SUSY Lectures in 2017 Summer

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Contents

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1 Introduction

1 [L1] Quantum Field Theory

: basic framework to study elementary particles and their interactions.

"special relativity + Quantum mechanics"

$$\text{On special relativity} \quad \begin{cases} \text{spacetime : } (ct, \vec{x}) \\ \text{force: mediated by fields. (e.g. EM)} \end{cases}$$

S: action
$\delta S = 0$: classical action(least action)

$$\langle f|i\rangle \sim \sum_{\text{path}} e^{iS/\hbar} \quad ; \text{path integral}$$
$$\begin{cases} \Delta S \gg \hbar & ; \text{Classical} \\ \Delta S \sim \hbar & ; \text{Quantum} \end{cases}$$

We use the natural unit, $\hbar = c = 1$.

$$\text{Theory} \quad \begin{cases} \text{action}(S) \\ \quad + \\ \text{regularization} \\ \quad + \\ \text{renormalization} \end{cases}$$

1-1 S(action)

$$S = \int \underbrace{d^4x}_{dtdxdydz} \; \mathcal{L}(\phi, \partial_\mu \phi)$$

where ϕ is a field, $\phi(x)$. Lagrange has a symmetry.

i) symmetry
- Important role to better understand natural.
- Defining an elementary particle according to the behavior of the corresponding field with respect to symmetries.
- Determining interactions among particles.
(can be hidden; spontaneous breaking)

Noether Theorem(classical field theory)
: continuous symmetry $\rightarrow$ conserved quantity

Symmetry

1. Spacetime Symmetry ($x \to x'$)
   (a) Poincaré Transformation: transformation leaving the line element invariant.
   $\to ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$
      i. Lorentz transformation; action conservation
      ii. Translations; energy-momentum conservation

   (b) General coordinate Transformation
   $\to$ general relativity

2. Internal Symmetry

$$\Phi^a(x) \to \Phi'^a(x) = M^a{}_b \Phi^b(x)$$

where $\Phi^a(x)$ is general field and a, b are internal indices.

$$M^a{}_b \begin{cases} \text{global symmetry: } M^a{}_b \text{ is spacetime-independent} \\ \text{local(gauge) symmetry: } M^a{}_b(x) \text{ is spacetime-dependent} \end{cases}$$

QFT: the most general symmetry of the S-matrix(Scattering matrix).

Note 1.1 *Coleman-Mandula(1967)*

$$\text{Poincaré symmetry} \bigotimes \text{Internal symmetry}$$

where $\bigotimes$ is a direct product.

- assumption on QFT (local, relativity, $4D$, ...)
  & scattering interacions.
- considered only bosonic generator.

Extended to include spinor generators(spin: $\frac{1}{2}$, $\frac{3}{2}$),

Note 1.2 *Haag-Lopuszański-Sohnius(1975)*

$$\text{Super-Poincaré symmetry} \bigotimes \text{Internal symmetry}$$

1-2 Regularization & Renormalization

1. Regularization
   (a) UV divergence
   : we need to regulate the theory.
   (Infinities $\to$ absorbed by appropriate counter terms.)
   e.g. cutoff regulation, dimensional regularization

2. Renormalization
   : determine how to absorb the infinities into counter terms.
   (arbitraty renormalization point μ(unphysical))

   (a) Perturbation Theory
       : expansion w.r.t $\underbrace{\text{renormalized parameters.}}_{\mu-\text{dep.}}$ $\rightarrow$ large logs: $\ln(\frac{E}{\mu}), \ln(\frac{M}{\mu})$
       $\rightarrow$ physical amplitude(independent of μ) $\rightarrow$ "set $\mu \sim E$"

   (b) RGE(Renormalization Group Equation)

2 [L2] Effective Field Theory

[Need Figure!] To make massive particle, we need high energy. So in low energy scale, we can see only small mass particles. We can't & don't need to see higher mass particles.

– Full Theory (UV)

$$\begin{aligned} &\mathcal{L}_{full} = \mathcal{L}_H(\phi_H, \phi_L) + \mathcal{L}(\phi_L) \\ &\quad \text{where } \begin{cases} \phi_H : \text{Fields describing particles with masses } > \Lambda \\ \phi_L : \text{Fields describing particles with masses } < \Lambda \end{cases} \end{aligned} \tag{1}$$

– Effective theory of light fields ϕ_L

: Integrating out ϕ_H in E.O.M. (Effects of all Heavy particle are in C_i)

$$\begin{aligned} &\mathcal{L}_{eff} = \mathcal{L}(\phi_L) + \sum_k \frac{C_i}{\Lambda^{k-4}} \mathcal{O}_i^{(k)}(\phi_L) \\ &\quad \text{where } \begin{cases} C_i : \text{Wilson Coefficient (dim=0)} \\ \Lambda : \text{Cut-off sclae of EFT} \end{cases} \end{aligned} \tag{2}$$

(1) Contribution of $\mathcal{O}_i^{(k)}$ to a process at energy scale $E \gg \Lambda$

$$C_i \left(\frac{E}{\Lambda}\right)^{k-4} \begin{cases} k = 4 : \text{Marginal} \\ k < 4 : \text{Relavent} \\ k > 4 : \text{Irrelavnt - (Almost non-renormalizable)} \end{cases}$$

(2) Matching (Around Λ) : Same physical predictions at low energy.

$$\lambda_{eff}(\mu) = \lambda_{full}(\mu) + \text{(threshold corrections)}$$

e.g. [Need Figure!]

3 [L2] Poincaré Symmetry

– Symmetry of the SM

$$\begin{cases} \text{Poincaré} \begin{cases} \text{Lorentz Inv.} \\ \text{Translation Sym.} \end{cases} \\ \\ \text{Internal} \begin{cases} \text{Global: Baryon number, Lepton number, Flavor Sym.} \\ \text{Gauge/Local: } SU(3)_c \times SU(2)_c \times U(1)_Y \end{cases} \end{cases}$$

3-1 Group

Definition 3.1 (Group) *A set of elements $G = \{g_1, g_2, \cdots\}$ with product operator $*$ s.t*

1. *Closure:* $\forall g_i, g_j \in G,\ g_i * g_j \in G$
2. *Unit element :* $\exists e \in G,\ e * g = g * e = g \quad \forall g \in G$
3. *Inverse element :* $\forall g \in G,\ \exists g^{-1} \in G \quad s.t \quad g * g^{-1} = g^{-1} * g = e$
4. *Associativity :* $\forall g_i, g_j, g_k \in G,\ g_i * (g_j * g_k) = (g_i * g_j) * g_k$

3-2 Representation

Definition 3.2 (Representation) *A map $G \to R$ where $R = \{D(g)\}$ such that*

1. $D(g_1)D(g_2) = D(g_1 * g_2)$
2. $D(e) = 1$
3. $D(g^{-1}) = D(g)^{-1}$

$D(g)$ is a linear operator acting on a vector space $V = \{v_1, v_2, \cdots\}$.

3-3 Poincaré group

– Coordinate trsf leaving $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ is invariant.

$$x' = \Lambda x + a \quad \begin{cases} a : \text{Translations} \\ \Lambda : \text{Lorentz trsf (rotations + boosts)} \end{cases}$$