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7i $\underbrace{0,9999\ldots}_{\text{dízima periódica}} = 1$ (V)

$$x = 0,999\ldots$$
$$10x = 9,999\ldots$$
$$9x = 9$$
$$\boxed{x = 1}$$

$$1,9999\ldots = 2$$
$$2,9999\ldots = 3$$

7j $\underbrace{0,999}_{\text{decimal exato}} = 1$ (F)

$$\frac{999}{1000}$$

(10) $y = (\underbrace{2,666\ldots\ldots}_{a} : \underbrace{1,666\ldots\ldots}_{b}) + \dfrac{2-\frac{1}{2}}{-3\cdot\frac{1}{2}}$

$a = 2,666\ldots\ldots$

$10a = 26,666\ldots$ (−)

$9a = 26-2$

$a = \dfrac{24}{9} = \dfrac{8}{3}$

$b = 1,666\ldots\ldots$

$10b = 16,666\ldots$

$9b = 16-1$

$b = \dfrac{15}{9} = \dfrac{5}{3}$

$y = \dfrac{\frac{8}{\cancel{3}}}{\frac{5}{\cancel{3}}} + \dfrac{\frac{4-1}{\cancel{2}}}{-\frac{3}{\cancel{2}}} \Rightarrow y = \dfrac{8}{5} - 1 = \dfrac{8-5}{5} = \dfrac{3}{5}$

11) $\frac{p}{z} = \dfrac{1+\frac{1}{5}}{2-\frac{2}{5}} = \dfrac{\frac{6}{5}}{\frac{8}{5}} = \frac{6}{8} = \frac{3}{4}$ fração irredutível

14) a) $0,2 \cdot 1,\overline{3} + 0,08 = \frac{\cancel{2}^{1}}{\cancel{10}_{5}} \cdot \frac{4}{3} + \frac{8}{100} = \frac{4}{15} + \frac{2}{25} = \frac{4 \cdot 5 + 2 \cdot 3}{75} =$

15) $\frac{19}{20}$, $\sqrt{2}$, $\sqrt{3}$, 1, $\sqrt{5}$ e $1,\overline{2} = \frac{11}{9}$

$1 < \frac{11}{9} < 2$

$\sqrt{2} = 1,4 \ldots\ldots$

$\sqrt{3} = 1,7 \ldots\ldots$

$\frac{19}{20} < 1 < 1,\overline{2} < \sqrt{2} < \sqrt{3} < \sqrt{5}$

$\underbrace{4}_{\text{pos}} < \underbrace{5}_{\text{pos}} < \underbrace{9}_{\text{pos}} \Rightarrow \sqrt{4} < \sqrt{5} < \sqrt{9} \Rightarrow 2 < \sqrt{5} < 3$

$4 < 5 < 6,25 \Rightarrow 2 < \sqrt{5} < 2,5$

$4,41 < 5 < 5,76 \Rightarrow 2,1 < \sqrt{5} < 2,4$

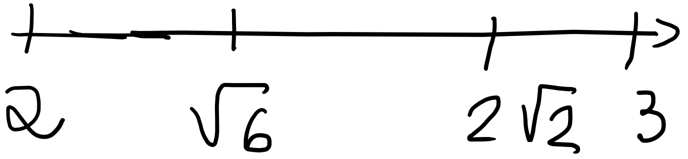
$\frac{625}{100} = \frac{25^2}{10^2}$

$2,1^2 = 4,41$

$2,4^2 = 5,76$

17) $\sqrt{6}$ $\qquad 2\sqrt{2} \cong 2{,}8 \qquad 5{,}76 < 6 < 6{,}25$

$2{,}4 < \sqrt{6} < 2{,}5$



a) 2,6 e 2,5

b) $\dfrac{\sqrt{6} + 2\sqrt{2}}{2}$ e $\underbrace{\sqrt{6} + \dfrac{1}{10}}_{\text{nº irracional}}$

c) não existe

18g) $\underbrace{(\sqrt{2}+1)}_{\text{irrac}} \cdot \underbrace{(\sqrt{2}-1)}_{\text{irrac}} = (\sqrt{2})^2 - 1^2 = 2 - 1 = \underbrace{1}_{\text{rac}}$

19 $\quad x \in \mathbb{R}^*$

a) $\underbrace{\text{o oposto de } x}_{-x}$ é negativo (F)

$-(-2) = 2 > 0$

$x > 0 \Rightarrow -x < 0$

$x < 0 \Rightarrow -x > 0$

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b) $x^2 > x$ $\quad$ F

$\frac{1}{4} = \left(\frac{1}{2}\right)^2 > \frac{1}{2}$? Não

$x = -2$

$(-2)^2 > -2$ (V)

$\left(-\frac{1}{2}\right)^2 > -\frac{1}{2}$ (V)

$x > 1 \Rightarrow x^2 > x$

$0 < x < 1 \Rightarrow 0 < x^2 < x < 1$

c) $2x < 3x \, , \forall x \in \mathbb{R}^*$

$x = -2$

$2 \cdot (-2) < 3 \cdot (-2)$ (F)

d) o inverso de x <u>pode ser</u> maior que x?

$x = 2 \qquad \frac{1}{x} = \frac{1}{2} \Rightarrow \frac{1}{2} < 2$

$x = \frac{1}{3} \qquad \frac{1}{x} = 3 \qquad \frac{1}{\frac{1}{3}} > \frac{1}{3}$ (V)

$0 < x < 1 \Rightarrow \frac{1}{x} > 1$

e) $x + 2 < x$ (F)

$x - x < -2$

$\underbrace{0 \cdot x}_{0} < -2$ F

(20)

b) $\underbrace{(\sqrt{2}+3)}_{\text{irrac}} + \underbrace{(-\sqrt{2}+5)}_{\text{irrac}} = \underbrace{8}_{\text{rac}}$ **pode ser**

$q, p \in \mathbb{Q}$ e $\alpha \in \mathbb{R} - \mathbb{Q}$ $\underbrace{(p+\alpha)}_{\text{irrac}} + \underbrace{(q-\alpha)}_{\text{irrac}} = \underbrace{p+q}_{\text{rac}}$

c) $p, q \in \mathbb{Q}$ e $\alpha \in \mathbb{R} - \mathbb{Q}$

supos $\underbrace{p}_{\text{rac}} + \underbrace{\alpha}_{\text{irrac}} = \underbrace{q}_{\text{rac}} \Rightarrow \underbrace{\alpha}_{\text{irrac}} = \underbrace{q - p}_{\text{rac}}$ absurdo

não pode ocorrer

e) $\alpha, \beta \in \mathbb{R} - \mathbb{Q}$ $\dfrac{\alpha}{\beta}$ irrac (F) $\dfrac{\sqrt{2}}{3\sqrt{2}} = \dfrac{1}{3}$

(21) $x = \frac{1}{0{,}05} = \frac{1}{\frac{5}{100}} = \frac{1}{\frac{1}{20}} = 20$ $\qquad$ $y = \frac{2}{0{,}2} = \frac{2}{\frac{2}{10}} = 10$

$A = \sqrt{\frac{20}{10}} = \sqrt{2}$ $\qquad$ $B = \sqrt{20 - \frac{20}{10}} = \sqrt{18} = \sqrt{2 \cdot 3^2} = 3\sqrt{2}$

$\therefore E = 1\sqrt{2} + 3\sqrt{2} = (1+3)\sqrt{2} = 4\sqrt{2}$

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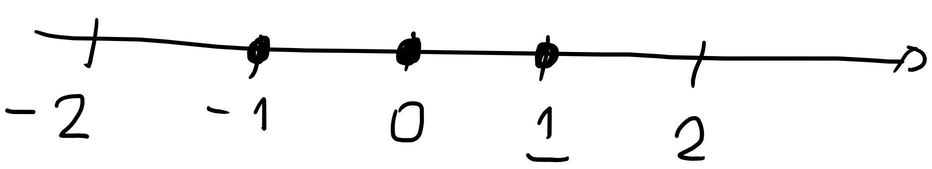
22e $E = \{ x \in \mathbb{Z} \mid |x| < 2 \} = \{-1, 0, 1\}$



23 b) $\{x \in \mathbb{R}_- \mid x^4 = 16\} = \{-2\}$

$2^4 = 16 \qquad (-2)^4 = 16$

$\boxed{x^2 \geqslant 0, \forall x \in \mathbb{R}}$

$x = 0 \Rightarrow 0^2 \geqslant 0$

$x = 3 \Rightarrow 3^2 \geqslant 0$

$x = -2 \Rightarrow (-2)^2 = 4 \geqslant 0$

$\underline{x^4 \geqslant 0, \forall x \in \mathbb{R}}$

$x \in \mathbb{R}$

x^{par}

$(-2)^2 = 4$

$2^2 = 4$

$2^2 = (-2)^2$

$x^4 = 16 \Rightarrow x = \sqrt[4]{16}$ ou $x = -\sqrt[4]{16} \Rightarrow x = 2$ ou $x = -2$

$x = \pm\sqrt[4]{16} \Rightarrow x = \pm 2$

$a > 0$, $\sqrt[2]{a}$ este símbolo representa um único nº real que é positivo

analogamente $\sqrt[4]{a}$ representa um único nº real que é positivo.

g) $\{x \in \mathbb{Q} \mid \frac{1}{x} = 2\} = \{\frac{1}{2}\}$

Axiomas $\mathbb{R}$ adição $(+)$ multiplicação $(\cdot$ ou $\times)$

$a, b, c \in \mathbb{R}$

① Comutativa

$a+b=b+a$

$ab=ba$

② Associativas

$(a+b)+c=a+(b+c)$

$(ab)c=a(bc)$

③ Elemento neutro

$a+0=a$

$1\cdot a=a$

4.1 Elemento oposto

$\forall a\in\mathbb{R}, \exists b\in\mathbb{R} \mid a+b=0$

b: oposto de a

notação: $b = \underbrace{-a}_{\text{oposto de } a}$

o oposto de zero é zero, $0+0=0$

o oposto de 2 é -2, pois $2+(-2)=0$

o oposto de -3 é 3, pois $-3+3=0$

Subtração

$$\boxed{a-b=a+(-b)}$$

$2-3=2+(-3)=$

$=-3+2$

4.2 Elemento inverso

$\forall a \in \mathbb{R}^*, \exists \underline{b} \in \mathbb{R}^* \mid ab = \underline{1}$

$\hookrightarrow$ inverso de a

$$b = a^{-1} = \frac{1}{a}$$

- $\frac{2}{3}$ é o inverso de $\frac{3}{2}$, pois

$$\frac{2}{3} \cdot \frac{3}{2} = \underline{1}$$

- $\frac{1}{4}$ é o inverso de 4, pois

$$\frac{1}{4} \cdot 4 = 1$$

- $\sqrt{2}-1$ é o inverso de $\sqrt{2}+1$, pois $(\sqrt{2}-1)(\sqrt{2}+1) = \underline{1}$

- $$\frac{1}{\frac{2}{3}} = 1 \cdot \frac{3}{2} = \frac{3}{2}$$

- o inverso de $\sqrt{2}-1$ é

$$\frac{1}{\sqrt{2}-1} = \frac{1}{(\sqrt{2}-1)} \cdot \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \sqrt{2}+1$$

$$\frac{1}{\sqrt{2}-1} = \sqrt{2}+1$$

Divisão em $\mathbb{R}$

$a \in \mathbb{R}$ e $b \in \mathbb{R}^*$

$$a : b = \frac{a}{b} = a \cdot \frac{1}{b}$$

⑤ Distributiva

$$a \cdot (b+c) = a \cdot b + a \cdot c$$

$\longrightarrow$ distributiva

colocar em evidência $\longleftarrow$

Testes

e

Complementares