Course: Física Teórica I (CDP7600)

Project: *Fibration building blocks of information-processing networks*

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Abstract

Fibration building blocks of a network represent the sets of nodes that are symmetric with respect to information processing. Here, we have reproduced important results concerning the identification and classification of the fibration building blocks of directed networks, constructed from real network data. More specifically, using the transcriptional regulatory network data of the *Escherichia Coli* bacteria, we quantify the groups of network nodes that synchronously process equivalent information and then we classify each fiber based on its specific topological features. This way, in order to consistently present the obtained results, in this report we first give a brief description of the theory concerning the graph fibration morphism and its main definitions related to information flow symmetries. Next, we detail the methods adopted to correctly identify and classify the network fibers. More specifically, to establish an optimal framework, I show the implementation details of the Minimal Balanced Coloring algorithm used to find the corresponding fibers in the network, presenting a slightly improvement for the algorithm complexity. At last, showing the proper methods for data preparation, I describe the results obtained concerning the fiber statistics for the specific case of the *Escherichia Coli* regulatory network, to properly compare with the recent results presented at *Morone et. al.* (2019).

Keywords: Graph Morphism; Biological Networks; Information Processing, Fibration, lectus

1 Brief Introduction

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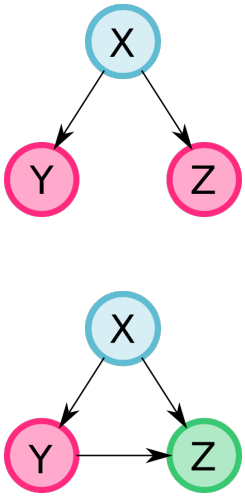

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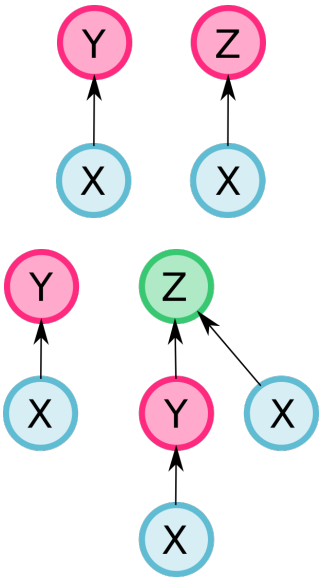

2 *Minimal Balanced Coloring* Algorithm

2.1 Algorithm description

For the proper identification of the fibers on the network we have to construct a graph partition that is equivalent to a coarse grain of the network with respect to the information flow passing through each node.

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2.2 Data preparation and algorithm implementation

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Table 1: Example table.

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Richard	Miles	5

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3 Conclusion

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