41176471V_2V_3\sin\left(\theta_2 - \theta_3\right)\right)^2 + \\
&\left(9.41176471V_2^2 - 9.41176471V_2V_3\cos\left(\theta_2 - \theta_3\right) - 2.35294118V_2V_3\sin\left(\theta_2 - \theta_3\right)\right)^2 \le 1^2 \\
&\left(2.35294118V_2^2 - 2.35294118V_2V_1\cos\left(\theta_2 - \theta_1\right) + 9.41176471V_2V_1\sin\left(\theta_2 - \theta_1\right)\right)^2 + \\
&\left(9.41176471V_2^2 - 9.41176471V_1V_2\cos\left(\theta_2 - \theta_1\right) - 2.35294118V_2V_1\sin\left(\theta_2 - \theta_1\right)\right)^2 \le 1^2 \\
&\left(2.35294118V_3^2 - 2.35294118V_3V_1\cos\left(\theta_3 - \theta_1\right) + 9.41176471V_3V_1\sin\left(\theta_3 - \theta_1\right)\right)^2 + \\
&\left(9.41176471V_3^2 - 9.41176471V_3V_1\cos\left(\theta_3 - \theta_1\right) - 2.35294118V_3V_1\sin\left(\theta_3 - \theta_1\right)\right)^2 \le 1^2 \\
&\left(2.35294118V_3^2 - 2.35294118V_3V_2\cos\left(\theta_3 - \theta_2\right) + 9.41176471V_3V_2\sin\left(\theta_3 - \theta_2\right)\right)^2 + \\
&\left(9.41176471V_3^2 - 9.41176471V_3V_2\cos\left(\theta_3 - \theta_2\right) - 2.35294118V_3V_2\sin\left(\theta_3 - \theta_2\right)\right)^2 \le 1^2
\end{aligned}$$

$$0 \le Pg_1 \le 4;\ 0 \le Pg_2 \le 4;\ 0 \le Pg_3 \le 4; -3 \le Qg_1 \le 3; -3 \le Qg_2 \le 3; -3 \le Qg_3 \le 3$$

$$-2\pi \le \theta_1 \le 2\pi; -2\pi \le \theta_2 \le 2\pi; -2\pi \le \theta_3 \le 2\pi$$

$$0.9^2 \le V_{r1}^2 + V_{i1}^2 \le 1.1^2;\ 0.9^2 \le V_{r1}^2 + V_{i1}^2 \le 1.1^2;\ 0.95^2 \le V_{r1}^2 + V_{i1}^2 \le 1.05^2$$

$$\theta_0 = 0$$