$$p \leftrightarrow q \equiv \underbrace{(p \to q)}_{P} \wedge \underbrace{(q \to p)}_{Q}$$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$P \wedge Q$				
V	V	V	V	V	V				
V	F	F	F	V	F				
~~V~~	~~V~~	~~V~~	~~V~~	~~V~~	~~V~~				
~~V~~	~~F~~	~~F~~	~~F~~	~~V~~	~~F~~				
F	V	F	V	F	F				
F	F	V	V	V	V				
~~F~~	~~V~~	~~F~~	~~V~~	~~F~~	~~F~~				
~~F~~	~~F~~	~~V~~	~~V~~	~~V~~	~~V~~				

tabelas verdade idênticas

Logo $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

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a) $\sim p \wedge (p \vee \sim q)$

b) $(p \wedge q) \vee (p \wedge \sim q)$

c) $(p \wedge q) \vee (p \wedge (\sim q \wedge r))$

d) $\sim (p \vee q) \wedge \sim (q \vee r)$

e) $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

f) $(q \wedge (p \wedge r)) \vee (p \wedge (q \wedge \sim r))$

p	T	p∧T	p∨T
V	V	V	V
F	V	F	V

$p \wedge T \equiv p$ | $p \vee T \equiv T$

p	C	p∧C	p∨C
V	F	F	V
F	F	F	F

$p \wedge C \equiv C$ $\quad$ $p \vee C \equiv p$

a) $\sim p \wedge (p \vee \sim q) \equiv \underbrace{(\sim p \wedge p)}_{\text{cont.}} \vee (\sim p \wedge \sim q) \equiv C \vee (\sim p \wedge \sim q) \equiv \sim p \wedge \sim q$

Distributiva

$p \to q \to r$

não associativo

b) $(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge \overbrace{(q \vee \sim q)}^{\text{Taut.}} \equiv p \wedge T \equiv p$

p em evidência

d) $\sim (p \vee q) \wedge \sim (q \vee r) \equiv \sim [(p \vee q) \vee (q \vee r)] \equiv \sim (p \vee \underbrace{(q \vee q)}_{q} \vee r) \equiv \sim (p \vee q \vee r) \equiv \sim p \wedge \sim q \wedge \sim r$

De Morgan

opção $(\sim p \wedge \sim q) \wedge (\sim q \wedge \sim r) \equiv \sim p \wedge \sim q \wedge \sim q \wedge \sim r \equiv \sim p \wedge \sim q \wedge \sim r$

prop. assoc.

$(a+b)+c = a+(b+c)$ $\quad$ a, b, c nºs

$a+b+c$

a) $\sim p \wedge (p \vee \sim q)$

b) $(p \wedge q) \vee (p \wedge \sim q)$

c) $(p \wedge q) \vee (p \wedge (\sim q \wedge r))$

d) $\sim (p \vee q) \wedge \sim (q \vee r)$

e) $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

f) $(q \wedge (p \wedge r)) \vee (p \wedge (q \wedge \sim r))$

e) $\underbrace{(p\wedge q)\vee(p\wedge \sim q)}\vee(\sim p\wedge \sim q)$

p em "evidência"

$(p\wedge \underbrace{(q\vee \sim q)}_{T})\vee(\sim p\wedge \sim q)$

$(p\wedge T)\vee(\sim p\wedge \sim q)$

$p\vee(\sim p\wedge \sim q)$ } distributiva

$(p\vee \sim p)\wedge(p\vee \sim q)$

$T\wedge(p\vee \sim q)$

$p\vee \sim q$

analogia $a\cdot b + a\cdot c = a\cdot(b+c)$

f) $(q\wedge(p\wedge r))\vee(p\wedge(q\wedge \sim r))$

$(\underbrace{(p\wedge q)}\wedge r)\vee(\underbrace{(p\wedge q)}\wedge \sim r)$ } comutativa associativa

"fator comum"

$(p\wedge q)\wedge\underbrace{(r\vee \sim r)}_{T}$

$p\wedge q$

$(p\wedge q)\wedge r$ mesmo operador prop. assoc.

distib $\cancel{(p\wedge q)\vee r = (p\wedge r)\vee(p\wedge q)}$

$(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$

$(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$

Distributiva

Analogia

$(a+b)c = ac+bc$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ associativa

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ associativa

$(a+b)+c = a+(b+c)$

posso escrever sem parênteses

$(ab)c = a.(bc)$

$p \wedge q \wedge r \wedge s$

$p \wedge q \vee r \wedge s$ precisa de parênteses

12. Elimine os operadores → e ↔ das proposições, por meio de equivalências lógicas que não

a) $p \rightarrow (p \vee q)$

b) $(p \rightarrow q) \rightarrow r$

c) $p \leftrightarrow q$

lembrando

$p \rightarrow q \equiv \sim p \vee q$

a) $p \rightarrow (p \vee q) \equiv \sim p \vee (p \vee q) \equiv \underbrace{(\sim p \vee p)}_{T} \vee q \equiv T$

c) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p) \equiv$

$\equiv (\sim p \wedge \sim q) \vee \underbrace{(\sim p \wedge p)}_{C} \vee \underbrace{(q \wedge \sim q)}_{C} \vee (q \wedge p)$

$\equiv (\sim p \wedge \sim q) \vee (p \wedge q)$

analogia

$(a+b) \cdot (c+d)$

$ac + ad + bc + ba$

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$p \to q \equiv \sim p \vee q$

h) $(p \wedge q) \to (\sim r \to \sim q)$

$(p \wedge q) \to (r \vee \sim q)$

$\sim (p \wedge q) \vee (r \vee \sim q)$

De Morgan

$(\sim p \vee \sim q) \vee (r \vee \sim q)$

assoc commut.

$\sim p \vee \sim q \vee \sim q \vee r$

$\sim p \vee \sim q \vee r$