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③  $a_n = \frac{1}{2+n^2}$ ,  $n \in \mathbb{N}^*$  termo geral

a)  $a_5 + a_4 = \frac{1}{2+5^2} + \frac{1}{2+4^2} = \frac{1}{27} + \frac{1}{18}$

$27 = 3^3$      $18 = 2 \cdot 3^2$      $\text{mmc}(27, 18) = 2 \cdot 3^3 = 54$

b)  $n?$   $\left( \dots, \underbrace{\frac{1}{123}}_{a_n}, \dots \right)$

$a_n = \frac{1}{123} \rightarrow \frac{1}{2+n^2} = \frac{1}{123} \Rightarrow n^2 = 121$

$n = 11$  ou  $n = -11$ , não convêm, pois  $n \in \mathbb{N}^*$

Resp  $\Rightarrow n = 11$

4b  $g: \mathbb{N}^* \rightarrow \mathbb{N}$

$$g(x) = x^2 - 2x + 4$$

$x \in \mathbb{N}^*$

$$g(1) = 1^2 - 2 \cdot 1 + 4 = 3$$

$$g(4) = 4^2 - 2 \cdot 4 + 4 = 12$$

$$g(2) = 2^2 - 2 \cdot 2 + 4 = 4$$

$$g(3) = 3^2 - 2 \cdot 3 + 4 = 7$$

a seq  $e^-$  ( 3, 4, 7, 12, ..... )

4c  $h: \mathbb{N}^* \rightarrow \mathbb{Q}$

$n \in \mathbb{N}^* \quad e \quad a_n \in \mathbb{Q}$

$n$	$a_n$
1	$2 \cdot \frac{1}{1} = 2$
2	$2 \cdot \frac{1}{2} = 1$
3	$2 \cdot \frac{1}{3} = \frac{2}{3}$

$$a_n = 2 \cdot \frac{1}{n}, n \in \mathbb{N}^*$$

$$a_n = \frac{2}{n}, n \in \mathbb{N}^*$$

a seq  $e' (2, 1, \frac{2}{3}, \dots)$

$$\textcircled{7} \quad a_n = -193 + 3n \quad b_n = 220 - 4n \quad n \in \mathbb{N}^*$$

$$a_n = b_n \Rightarrow -193 + 3n = 220 - 4n$$

$$-413 = -7n \Rightarrow n = 59$$

$$\therefore a_{59} = b_{59} = -16$$

—//—

$$a_n = 3n - 193, n \in \mathbb{N}^*$$

$$b_n = 220 - 4n, n \in \mathbb{N}^*$$

$$a_n = b_n$$

$$-193 + 3n = 220 - 4n$$

$$3n = 413 - 4n$$

$$n = \frac{413 - 4n}{3}, n, n \in \mathbb{N}^*$$

atribuiri valor p/n e calcula  
n

$$m = \frac{413 - 4m}{3}, m, m \in \mathbb{N}^*$$

$$\textcircled{1} m \in \mathbb{N}^* \Rightarrow \frac{413 - 4m}{3} > 0$$

$$\Rightarrow 413 - 4m > 0 \rightarrow -4m > -413$$

$$m < \frac{413}{4} = 103.25 \quad m \in \mathbb{N}^*$$

$$m \in \{1, 2, 3, \dots\}, 103 \text{ candidates}$$

$$m = \frac{411 + 2 - 4m}{3} = \frac{411}{3} + \frac{2 - 4m}{3} = 137 + \frac{2 - 4m}{3}$$

$$M = 137 + \underbrace{\frac{2(1 - 2m)}{3}}_{\text{natural}} \quad m \in \mathbb{N}^*$$

$$a_{135} = b_2, a_{131} = b_5, \text{ etc}$$

m	m
<del>1</del>	<del><math>\frac{409}{3}</math></del>
2	$\frac{405}{3} = 135$
<del>3</del>	<del><math>\frac{401}{3}</math></del>
<del>4</del>	<del><math>\frac{397}{3}</math></del>
5	$\frac{393}{3} = 131$
...	...

$$\underline{\underline{7c}} \quad b_n = 220 - 4n, n \in \mathbb{N}^*$$

$$b_n < 0?$$

$$220 - 4n < 0$$

$$-4n < -220$$

$$n > \frac{220}{4}$$

$$n > 55 \text{ e } n \in \mathbb{N}^*$$

$$b_n < 0, \text{ para } n > 55 \text{ e } n \in \mathbb{N}^*$$

$$n \in \{56, 57, \dots\}$$

1º termo negativo

$$b_{56} = 220 - 4 \cdot 56$$

$$b_{56} = 220 - 224$$

$$\boxed{b_{56} = -4}$$

$$\boxed{-4m < -220} \xrightarrow{\cdot (-1)} 4m > 220 \xrightarrow{\div 4} m > \frac{220}{4}$$

$$2 < 3 \text{ e } 5 > 0 \Rightarrow \underbrace{2 \cdot 5}_{10} < \underbrace{3 \cdot 5}_{15} \quad (v)$$

$$2 < 3 \text{ e } -5 < 0 \Rightarrow \underbrace{2 \cdot (-5)}_{-10} > \underbrace{3 \cdot (-5)}_{-15} \quad (v)$$

$$\boxed{\begin{array}{l} x < y \text{ e } a > 0 \Leftrightarrow ax < ay, a > 0 \\ x < y \text{ e } a < 0 \Leftrightarrow ax > ay, a < 0 \end{array}}$$

$$\begin{array}{c} \text{---|---|---|--->} \\ -3 \quad -2 \quad 2 \quad 3 \\ -3 < -2 \quad \text{e} \quad 3 > 2 \end{array}$$

$$\left| \begin{array}{l} -4m < -220 \\ m > \frac{-220}{-4} \end{array} \right| \xrightarrow{\div (-4)} \left| \begin{array}{l} -4m < -220 \\ m > \frac{220}{4} \end{array} \right|$$

$$\textcircled{8} \quad a_n = 4n^2 - n + 9, \quad n \in \mathbb{N}^*$$

$$a_1 = 4 - 1 + 9 = 12$$

$$a_2 = 4 \cdot 2^2 - 2 + 9 = 23$$

$$a_3 = 4 \cdot 3^2 - 3 + 9 = 42$$

$\Leftrightarrow$

$$(12, 23, a_3, a_4, 104, \dots)$$




# Progressão Aritmética (PA)

Def  $\left\{ \begin{array}{l} a_1 \\ a_n = a_{n-1} + r, n \geq 2 \end{array} \right., n \in \mathbb{N}^* \text{ e } r \in \mathbb{R}$   
 $r$  razão da PA (constante)

Em particular  $a_n - a_{n-1} = r, \forall n \in \mathbb{N}^*, n \geq 2$

①  $(-7, -4, -1, 2, 5, \dots)$  PA de razão  $r = 3$   
 $a_1 = -7$



The diagram shows the sequence  $-7, -4, -1, 2, 5$ . Below the first four terms, there are four horizontal arrows pointing to the right, each labeled  $+3$ . Vertical arrows point from each term to the arrow below it: from  $-7$  to the first  $+3$ , from  $-4$  to the second  $+3$ , from  $-1$  to the third  $+3$ , and from  $2$  to the fourth  $+3$ .

$$\left\{ \begin{array}{l} a_1 = -7 \\ a_n = a_{n-1} + 3 \end{array} \right. \quad \left\{ \begin{array}{l} a_1 < a_2 < a_3 < a_4 < \dots \\ \text{PA crescente} \end{array} \right.$$

$$\textcircled{2} \quad \left\{ \begin{array}{l} a_1 = 5 \\ e \\ a_n = a_{n-1} + 4, n \geq 2 \end{array} \right.$$

$$a_2 = a_1 + 4 = 5 + 4 = 9$$

$$a_3 = a_2 + 4 = 9 + 4 = 13$$

$$a_4 = a_3 + 4 = 13 + 4 = 17$$

$$(5, 9, 13, 17, \dots)$$

$$a_1 = 5 \text{ e } r = 4 > 0$$

PA crescente

$$\textcircled{3} \quad \begin{array}{cccc} -4 & -4 & -4 & -4 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{array}$$

$$(10, 6, 2, -2, -6, \dots)$$

$$\underbrace{a_2 - a_1}_{-4} = \underbrace{a_3 - a_2}_{-4} = \underbrace{a_4 - a_3}_{-4} = \dots = -4$$

$$a_4 - a_3 = -2 - 2 = -4$$

$$a_5 - a_4 = -6 - (-2) = -4$$

$$\text{PA } a_1 = 10 \text{ e } r = -4 < 0$$

e' decrescente

④  $(2, 2, 2, 2, 2, \dots)$  PA de  $a_1 = 2$  e  $r = 0$

$$\begin{cases} a_1 = 2 \\ a_m = a_{m-1} + r, m \geq 2 \end{cases}$$

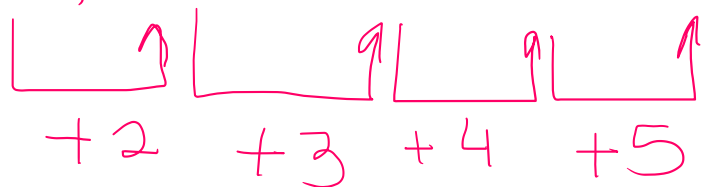
$a_m - a_{m-1} = r$  constante

$$\begin{cases} a_2 - a_1 = 0 \\ a_3 - a_2 = 0 \\ a_4 - a_3 = 0 \\ a_5 - a_4 = 0 \end{cases}$$

PA razão  $r$

- crescente  $r > 0$
- decrescente  $r < 0$
- constante  $r = 0$

⑤  $(1, 3, 6, 10, 15, \dots)$  não é PA



$$a_2 - a_1 = 2$$

$$a_3 - a_2 = 3$$

$$a_4 - a_3 = 4$$

$$a_5 - a_4 = 5$$

etc

não diferentes

$$a_n = a_{n-1} + \underbrace{(n+1)}_{\text{razão constante}}$$

$$(a_1, a_2, a_3, a_4, \dots)$$

Diagram illustrating the sequence  $(a_1, a_2, a_3, a_4, \dots)$  with arrows indicating a constant difference  $+r$ .

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