PA 1º termo a_1 e razão r

$$\left(\overset{a_1}{a_1},\ \overset{a_2}{a_1+1r},\ \overset{a_3}{a_1+2r},\ \overset{a_4}{a_1+3r},\ \ldots,\ \overset{a_n}{a_1+(n-1).r},\ \ldots\right)$$

$+r \quad +r \quad +r$

$$a_n = a_1 + (n-1).r\ ,\ n \in \mathbb{N}^*$$

11) PA $(-33, -29, -25, -21, \ldots)$

$$r = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -29 - (-33) = -29 + 33 = 4$$

$$a_n = -33 + (n-1).4$$

$$a_n = -33 + 4n - 4$$

$$\boxed{a_n = 4n - 37,\ n \in \mathbb{N}^*}$$

$n = 1 \Rightarrow a_1 = 4 - 37 = -33$

$n = 2 \Rightarrow a_2 = 4.2 - 37 = -29$

etc...

28b $\left(2, \frac{7}{3}, \frac{8}{3}, \dots, 18\right)$ PA

$$r = \frac{7}{3} - 2 = \frac{8}{3} - \frac{7}{3} = \frac{1}{3}$$

$$a_n = a_1 + (n-1) \cdot r$$

$$18 = 2 + (n-1) \cdot \frac{1}{3}$$

$$18 - 2 = (n-1) \cdot \frac{1}{3}$$

$$16 \cdot 3 = n - 1$$

$$\boxed{n = 49}$$

Carl F. Gauss (1777 - 1855) alemão

$$S = 1 + 2 + 3 + \cdots + 98 + 99 + 100$$

$$S = 100 + 99 + 98 + \cdots + 3 + 2 + 1 \quad (+)$$

$$2S = \underbrace{101 + 101 + 101 + \cdots + 101 + 101 + 101}_{100 \text{ parcelas}}$$

$$2S = 100 \cdot 101$$

$$S = \frac{100 \cdot 101}{2}$$

$$S = \frac{(\overset{a_1}{1} + \overset{a_{100}}{100}) \cdot \overset{n^{\underline{o}}\ \text{termos}}{\overline{100}}}{2}$$

$$\boxed{S = 5050}$$

seq. finita

termos equidistantes dos extremos

$(a_1, a_2, a_3, \ldots, a_k, a_{k+1}, \ldots, a_{n-k}, \ldots, a_{n-2}, a_{n-1}, a_n)$

termos equidistantes dos extremos

extremos

PA finita 1º termo a_1 e razão r

$$(a_1, a_2, a_3, \ldots, a_{k+1}, \ldots, a_{n-k}, \ldots, a_{n-2}, a_{n-1}, a_n)$$

termos equidistantes dos extremos

$$a_{k+1} = a_1 + \underbrace{(k+1-1)}_{k} \cdot r$$

$$a_k + a_{n-k} = [a_1 + (k+1-1) \cdot r] + [a_1 + (n-k-1) \cdot r]$$

$$a_k + a_{n-k} = a_1 + k \cdot r + a_1 + \underbrace{(n-k-1) \cdot r}$$

$$a_k + a_{n-k} = a_1 + a_1 + (\cancel{k} + n - \cancel{k} - 1) \cdot r$$

$$a_k + a_{n-k} = a_1 + \underbrace{a_1 + (n-1) \cdot r}$$

$$a_k + a_{n-k} = a_1 + a_n$$

Em toda PA finita, a soma de termos equidistantes dos extremos é igual a soma dos extremos.

termo central

$$(3,\ 8,\ 13,\ \overset{\frown}{18},\ 23,\ 28,\ 33)$$

$13 + 23 = 36$

$8 + 28 = 36$

$3 + 33 = 36$

7 termos

$a_4 = 18$

$a_4 + a_4 = 36$

$$S = 3 + 8 + 13 + 18 + 23 + 28 + 33$$
$$S = 33 + 28 + 23 + 18 + 13 + 8 + 3$$

36 36 36 36 36 36 36

$(a_1, a_2, a_3, \ldots, a_n, \ldots)$ PA

S_n: soma dos n primeiros termos da PA

$S_n = a_1 + a_2 + a_3 + \ldots + a_{n-2} + a_{n-1} + a_n$ +

$S_n = a_n + a_{n-1} + a_{n-2} + \ldots + a_3 + a_2 + a_1$

$2S_n = (a_1 + a_n) + \underbrace{(a_2 + a_{n-1})}_{a_1 + a_n} + \underbrace{(a_3 + a_{n-2})}_{a_1 + a_n} + \ldots + \underbrace{(a_{n-1} + a_2)}_{a_1 + a_n} + (a_n + a_1)$

$2S_n = n \cdot (a_1 + a_n)$

$$\boxed{S_n = \frac{(a_1 + a_n) \cdot n}{2}, \ n \in \mathbb{N}^*}$$

Gauss

$$\frac{(1+100) \cdot 100}{2}$$

① PA $(3, 7, 11, 15, \ldots)$ soma dos 20 primeiros termos

$a_1 = 3 \quad r = 4 \quad a_{20} = 3 + 19 \cdot 4 = 79$

$$S_{20} = \frac{(a_1 + a_{20}) \cdot 20}{2} = \frac{(3 + 79) \cdot 20}{2} \Rightarrow \boxed{S_{20} = 820}$$

— // —

② PA $(6, 10, 14, 18, \ldots)$ soma dos n primeiros termos

$a_1 = 6 \quad r = 4 \quad a_n = 6 + (n-1) \cdot 4 \Rightarrow a_n = 6 + 4n - 4$

$a_n = 2 + 4n$

$$S_n = \frac{(6 + 2 + 4n) \cdot n}{2} = \frac{(8 + 4n) \cdot n}{2} = \frac{\cancel{2} \cdot (4 + 2n) \cdot n}{\cancel{2}}$$

$$\boxed{S_n = n \cdot (4 + 2n), \; n \in \mathbb{N}^*}$$

$\boxed{\text{OBS} \quad S_1 = a_1}$

$\sum$ letra grega sigma maiúscula

$$\sum_{i=1}^{7} 2i = \underbrace{(2\cdot 1) + (2\cdot 2) + (2\cdot 3) + (2\cdot 4) + (2\cdot 5) + (2\cdot 6) + (2\cdot 7)}_{\text{soma dos 7 primeiros termos de uma PA}}$$

termo geral da parcela

$$\sum_{i=1}^{7} 2i = \frac{(2+14)\cdot 7}{2} = 56$$

$$\sum_{i=1}^{40}(3i-1) = \underbrace{(3\cdot 1-1)}_{2} + \underbrace{(3\cdot 2-1)}_{5} + \underbrace{(3\cdot 3-1)}_{8} + \dots + \underbrace{(3\cdot 40-1)}_{119}$$

soma dos 40 primeiros termos da PA

$$\sum_{i=1}^{40}(3i-1) = \frac{(2+119)\cdot \cancel{40}^{20}}{\cancel{2}} = 2420$$

— // —

$$a_i = 3i-1$$

$$a_{i+1} = 3(i+1)-1 = 3i+2$$

$$a_{i+1} - a_i = 3i+2-3i+1$$

$$a_{i+1} - a_i = \underset{\text{cte}}{3}, \ \forall i \in \mathbb{N}^*$$

Logo, a seq. é PA

Pag 177 do 37 ao 51

nº exec $51 - 37 + 1 = 15$

$a_n - a_1 = (n-1) \cdot r$

$n - 1 = \dfrac{a_n - a_1}{r}$

de 1 a 10 → 10 nºs

$10 - 1 + 1 = 10$

$a, b \in \mathbb{R}^*$

$$a = b \quad \Big]\, \times a$$

$$a^2 = ab \quad \Big]\, -b^2$$

$$a^2 - b^2 = ab - b^2$$

$$\underbrace{(a-b)}_{=0}(a+b) = \underbrace{(a-b)}_{=0} \cdot b$$

$$a + b = b$$

Como $a = b$, então

$$2b = 1b$$

$$2 = 1$$

absurdo!

sofisma ou falácia

$$\boxed{2}x = \boxed{2}y \Rightarrow x = y$$

$$2 \neq 0$$

Propriedade do cancelamento

$$x + a = y + a \Leftrightarrow x = y$$

$$ab = ac \text{ e } a \neq 0 \Rightarrow b = c$$

$$0 \cdot 2 = 0 \cdot 3 \not\Rightarrow 2 = 3$$

ex $x^2 = x \Rightarrow \underbrace{x \cdot \cancel{x} = \cancel{x}\, 1}_{\text{não!}}$

$$x^2 - x = 0$$

$$x \cdot (x-1) = 0$$

$$x = 0 \text{ ou } x - 1 = 0$$

$$x = 0 \text{ ou } x = 1$$

$$36+16=16+36$$

$$\underbrace{36}-48+16=16-48+36$$

$$6^2-2\cdot6\cdot4+4^2=4^2-2\cdot6\cdot4+6^2$$

$$\cancel{(6-4)^2}=\cancel{(4-6)^2}$$ erro!

$$(2)^2=(-2)^2$$

$$6-4=4-6$$

$$2=-2$$

$$a^2=b^2 \Rightarrow a=b \text{ ou } a=-b$$

$$2 = \sqrt[4]{16} = \sqrt[4]{2^4} = \sqrt[\cancel{4}]{(-2)^{\cancel{4}}} \neq -2$$

$n, p \in \mathbb{N}^*$ e $m \in \mathbb{Z}$

$$\sqrt[m \cdot \cancel{p}]{a^{m \cdot \cancel{p}}} = \sqrt[n]{a^m} \quad \text{e} \quad a \geq 0$$

$$\sqrt[4]{16} = \sqrt[\cancel{4}]{2^{\cancel{4}}} = 2$$