

SUSY Lectures in 2017 Summer

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1 Introduction

1 [L1] Quantum Field Theory

: basic framework to study elementary particles and their interactions.

”special relativity + Quantum mechanics”

On special relativity $\left\{ \begin{array}{l} \text{spacetime : } (ct, \vec{x}) \\ \text{force: mediated by fields. (e.g. EM)} \end{array} \right.$

S : action

$\delta S = 0$: classical action(least action)

$$\langle f|i \rangle \sim \sum_{\text{path}} e^{iS/\hbar} \quad ; \text{ path integral}$$

$$\left\{ \begin{array}{l} \Delta S \gg \hbar \quad ; \text{ Classical} \\ \Delta S \sim \hbar \quad ; \text{ Quantum} \end{array} \right.$$

We use the natural unit, $\hbar = c = 1$.

$$\text{Theory} \left\{ \begin{array}{l} \text{action}(S) \\ + \\ \text{regularization} \\ + \\ \text{renormalization} \end{array} \right.$$

1-1 $S(\text{action})$

$$S = \int \underbrace{d^4x}_{dtdxdydz} \mathcal{L}(\phi, \partial_\mu \phi)$$

where ϕ is a field, $\phi(x)$. Lagrange has a symmetry.

i) symmetry

- Important role to better understand natural.
 - Defining an elementary particle according to the behavior of the corresponding field with respect to symmetries.
 - Determining interactions among particles.
- (can be hidden; spontaneous breaking)

Noether Theorem(classical field theory)

: continuous symmetry \rightarrow conserved quantity

Symmetry

1. Spacetime Symmetry ($x \rightarrow x'$)

- (a) Poincaré Transformation: transformation leaving the line element invariant.
 $\rightarrow ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$
 - i. Lorentz transformation; action conservation
 - ii. Translations; energy-momentum conservation
- (b) General coordinate Transformation
 \rightarrow general relativity

2. Internal Symmetry

$$\Phi^a(x) \rightarrow \Phi'^a(x) = M^a_b \Phi^b(x)$$

where $\Phi^a(x)$ is general field and a, b are internal indices.

$$M^a_b \begin{cases} \text{global symmetry: } M^a_b \text{ is spacetime-independent} \\ \text{local(gauge) symmetry: } M^a_b(x) \text{ is spacetime-dependent} \end{cases}$$

QFT: the most general symmetry of the S-matrix(Scattering matrix).

Note 1.1 *Coleman-Mandula(1967)*

$$\text{Poincaré symmetry} \otimes \text{Internal symmetry}$$

where \otimes is a direct product.

- assumption on QFT (local, relativity, $4D$, ...)
 & scattering interactions.
- considered only bosonic generator.

Extended to include spinor generators(spin: $\frac{1}{2}, \frac{3}{2}$),

Note 1.2 *Haag-Lopuszański-Sohnius(1975)*

$$\text{Super-Poincaré symmetry} \otimes \text{Internal symmetry}$$

1-2 Regularization & Renormalization

1. Regularization

- (a) UV divergence
 : we need to regulate the theory.
 (Infinites \rightarrow absorbed by appropriate counter terms.)
 e.g. cutoff regulation, dimensional regularization

2. Renormalization

: determine how to absorb the infinities into counter terms.
(arbitrary renormalization point μ (unphysical))

(a) Perturbation Theory

: expansion w.r.t $\underbrace{\text{renormalized parameters}}_{\mu\text{-dep.}} \rightarrow \text{large logs: } \ln(\frac{E}{\mu}), \ln(\frac{M}{\mu})$
 $\rightarrow \text{physical amplitude (independent of } \mu) \rightarrow \text{"set } \mu \sim E\text{"}$

(b) RGE(Renormalization Group Equation)

2 [L2] Effective Field Theory

[Need Figure!] To make massive particle, we need high energy. So in low energy scale, we can see only small mass particles. We can't & don't need to see higher mass particles.

– Full Theory (UV)

$$\mathcal{L}_{full} = \mathcal{L}_H(\phi_H, \phi_L) + \mathcal{L}(\phi_L)$$

$$\text{where } \begin{cases} \phi_H : \text{Fields describing particles with masses} > \Lambda \\ \phi_L : \text{Fields describing particles with masses} < \Lambda \end{cases}$$

– Effective theory of light fields ϕ_L

: Integrating out ϕ_H in E.O.M. (Effects of all Heavy particle are in C_i)

$$\mathcal{L}_{eff} = \mathcal{L}(\phi_L) + \sum_k \frac{C_i}{\Lambda^{k-4}} \mathcal{O}_i^{(k)}(\phi_L)$$

$$\text{where } \begin{cases} C_i : \text{Wilson Coefficient (dim=0)} \\ \Lambda : \text{Cut-off scale of EFT} \end{cases}$$

(1) Contribution of $\mathcal{O}_i^{(k)}$ to a process at energy scale $E \gg \Lambda$

$$C_i \left(\frac{E}{\Lambda} \right)^{k-4} \begin{cases} k = 4 : \text{Marginal} \\ k < 4 : \text{Relevant} \\ k > 4 : \text{Irrelevant} - (\text{Almost non-renormalizable}) \end{cases}$$

(2) Matching (Around Λ) : Same physical predictions at low energy.

$$\lambda_{eff}(\mu) = \lambda_{full}(\mu) + (\text{threshold corrections})$$

e.g. [Need Figure!]

3 [L2] Poincaré Symmetry

– Symmetry of the SM

$$\left\{ \begin{array}{l} \text{Poincaré} \left\{ \begin{array}{l} \text{Lorentz Inv.} \\ \text{Translation Sym.} \end{array} \right. \\ \\ \text{Internal} \left\{ \begin{array}{l} \text{Global: Baryon number, Lepton number, Flavor Sym.} \\ \text{Gauge/Local: } SU(3)_c \times SU(2)_c \times U(1)_Y \end{array} \right. \end{array} \right.$$

3-1 Group

Definition 3.1 (Group) A set of elements $G = \{g_1, g_2, \dots\}$ with product operator $*$ s.t

1. Closure: $\forall g_i, g_j \in G, g_i * g_j \in G$
2. Unit element : $\exists e \in G, e * g = g * e = g \quad \forall g \in G$
3. Inverse element : $\forall g \in G, \exists g^{-1} \in G \quad \text{s.t} \quad g * g^{-1} = g^{-1} * g = e$
4. Associativity : $\forall g_i, g_j, g_k \in G, g_i * (g_j * g_k) = (g_i * g_j) * g_k$

3-2 Representation

Definition 3.2 (Representation) A map $G \rightarrow R$ where $R = \{D(g)\}$ such that

1. $D(g_1)D(g_2) = D(g_1 * g_2)$
2. $D(e) = 1$
3. $D(g^{-1}) = D(g)^{-1}$

$D(g)$ is a linear operator acting on a vector space $V = \{v_1, v_2, \dots\}$.

3-3 Poincaré group

– Coordinate trsf leaving $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ is invariant.

$$x' = \Lambda x + a \quad \left\{ \begin{array}{l} a : \text{Translations} \\ \Lambda : \text{Lorentz trsf (rotations + boosts)} \end{array} \right.$$

– **Infinitesimal transformation** : (Λ, a)

$$\delta x^\mu = \epsilon^\mu + \omega^\mu{}_\nu x^\nu \quad (\omega^{\mu\nu} = -\omega^{\nu\mu}; \text{ anti-sym})$$

$$\Rightarrow \delta X^\mu = -i \left(\epsilon_\sigma P^\sigma + \frac{1}{2} \omega_{\rho\sigma} M^{\rho\sigma} \right)^\mu{}_\nu X^\nu \quad (X \text{ in Vector space})$$

In this representations, $\begin{cases} P_\mu = i\partial_\mu \\ M_{\mu\nu} = i(X_\mu\partial_\nu - X_\nu\partial_\mu) \end{cases} \rightarrow \text{the form of } P \text{ and } M \text{ depends on the reps.}$

– **Poincaré algebra**

From the composition rule : $(\Lambda_2, a_2)(\Lambda_1, a_1) = (\Lambda_2\Lambda_1, \Lambda_2 a_1 + a_2)$, we can derive below algebra.

1. $[P^\mu, P^\nu] = 0$
2. $[M^{\mu\nu}, P^\sigma] = i(P^\mu\eta^{\nu\sigma} - P^\nu\eta^{\mu\sigma})$
3. $[M^{\mu\nu}, M^{\rho\sigma}] = i(M^{\mu\sigma}\eta^{\nu\rho} + M^{\nu\rho}\eta^{\mu\sigma} - M^{\mu\rho}\eta^{\nu\sigma} - M^{\nu\sigma}\eta^{\mu\rho})$

– **Representation on fields** : $(x' = \Lambda x + a)$

- As a vector, $\phi^a(x) \rightarrow \phi'^a(x') = R^a{}_b(\Lambda)\phi^b(x)$ (ϕ is regarded as scalar for translations.)
- As a field(quantum) operator, $\hat{\phi}^a(x)$
: Consider transformation of state vector $|\psi\rangle \rightarrow |\psi'\rangle = \hat{U}(\Lambda, a)|\psi\rangle$ ($\hat{U}(\Lambda, a)$ is linear & unitary.)

$$\hat{U}(\Lambda, a) = 1 - i\epsilon_\mu \hat{P}^\mu - \frac{i}{2}\omega_{\mu\nu}\hat{M}^{\mu\nu} + \dots$$

Now consider field operator, we can get below equations.

$$\begin{cases} \varphi^a(x) = \langle f | \hat{\phi}^a(x) | g \rangle \quad \text{then} \quad \varphi'^a(x) = \langle f' | \hat{\phi}^a(x) | g' \rangle = \langle f | \hat{U}^\dagger \hat{\phi}^a(x) \hat{U} | g \rangle \\ \varphi'^a(x') = R^a{}_b(\Lambda)\varphi^b(x) = R^a{}_b \langle f | \hat{\phi}^b(x) | g \rangle \end{cases}$$

$$\therefore \hat{\phi}'^a(x') = R^a{}_b \hat{\phi}^b(x) = \hat{U}^\dagger \hat{\phi}^a(x') \hat{U}$$

(Caution : It's not unitary equivalence!)