Homework 1 - Due on Sep 22

MAS 632 Fall 2023

Problem 1

Consider the following LP:

$$\begin{aligned} \min\ z = 3x_1 - x_2 \\ \text{subject to} \\ 2x_1 + x_2 &\geq 4 \\ 2x_1 + 3x_2 &\leq 10 \\ x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(a) Solve the LP in Excel.

(b) Verify your solution with Python.

Problem 2 (LP with 2 Variables)

The Denver advertising agency, promoting the new Breem dishwashing detergent, wants to get the best exposure possible for the product within the $100,000 advertising budget ceiling placed on it. To do so, the agency needs to decide how much of the budget to spend on each of its two most effective media: (1) television spots during the afternoon hours and (2) large ads in the Sunday newspaper. Each television spot costs $3,000; each Sunday newspaper ad costs $1,250. The expected exposure, based on industry ratings, is 35,000 viewers for each TV commercial and 20,000 readers for each newspaper advertisement. The agency director, Deborah Kellogg, knows from experience that it is important to use both media in order to reach the broadest spectrum of potential Breem customers. She decides that at least 5 but no more than 25 television spots should be ordered, and that at least 10 newspaper ads should be contracted. How many times should each of the two media be used to obtain maximum exposure while staying within the budget?

(a) Formulate the optimization problem as a linear programming problem. Clearly define the decision variables, the objective function, the constraints.

(b) Create a Spreadsheet model for the optimization problem and solve it.

(c) Verify your solution with Python.

Problem 3 (Production Planning)

A company manufactures two products, P_1 and P_2. Each product must be processed either by machine M_1 or by machine M_2. The goal is to process 20 units of product P_1 and 30 units of

	M_1	M_2
P_1	15 min	10 min
P_2	25 min	20 min

Table 1: Processing times per unit

	M_1	M_2
P_1	4	7
P_2	12	15

Table 2: Processing costs per unit

product P_2 within 450 minutes, at minimum cost. The processing time and the processing cost of one unit of each product at each machine are shown in the tables below:

Machine M_2 should be used for at least as much time as machine M_1. We assume that both machines can process one unit at a time, and that they can work independently and in parallel.

(a) Formulate the optimization problem as a linear programming problem with 4 decision variables. Clearly define the decision variables, the objective function, the constraints.

(b) Use either Python or Excel to solve the problem.

Problem 4 (Robust Predictions)
Let Y be a discrete random variable (for example random demand for a product or service) taking values on the set $\{1,2,3,4,5,6,7\}$. We have limited information about the behavior of Y. We do not know its exact probability distribution. We do know however that Y has a variance of 5 and an expected value of 4. No realization of Y happens with probability higher than 0.25.

(a) Formulate two Linear Programming problems that provide robust predictions (an upper bound and a lower bound) for the probability $\Pr(Y \leq 2)$.

(b) Use either Python or Excel to solve the two problems.

Problem 5 (Robust Predictions)
Let Z be a discrete random variable (for example random demand for a product or service) taking values on the set $\{0,1,2,3,4,5\}$. We have limited information about the behavior of Z. We do not know its exact probability distribution. We do know, however, that Z has an expected value of 3, and that each realization of Z happens with probability at least 0.10. We also know that $\Pr(Z \geq 4)$ is at least 0.4.

(a) Formulate two Linear Programming problems that will help provide an upper bound and a lower bound on the **variance** of Z.

(b) Use either Python or Excel to solve the two problems.

Problem 6 (Chapter 6, exercise 16 - easy BP)

A developer of video game software has seven proposals for new games. Unfortunately, the company cannot develop all the proposals because its budget for new projects is limited to $950,000, and it has only 20 programmers to assign to new projects. The financial requirements, returns, and the number of programmers required by each project are summarized in the following table. Projects 2 and 6 require specialized programming knowledge that only one of the programmers has. Both of these projects cannot be selected because the programmer with the necessary skills can be assigned to only one of the projects. (Note: All dollar amounts represent thousands.)

Project	Programmers Required	Capital Required	Estimated NPV
1	7	$250	$650
2	6	$175	$550
3	9	$300	$600
4	5	$150	$450
5	6	$145	$375
6	4	$160	$525
7	8	$325	$750

Formulate an optimization problem that maximizes the total estimated NPV subject to constraints. Crate a spreadsheet model and solve the problem. What is the optimal solution?

Problem 7 (Multi-Period Knapsack problem)

(a) Consider the following multi-period capital budgeting problem. An investor is considering 8 independent investment projects. Each project requires full participation, no partial participation is allowed. If selected, a project may require cash contributions yearly over the next 3 years, as in the following table. There is an external fixed amount of \$1,000,000 available for these investments in each of the following 3 years. Each investment project is expected to have a net return as shown in Table 3. The investor needs to determine the subset of projects to invest in, so as to maximize the total expected net returns. For simplicity, assume that:

- Unutilized funds are not carried forward to the next period and are not available for investments.
- Cash flows are not discounted for the time value of money (we could consider discounting of cash flows, but the problem becomes considerably more difficult to formulate).

	cash out-flows								Funds Available
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	
Year 1	20	40	50	25	15	7	23	13	100
Year 2	30	20	30	25	25	22	23	28	100
Year 3	10	0	10	35	30	23	23	15	100
Expected Net Return	10	15	20	20	15	13	13	14	

Table 3: All figures in \$10,000's of dollars

Formulate the problem as a binary programming problem, and use either Python or Excel to solve it.

Hint: Introduce 8 binary variables and consider 3 constraints (knapsacks).

(b) Consider the same setting as in Problem 7(a), but relax the first assumption, i.e., unused funds **can be** carried forward to the next period and **are** available for cash out-flows. Formulate the problem as an integer programming problem, and use either Python or Excel to solve it. Compare your solution to the one from Problem 7(a).

Hint: Introduce additionally 3 auxiliary variables y_1, y_2, y_3, corresponding to unutilized funds in years 1,2,3 respectively. Then the available funds for period 1 are \$1,000,000. The available funds for period 2 are \$1,000,000 $+y_1$. The available funds for period 3 are \$1,000,000 $+y_2$. Apply ideas from the multi-period models to capture the cash-flow dynamics.

Problem 8 (Assignment Problem) Consider the assignment problem for the 15×10 "cost matrix" contained in the attached file "cost_matrix.xlsx". Your task is to assign 10 out of 15 resources to perform 10 tasks. Each resource will undertake at most one task and every task must be completed by exactly one resource. Solve the problem both in Excel and Python.