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③ $a_n = \dfrac{1}{2+n^2}$, $n \in \mathbb{N}^*$ termo geral

a) $a_5 + a_4 = \dfrac{1}{2+5^2} + \dfrac{1}{2+4^2} = \dfrac{1}{27} + \dfrac{1}{18}$

$27 = 3^3 \quad 18 = 2 \cdot 3^2 \quad mmc(27, 18) = 2 \cdot 3^3 = 54$

b) n?

$\left(\ldots, \underbrace{\dfrac{1}{123}}_{a_n}, \ldots \right)$

$a_n = \dfrac{1}{123} \longrightarrow \dfrac{1}{2+n^2} = \dfrac{1}{123} \Rightarrow n^2 = 121$

$n = 11$ ou $n = -11$, não convém, pois $n \in \mathbb{N}^*$

Resp $\Rightarrow n = 11$

4b $g: \mathbb{N}^* \longrightarrow \mathbb{N}$

$g(x) = x^2 - 2x + 4$ $\quad$ $x \in \mathbb{N}^*$

$g(1) = 1^2 - 2 \cdot 1 + 4 = 3$ $\quad$ $g(4) = 4^2 - 2 \cdot 4 + 4 = 12$

$g(2) = 2^2 - 2 \cdot 2 + 4 = 4$

$g(3) = 3^2 - 2 \cdot 3 + 4 = 7$

a seq é $(3, 4, 7, 12, \ldots)$

4c $h: \mathbb{N}^* \longrightarrow \mathbb{Q}$

$n \in \mathbb{N}^*$ e $a_n \in \mathbb{Q}$

n	Regra
1	$2 \cdot \frac{1}{1} = 2$
2	$2 \cdot \frac{1}{2} = 1$
3	$2 \cdot \frac{1}{3} = \frac{2}{3}$

$a_n = 2 \cdot \frac{1}{n}, \; n \in \mathbb{N}^*$

$$\boxed{a_n = \frac{2}{n}, \; n \in \mathbb{N}^*}$$

a seq é $\left(2, 1, \frac{2}{3}, \ldots\right)$

7) $a_n = -193 + 3n \qquad b_m = 220 - 4m \qquad m \in \mathbb{N}^*$

$a_n = b_m \Rightarrow -193 + 3n = 220 - 4n$

$-413 = -7n \Rightarrow n = 59$

$\therefore\ a_{59} = b_5 = -16$

—//—

$a_n = 3n - 193,\ n \in \mathbb{N}^*$

$b_m = 220 - 4m,\ m \in \mathbb{N}^*$

$a_n = b_m$

$-193 + 3n = 220 - 4m$

$3n = 413 - 4m$

$n = \dfrac{413 - 4m}{3},\ n, m \in \mathbb{N}^*$

atribuir valor p/ m e calcular n

$n = \frac{413 - 4m}{3}$, $n, m \in \mathbb{N}^*$

m	n
~~1~~	~~$\frac{409}{3}$~~
2	$\frac{405}{3} = 135$
~~3~~	~~$\frac{401}{3}$~~
~~4~~	~~$\frac{397}{3}$~~
5	$\frac{393}{3} = 131$
$\vdots$	$\vdots$

1) $m \in \mathbb{N}^* \Rightarrow \frac{413 - 4m}{3} > 0$

$\Rightarrow 413 - 4m > 0 \longrightarrow -4m > -413$

$m < \frac{413}{4} = 103{,}25$ $\quad m \in \mathbb{N}^*$

$m \in \{1, 2, 3, \dots, 103\}$ candidatos

$n = \frac{411 + 2 - 4m}{3} = \frac{411}{3} + \frac{2 - 4m}{3} = 137 + \frac{2 - 4m}{3}$

$m \in \mathbb{N}^*$

$n = 137 + \underbrace{\frac{2(1 - 2m)}{3}}_{\text{natural}}$

$a_{135} = b_2$, $a_{131} = b_5$, etc

7c $b_n = 220 - 4n \,,\, n \in \mathbb{N}^*$

$b_n < 0$?

$220 - 4n < 0$

$-4n < -220$

$n > \frac{220}{4}$

$n > 55$ e $n \in \mathbb{N}^*$

$b_n < 0$, para $n > 55$ e $n \in \mathbb{N}^*$

$n \in \{56, 57, \ldots\}$

1.º termo negativo

$b_{56} = 220 - 4 \cdot 56$

$b_{56} = 220 - 224$

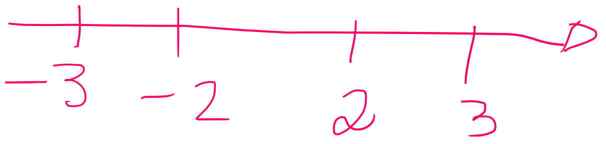
$\boxed{b_{56} = -4}$

$$\boxed{-4m < -220} \xrightarrow{\times(-1)} 4m > 220 \xrightarrow{\div 4} m > \frac{220}{4}$$

$$2 < 3 \text{ e } 5 > 0 \Longrightarrow \underbrace{2\cdot 5}_{10} < \underbrace{3\cdot 5}_{15} \quad (V)$$

$$2 < 3 \text{ e } -5 < 0 \Longrightarrow \underbrace{2\cdot(-5)}_{-10} > \underbrace{3\cdot(-5)}_{-15} \quad (V)$$

$$\boxed{\begin{array}{l} x<y \text{ e } a>0 \iff ax<ay,\ a>0 \\ x<y \text{ e } a<0 \iff ax>ay,\ a<0 \end{array}}$$



$-3 < -2$ e $3 > 2$

$$-4m < -220 \xrightarrow{\div(-4)} m > \frac{-220}{-4}$$

$$m > \frac{220}{4}$$

8) $a_n = 4n^2 - n + 9, \; n \in \mathbb{N}^*$

$a_1 = 4 - 1 + 9 = 12$

$a_2 = 4 \cdot 2^2 - 2 + 9 = 23$

$a_3 = 4 \cdot 3^2 - 3 + 9 = 42$

$(12, 23, a_3, a_4, 104, \ldots)$

Progressão Aritmética (PA)

Def

$$\begin{cases} a_1 \\ a_n = a_{n-1} + r, \ n \geq 1 \end{cases}, \quad n \in \mathbb{N}^* \text{ e } r \in \mathbb{R}$$

r razão da PA (constante)

Em particular $\quad a_n - a_{n-1} = r \quad , \forall n \in \mathbb{N}^*, n \geq 2$

① $(-7, \underbrace{-4}_{+3}, \underbrace{-1}_{+3}, \underbrace{2}_{+3}, \underbrace{5}_{+3}, \ldots\ldots)$ PA de razão $r = 3$

$a_1 = -7$

$$\begin{cases} a_1 = -7 \\ a_n = a_{n-1} + 3 \end{cases}$$

$a_1 < a_2 < a_3 < a_4 < \ldots\ldots$

PA crescente

② $\begin{cases} a_1 = 5 \\ e \\ a_n = a_{n-1} + 4, \ n \geq 2 \end{cases}$

$a_2 = a_1 + 4 = 5 + 4 = 9$

$a_3 = a_2 + 4 = 9 + 4 = 13$

$a_4 = a_3 + 4 = 13 + 4 = 17$

$(5, 9, 13, 17, \ldots)$

$a_1 = 5$ e $r = 4 > 0$

PA crescente

③ $(\overset{-4}{10}, \overset{-4}{6}, \overset{-4}{2}, \overset{-4}{-2}, -6, \ldots)$

$\underbrace{a_2 - a_1}_{-4} = \underbrace{a_3 - a_2}_{-4} = \underbrace{a_4 - a_3}_{-4} = \ldots = -4$

$a_4 - a_3 = -2 - 2 = -4$

$a_5 - a_4 = -6 - (-2) = -4$

PA $a_1 = 10$ e $r = -4 < 0$

é decrescente

4) $(2, 2, 2, 2, 2, \ldots)$ PA de $a_1 = 2$ e $r = 0$

$$\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + r, \ n \geq 2 \end{cases}$$

$$\begin{cases} a_2 - a_1 = 0 \\ a_3 - a_2 = 0 \\ a_4 - a_3 = 0 \\ a_5 - a_4 = 0 \end{cases}$$

$\underbrace{a_n - a_{n-1} = r}$ constante

$$\text{PA razão } r \begin{cases} \text{crescente} & r > 0 \\ \text{decrescente} & r < 0 \\ \text{constante} & r = 0 \end{cases}$$

5) $(1, 3, 6, 10, 15, \ldots)$ não é PA

$$\underbrace{1, 3}_{+2}\ \underbrace{3, 6}_{+3}\ \underbrace{6, 10}_{+4}\ \underbrace{10, 15}_{+5}$$

$a_2 - a_1 = 2$

$a_3 - a_2 = 3$

$a_4 - a_3 = 4$

$a_5 - a_4 = 5$

etc

são diferentes

$$a_n = a_{n-1} + \underbrace{r}_{\text{razão}}$$

constante

$$(a_1, a_2, a_3, a_4, \ldots)$$

$$\underbrace{a_1, a_2}_{+r}\ \underbrace{a_2, a_3}_{+r}\ \underbrace{a_3, a_4}_{+r}$$

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