MAS 632

The Wolverine Retirement Fund (Case 3.3 from the textbook)

Kelly Jones is a financial analyst for Wolverine Manufacturing, a company that produces engine bearings for the automotive industry. Wolverine is in the process of hammering out a new labor agreement with its unionized workforce. One of the major concerns of the labor union is the funding of Wolverine's retirement plan for its hourly employees. The union believes the company has not been contributing enough money to this fund to cover the benefits it will need to pay to retiring employees. Because of this, the union wants the company to contribute approximately \$1.5 million dollars in additional money to this fund over the next 20 years. These extra contributions would begin with an extra payment of \$20,000 at the end of 1 year with annual payments increasing by 12.35% per year for the next 19 years.

The union has asked the company to set up a sinking fund to cover the extra annual payments to the retirement fund. The Wolverines' CFO and the union's chief negotiator have agreed that AAA rated bonds recently issued by three different companies may be used to establish this fund. The following table summarizes the provisions of these bonds.

Company	Maturity	Coupon Payment	Price	Par Value
AC&C	15 years	\$80	\$847.88	\$1,000
IBN	10 years	\$90	\$938.55	\$1,000
MicroHard	20 years	\$85	\$872.30	\$1,000

According to this table, Wolverine may buy bonds issued by AC&C for \$847.88 per bond. Each AC&C bond will pay the bondholder \$80 per year for the next 15 years, plus an extra payment of \$1,000 (the par value) in the fifteenth year. Similar interpretations apply to the information for the IBN and MicroHard bonds. A money market fund yielding 5% may be used to hold any coupon payments that are not needed to meet the company's required retirement fund payment in any given year.

Wolverine's CFO has asked Kelly to determine how much money the company would have to invest and which bonds the company should buy in order to meet the labor union's demands.

1. If you were Kelly, what would you tell the CFO?
2. Suppose the union insists on including one of the following stipulations in the agreement:

a) No more than half of the total number of bonds purchased may be purchased from a single company.

b) At least 10% of the total number of bonds must be purchased from each of the companies. Which stipulation should Wolverine agree to?

Solution: *Mathematical Formulation* is as follows:

Goal: Contribute \$1.5 million to the Wolverine's retirement fund over the next 20 years. Achieve this goal with the minimum possible amount of initial investments in the bonds (sinking fund). The contribution amount for the year t is $(\$20,000 \times 1.1235^{t-1})$. Is it true that

$$\sum_{t=1}^{20} \$20,000 \times 1.1235^{t-1} = \$1,500,000?$$

Decision Variables:

x_1 = number of AC&C bonds to buy
x_2 = number of IBN bonds to buy
x_3 = number of MicroHard bonds to buy

We introduce following additional variables to simply our formulation. Sometimes they are called auxiliary variables because they are not direct decisions; they are the outcome of our decisions. They simplify modeling and analysis.

Auxiliary Variables:

C_t = closing balance at the end of year t; $\quad t = 1, 2, \ldots, 20$
= money carried forward from year t to year $t+1$

Sink Fund

Objective: min $z = 847.88x_1 + 938.55x_2 + 872.30x_3$

Constraints: For every year t, we have the following Cash-flow equation:

$$\text{(Net Cash flow)}_t = \text{(Cash Inflow)}_t - \text{(Cash outflow)}_t$$
$$\Rightarrow \text{(Closing Balance)}_t - \text{(Opening Balance)}_t = \text{(Cash Inflow)}_t - \text{(Cash Outflow)}_t$$
$$\Rightarrow \text{(Closing Balance)}_t - \text{(Closing Balance)}_{t-1} = \text{(Cash Inflow)}_t - \text{(Cash Outflow)}_t$$
$$\Rightarrow \text{(Closing Balance)}_t - \text{(Closing Balance)}_{t-1} = \text{(Coupon Payments)}_t + \text{(Par-Value Payments)}_t + \begin{pmatrix} 5\% \text{ interest from} \\ \text{Money-Market fund} \end{pmatrix}_t - \begin{pmatrix} \text{Annual Contribution} \\ \text{to Retirement fund} \end{pmatrix}_t$$

Auxiliary Variables

Where we have:

$$(\text{Closing Balance})_t = C_t$$

$$\begin{pmatrix}\text{Coupon}\\ \text{Payments}\end{pmatrix}_t = \begin{cases} 80x_1 + 90x_2 + 85x_3 & \text{if } 1 \le t \le 10 \\ 80x_1 \quad\quad + 85x_3 & \text{if } 11 \le t \le 15 \\ \quad\quad\quad 85x_3 & \text{if } 16 \le t \le 20 \end{cases}$$

$$\begin{pmatrix}\text{Par-Value}\\ \text{Payments}\end{pmatrix}_t = \begin{cases} 1000x_2 & \text{if } t = 10 \\ 1000x_1 & \text{if } t = 15 \\ 1000x_3 & \text{if } t = 20 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{pmatrix}\text{5\% interest from}\\ \text{Money-Market fund}\end{pmatrix}_t = 0.05C_{t-1}$$

$$\begin{pmatrix}\text{Annual Contribution}\\ \text{to Retirement fund}\end{pmatrix}_t = 20000 \times 1.1235^{t-1}$$

$x_1, x_2, x_3 \ge 0$

$C_1, C_2, \ldots, C_{20} \ge 0$

This constraint gurantees that we will be able to make the annual contributions