Cal 2

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1 Applications of integration

Area between curve:

$$Area = \int_a^b \mid f(x) - g(x) \mid dx$$

Volume of a solid:

$$Volume = \int_a^b A(x)dx$$

A(x) is the cross-sectional area of the solid at x.
Volume of f(x) rotated around y - axis:

$$Volume = \int_a^b 2\pi x f(x)dx$$

Length of curve:

$$Length = \int_a^b \sqrt{1 + (f'(x))^2}dx$$

Average value of f(x):

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$

Calc center of mass: *Split f(x) into many small pieces, each piece has* Δx *and f(x*) is the height of that piece.*

$$My = \int_a^b \rho x f(x)dx$$

$\rho \Delta x f(x)$ is the mass of that piece and dx is approximately to Δx when in integration.

$$Mx = \int_a^b \rho f(x)\frac{1}{2}f(x)dx$$

Center of mass:

$$(\frac{My}{M}, \frac{Mx}{M})$$

Consumer surplus:

$$CS = \int_0^Q (D(p) - K)dp$$

D(p) is the demand function, K = D(Q) is the cost of production.

2 Polar coordinates

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

$$r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$

Area in polar coordinates:

$$Area = \frac{1}{2}\int_\alpha^\beta r^2 d\theta$$

Arc length in polar coordinates:

$$Length = \int_\alpha^\beta \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta = \int_\alpha^\beta \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2} d\theta$$

Parabola: $x^2 = 4py$ *or* $y^2 = 4px$
Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
or $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
First case is horizontal, second case is vertical.
Conic section in polar coordinates: $r = \frac{ed}{1 \pm e\cos\theta}$
or $r = \frac{ed}{1 \pm e\sin\theta}$
e is the eccentricity, d is the distance from the directrix to the focus. if conic section synmetric with x-axis, use cos, if synmetric with y-axis, use sin. if the curve is go to the opposite direction, use + . Similarity, the directrix x (or y) = + d.
Distance between two points in polar coordinates:

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$

3 Differential equations

Euler method:

$$y_{n+1} = y_n + hf(x_n, y_n)$$