**lecture03 Word Window Classification,Neural Networks, and Matrix Calculus**

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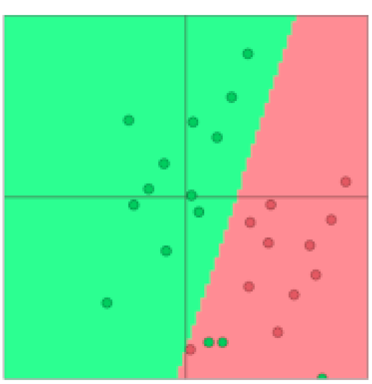
# Classification setup and notation

一个包含如下样本的训练集：


xi：输入（词、向量、索引、句子、文档等），维度为d
yi：试图预测的标签（类别，共有C类）



Visualizations with ConvNetJS by Karpathy!
http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html

the learned line is our classifier。

传统ML中：xi固定，训练softmax/logistic回归的权重


来决定边界

- softmax classifier

1. 将 $W$ 的 $y^{th}$ 行和 $x$ 中的对应行相乘得到分数

$$W_y \cdot x = \sum_{i=1}^{d} W_{yi} x_i = f_y$$

计算所有的 $f_c, for\ c = 1, \ldots, C$

2. 使用softmax函数获得归一化的概率

$$p(y|x) = \frac{\exp(f_y)}{\sum_{c=1}^{C} \exp(f_c)} = \mathrm{softmax}(f_y)$$

对于每个训练样本，目标是最大化正确类y的概率，或最小化负对数概率

$$-\log p(y|x) = -\log\left(\frac{\exp(f_y)}{\sum_{c=1}^{C} \exp(f_c)}\right)$$

### Background: What is “cross entropy” loss/error?

- 交叉熵概念来源于信息论
- 真实概率分布p
- 计算的模型概率q
- 交叉熵为：

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- 假定目标的概率分布在正确的类上是1，在其他为0。p=[0,...,0,1,0,...,0]为独热向量，唯一剩下的项是真实类的负对数概率

## Classification over a full dataset

整个数据集上的交叉熵损失函数

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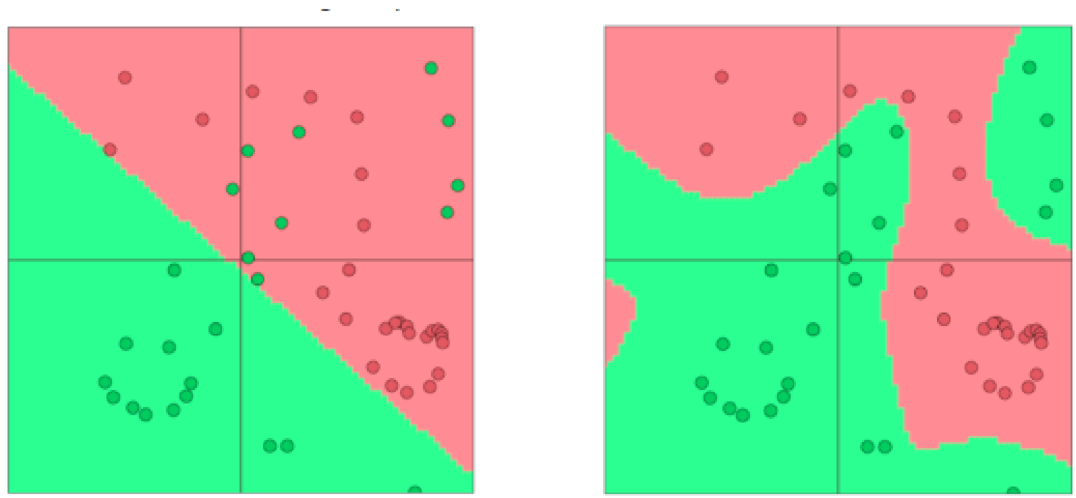
## Traditional ML optimization

一般机器学习的学习参数由权重的列组成


只通过以下方式更新决策边界


## Neural Network Classifiers

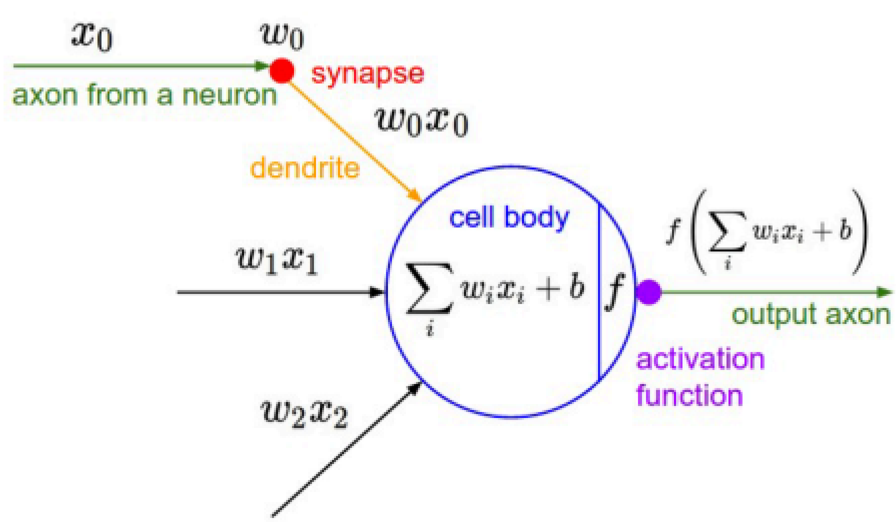
简单的线性分类器，并不强大，当问题复杂时是无用的
而神经网络可以学习更复杂的函数和非线性决策边界



## Classification difference with word vectors

DL中我们学习权重矩阵W和词向量x，传统参数和表示
需要对其进行更新优化分类器

## An artificial neuron




f = nonlinear activation fct. (e.g. sigmoid), w = weights, b = bias, h = hidden, x = inputs

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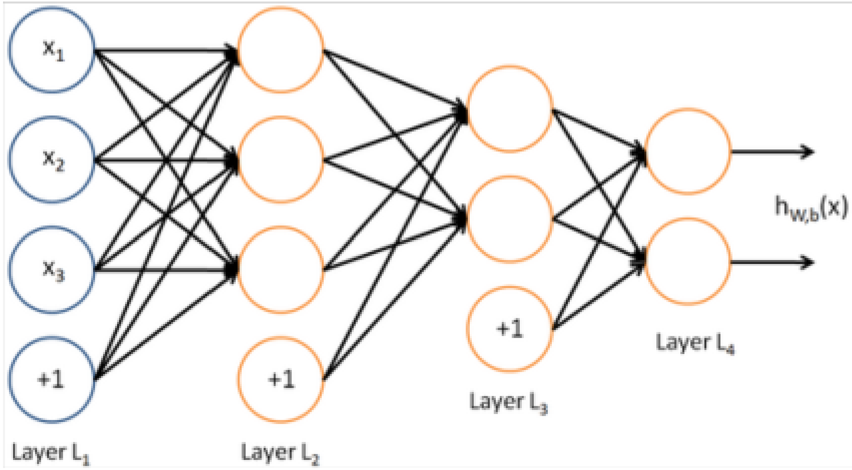
### A neural network= running several logistic regressions at the same time

如果我们输入一个向量通过一系列逻辑回归函数，那么我们得到一个输出向量，但是我们不需要提前决定这些逻辑回归试图预测的变量是什么。
我们可以输入另一个逻辑回归函数。损失函数将指导中间隐藏变量应该是什么，以便更好地预测下一层的目标。我们当然可以使用更多层的神经网络。




we have:

$a1=f(W11x1+W12x2+W13x3+b1)$
$a2=f(W21x1+W22x2+W23x3+b2)$

In matrix notation

$z=Wx+b$
$a=f(z)$激活函数

Activation f is applied element-wise:

$f([z1,z2,z3])=[f(z1),f(z2),f(z3)]$

when we apply that to a vector, we apply it to each element of the vecotr element-wise.

### Non-linearities (aka “f ”): Why they’re needed

- 没有非线性，深度神经网络只能做线性变换，且多个线性变换仅组成另一个简单线性变换
- 对于非线性函数，使用更多层可以近似更复杂的函数

## Named Entity Recognition (NER)

任务：查找和分类文件中的实体名称
可能用途：

1. 跟踪文档中提到的特定的实体
2. 问题回答的答案常为命名实体
3. 很多需要的信息为命名实体间的联系
4. 其他分类

### Why might NER be hard?

- 很难计算出实体边界
- 很难判断是否为实体
- 很难知道未知/新奇实体的类别

- 实体类别模棱两可，依赖于上下文

## Binary word window classification

为在上下文中的语言构建分类器

### Window classification

在相邻的上下文窗口对一个词进行分类，简单的方法是对窗口中的词向量取平均，并对平均向量进行分类，但是这会丢失位置信息

### Softmax

例子: Not all museums in Paris are amazing .

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将中心词上下文窗口大小至少为2的所有单词向量组成一个窗口列向量

- With $x = x_{window}$ we can use the same softmax classifier as before

predicted model
output probability

$$\boxed{\hat{y}_y} = p(y|x) = \frac{\exp(\boxed{W_y.x})}{\sum_{c=1}^{C} \exp(W_c.x)}$$

- With cross entropy error as before:

same

$$J(\theta) = \frac{1}{N} \sum_{i=1}^{N} -\log \left( \frac{e^{\boxed{f_{y_i}}}}{\sum_{c=1}^{C} e^{f_c}} \right)$$

同样利用求导优化来更新向量

我们希望窗口为 [museums in Paris are amazing] 中心词为地点类别的（真正窗口）比窗口 [Not all museums in Paris] 不是地点类别的（损坏窗口）得分要高

## Neural Network Feed-forward Computation

神经网络顶层a，与向量U点乘，返回即为得分

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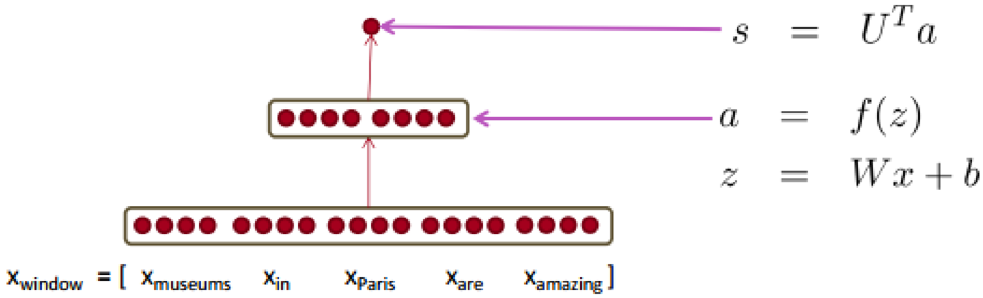
下图为计算有三层神经网络的窗口得分

- $s$ = *score*("museums in Paris are amazing”)

$$s = U^T f(Wx + b)$$

$$x \in \mathbb{R}^{20\times 1}, W \in \mathbb{R}^{8\times 20}, U \in \mathbb{R}^{8\times 1}$$

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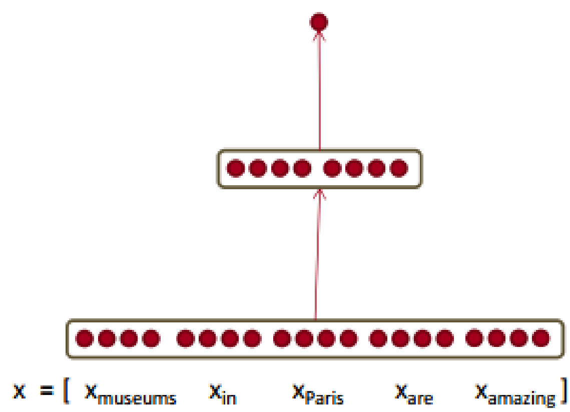
$$s = \boldsymbol{u}^T \boldsymbol{h}$$

$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b})$$

$$\boldsymbol{x} \quad \text{(input)}$$




## Gradients

- 给定一个函数，有1个输出和 n 个输入
  f(x)=f(x1,x2,...,xn)
  梯度是关于每个输入的偏导数的向量


- 给定一个函数，有 m 个输出和 n 个输入
  f(x)=[f1(x1,x2,...,xn),...,fm(x1,x2,...,xn)]
  其雅可比矩阵是一个m×n的偏导矩阵
  ∂f∂x=⎡⎣⎢⎢⎢⎢∂f1∂x1⋯∂f1∂xn⋮⋱⋮∂fm∂x1⋯∂fm∂xn⎤⎦⎥⎥⎥⎥∂f∂x=[∂f1∂x1⋯∂f1∂xn⋮⋱⋮∂fm∂x1⋯∂fm∂xn]

(∂f∂x)ij=∂fi∂xj

$$\boldsymbol{h} = f(\boldsymbol{z}), \text{what is } \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}}? \qquad \boldsymbol{h}, \boldsymbol{z} \in \mathbb{R}^n$$
$$h_i = f(z_i)$$

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$$\left(\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}}\right)_{ij} = \frac{\partial h_i}{\partial z_j} = \frac{\partial}{\partial z_j} f(z_i) \qquad \text{definition of Jacobian}$$

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$$\begin{aligned}\left(\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}}\right)_{ij} &= \frac{\partial h_i}{\partial z_j} = \frac{\partial}{\partial z_j} f(z_i) && \text{definition of Jacobian} \\ &= \begin{cases} f'(z_i) & \text{if } i = j \\ 0 & \text{if otherwise} \end{cases} && \text{regular 1-variable derivative}\end{aligned}$$

$$\boldsymbol{h} = f(\boldsymbol{z}), \text{what is } \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}}? \qquad \boldsymbol{h}, \boldsymbol{z} \in \mathbb{R}^n$$
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$$= \begin{cases} f'(z_i) & \text{if } i = j \\ 0 & \text{if otherwise} \end{cases} \qquad \text{regular 1-variable derivative}$$

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$$\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}} = \begin{pmatrix} f'(z_1) & & 0 \\ & \ddots & \\ 0 & & f'(z_n) \end{pmatrix} = \text{diag}(\boldsymbol{f}'(\boldsymbol{z}))$$

$$\frac{\partial}{\partial \boldsymbol{x}}(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b}) = \boldsymbol{W}$$
$$\frac{\partial}{\partial \boldsymbol{b}}(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b}) = \boldsymbol{I} \text{ (Identity matrix)}$$
$$\frac{\partial}{\partial \boldsymbol{u}}(\boldsymbol{u}^T\boldsymbol{h}) = \boldsymbol{h}^{\boldsymbol{T}}$$

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$$s = \boldsymbol{u}^T\boldsymbol{h}$$
$$\boldsymbol{h} = f(\boldsymbol{z})$$
$$\boldsymbol{z} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b}$$
$$\boldsymbol{x} \quad \text{(input)}$$

$$\frac{\partial s}{\partial \boldsymbol{b}} = \frac{\partial s}{\partial \boldsymbol{h}} \quad \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}} \quad \frac{\partial \boldsymbol{z}}{\partial \boldsymbol{b}}$$
$$= \boldsymbol{u}^{\boldsymbol{T}} \text{diag}(f'(\boldsymbol{z}))\boldsymbol{I}$$
$$= \boldsymbol{u}^{\boldsymbol{T}} \circ f'(\boldsymbol{z})$$

Useful Jacobians from previous slide

$$\frac{\partial}{\partial \boldsymbol{u}}(\boldsymbol{u}^T\boldsymbol{h}) = \boldsymbol{h}^T$$
$$\frac{\partial}{\partial \boldsymbol{z}}(f(\boldsymbol{z})) = \text{diag}(f'(\boldsymbol{z}))$$
$$\frac{\partial}{\partial \boldsymbol{b}}(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b}) = \boldsymbol{I}$$

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同理，当计算s对W的导数时：

$$\frac{\partial s}{\partial \boldsymbol{W}} = \frac{\partial s}{\partial \boldsymbol{h}} \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}} \frac{\partial \boldsymbol{z}}{\partial \boldsymbol{W}}$$
$$\frac{\partial s}{\partial \boldsymbol{b}} = \frac{\partial s}{\partial \boldsymbol{h}} \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}} \frac{\partial \boldsymbol{z}}{\partial \boldsymbol{b}}$$

$$\frac{\partial s}{\partial \boldsymbol{W}} = \boldsymbol{\delta}\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{W}}$$

$$\frac{\partial s}{\partial \boldsymbol{b}} = \boldsymbol{\delta}\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{b}} = \boldsymbol{\delta}$$

$$\boldsymbol{\delta} = \frac{\partial s}{\partial \boldsymbol{h}}\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{z}} = \boldsymbol{u}^T \circ f'(\boldsymbol{z})$$

$\delta$ is local error signal

δ 是局部误差符号

∂s/∂W的形状是 n×m

- Remember $\frac{\partial s}{\partial \boldsymbol{W}} = \boldsymbol{\delta}\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{W}}$
  - $\boldsymbol{\delta}$ is going to be in our answer
  - The other term should be $\boldsymbol{x}$ because $\boldsymbol{z} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b}$
- It turns out $\frac{\partial s}{\partial \boldsymbol{W}} = \boldsymbol{\delta}^T \boldsymbol{x}^T$

$\delta$ is local error signal at $z$

$x$ is local input signal

### Why the Transposes?

$$\frac{\partial s}{\partial \boldsymbol{W}} = \boldsymbol{\delta}^T \boldsymbol{x}^T = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_n \end{bmatrix} [x_1, ..., x_m] = \begin{bmatrix} \delta_1 x_1 & \dots & \delta_1 x_m \\ \vdots & \ddots & \vdots \\ \delta_n x_1 & \dots & \delta_n x_m \end{bmatrix}$$

$$\underset{[n \times m]}{\frac{\partial s}{\partial \boldsymbol{W}}} = \underset{[n \times 1]}{\boldsymbol{\delta}^T} \underset{[1 \times m]}{\boldsymbol{x}^T}$$

详见note