

MATH 145 Calculus for Engineering and Science I

Recitation 3

October 27th, 2025

1. Find the following limits

1. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x + 1}$

2. $\lim_{x \rightarrow 3} \frac{x^3 - 8}{x - 2}$

3. $\lim_{h \rightarrow 0} \frac{\sqrt{a+h} - \sqrt{a}}{h}$

2. In each of the following cases, determine the limit ℓ for the given a , and prove that it is the limit by showing how to find a δ such that $|f(x) - \ell| < \epsilon$ for all x satisfying $|x - a| < \delta$.

1. $f(x) = x[3 - \cos(x^2)]$, $a = 0$

2. $f(x) = x^2 + 5x - 2$, $a = 2$

3. $f(x) = \frac{x}{2 - \sin^2 x}$, $a = 0$

3. Find the following limits in terms of the number $\alpha = \lim_{x \rightarrow 0} (\sin x)/x$.

1. $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

2. $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$

4. Prove that $\lim_{x \rightarrow a} f(x) = \lim_{h \rightarrow 0} f(a + h)$.

5. For which of the following functions f is there a continuous function F with domain \mathbb{R} such that $F(x) = f(x)$ for all x in the domain of f ?

1. $f(x) = \frac{x^2 - 4}{x - 2}$

2. $f(x) = \frac{|x|}{x}$

6. Suppose that f is continuous at a and $f(a) = 0$. Prove that if $\alpha \neq 0$, then $f + \alpha$ is nonzero in some open interval containing a .