INTERNATIONAL STANDARD

ISO 17123-3

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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 3: Theodolites

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 3: Théodolites

Reference number
ISO 17123-3:2001(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this part of ISO 17123 may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 17123-3 was prepared by Technical Committee ISO/TC 172, *Optics and optical instruments*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This first edition of ISO 17123-3 cancels and replaces ISO 8322-4:1991 and ISO 12857-2:1997, which have been technically revised.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*:

— *Part 1: Theory*

— *Part 2: Levels*

— *Part 3: Theodolites*

— *Part 4: Electro-optical distance meters (EDM instruments)*

— *Part 5: Electronic tacheometers*

— *Part 6: Rotating lasers*

— *Part 7: Optical plumbing instruments*

Annexes A, B and C of this part of ISO 17123 are for information only.

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 3: Theodolites

1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of theodolites and their ancillary equipment when used in building and surveying measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

This part of ISO 17123 can be thought of as one of the first steps in the process of evaluating the uncertainty of a measurement (more specifically a measurand). The uncertainty of a result of a measurement is dependent on a number of factors. These include among others: repeatability (precision), reproducibility (between day repeatability), traceability (an unbroken chain to national standards) and a thorough assessment of all possible error sources, as prescribed by the ISO Guide to the expression of uncertainty in measurement (GUM).

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposefully designed to minimize atmospheric influences.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 17123. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 17123 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

GUM, *Guide to the expression of uncertainty in measurement*

VIM, *International vocabulary of basic and general terms in metrology*

3 Terms and definitions

For the purposes of this part of ISO 17123, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, GUM and VIM apply.

4 General

4.1 Requirements

Before commencing surveying, it is important that the operator investigates that the precision in use of the measuring equipment is appropriate to the intended measuring task.

The theodolite and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's handbook, and used with tripods as recommended by the manufacturer.

The results of these tests are influenced by meteorological conditions, especially by the gradient of temperature. An overcast sky and low wind speed guarantee the most favourable weather conditions. The particular conditions to be taken into account may vary depending on where the tasks are to be undertaken. Note should also be taken of the actual weather conditions at the time of measurement and the type of surface above which the measurements are made. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high, and therefore they are not practicable for most users. In addition, laboratory tests yield precisions much higher than those that can be obtained under field conditions.

The measure of precision of theodolites is expressed in terms of the experimental standard deviation (root mean square error) of a horizontal direction (HZ), observed once in both face positions of the telescope or of a vertical angle (V) observed once in both face positions of the telescope.

This part of ISO 17123 describes two different field procedures both for the measurement of horizontal directions and vertical angles as given in clauses 5 and 6. The operator shall choose the procedure which is most relevant to the project's particular requirements.

4.2 Procedure 1: Simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given theodolite is within the specified permitted deviation, according to ISO 4463-1.

This test procedure is normally intended for checking whether the measure of precision in use of the measuring equipment in conjunction with its operator is appropriate to carry out the measurement to the specified measure of precision requirement.

This simplified test procedure is based on a limited number of measurements and, therefore, the experimental standard deviation calculated can only be indicative of the order of the measure of precision achievable in common use. If a more precise assessment of the measuring instrument and its ancillary equipment under field conditions is required, it is recommended to adopt the more rigorous full test procedure. Statistical tests based on the simplified test procedure are not proposed.

4.3 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of a particular theodolite and its ancillary equipment under field conditions.

The full test procedure is intended for determining the experimental standard deviation of a horizontal direction or a vertical angle observed once in both face positions of the telescope:

$s_{\text{ISO-THEO-HZ}}$ and $s_{\text{ISO-THEO-V}}$

Further, this procedure may be used to determine:

— the measure of precision in use of theodolites by a single survey team with a single instrument and its ancillary equipment at a given time;

— the measure of precision in use of a single instrument over time;

— the measure of precision in use of each of several theodolites in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviation, s, obtained belongs to the population of the instrumentation's theoretical standard deviation, σ, whether two tested samples belong to the same population and whether the vertical index error, δ, is equal to zero or has not changed (see 5.4 and 6.4).

5 Measurement of horizontal directions

5.1 Configuration of the test field

Fixed targets (4 targets for the simplified test procedure and 5 targets for the full test procedure) shall be set up located approximately in the same horizontal plane as the instrument, between 100 m to 250 m away, and situated at intervals around the horizon as regular as possible. Targets shall be used which can be observed unmistakably, preferably target plates.

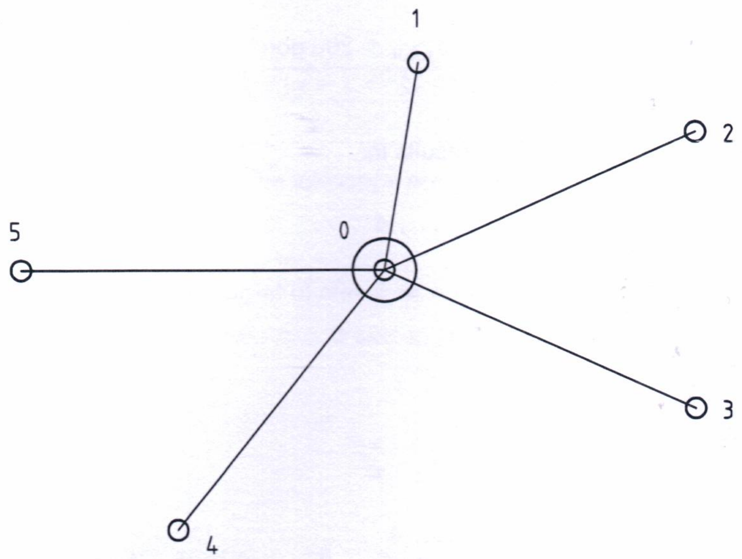

Figure 1 — Test configuration for measurement of horizontal directions

5.2 Measurements

For the simplified test procedure, $m = 1$ series of measurements shall be taken.

For the full test procedure, $m = 4$ series of measurements shall be taken under various but not extreme weather conditions.

Each series (i) of measurements shall consist of $n = 3$ sets (j) of directions to the $t = 4$ or $t = 5$ targets (k).

For the full test procedure, when setting up the theodolite for different series of measurements, special care shall be taken when centring above the ground point. Achievable accuracies of centring expressed in terms of experimental standard deviations are the following:

— plumb bob: 1 mm to 2 mm (worse in windy weather),

— optical or laser plummet: 0,5 mm (the adjustment shall be checked according to the manufacturer's handbook),

— centring rod: 1 mm.

NOTE With targets at 100 m distance, a miscentring of 2 mm could affect the observed direction by up to $4''$(1,3 mgon). The shorter the distance, the greater the effect.

The targets shall be observed in each set in face position I of the telescope in clockwise sequence, and in face position II of the telescope in anticlockwise sequence. The graduated circle shall be changed by 60° (67 gon) after each set. If physical rotation of the graduated circle is not possible, as e.g. for electronic theodolites, the lower part of the theodolite may be turned by approximately 120° (133 gon) on the tribrach.

5.3 Calculation

5.3.1 Simplified test procedure

The evaluation of the measured values is a least squares adjustment of observation equations. One direction is marked by $x_{j,k,\mathrm{I}}$ or $x_{j,k,\mathrm{II}}$, the index j being the number of the set and the index k being the number of the target. I and II indicate the face position of the telescope.

First of all, the mean values of the readings in both face positions I and II of the telescope are calculated:

$$x_{j,k} = \frac{x_{j,k,\mathrm{I}} + x_{j,k,\mathrm{II}} \pm 180^\circ}{2} \left(= \frac{x_{j,k,\mathrm{I}} + x_{j,k,\mathrm{II}} \pm 200 \text{ gon}}{2} \right); \quad j = 1, 2, 3; \quad k = 1, \ldots, 4 \tag{1}$$

Reduction into the direction of the target No. 1 results in:

$$x'_{j,k} = x_{j,k} - x_{j,1}; \quad j = 1, 2, 3; \quad k = 1, \ldots, 4 \tag{2}$$

The mean values of the directions resulting from $n = 3$ sets to target No. k are:

$$\overline{x}_k = \frac{x'_{1,k} + x'_{2,k} + x'_{3,k}}{3}; \quad k = 1, \ldots, 4 \tag{3}$$

From the differences

$$d_{j,k} = \overline{x}_k - x'_{j,k}; \quad j = 1, 2, 3; \quad k = 1, \ldots, 4 \tag{4}$$

for each set of measurements the arithmetic mean values result in:

$$\overline{d}_j = \frac{d_{j,1} + d_{j,2} + d_{j,3} + d_{j,4}}{4}; \quad j = 1, 2, 3 \tag{5}$$

from which the residuals result:

$$r_{j,k} = d_{j,k} - \overline{d}_j; \quad j = 1, 2, 3; \quad k = 1, \ldots, 4 \tag{6}$$

Except for the rounding errors, each set must meet the condition:

$$\sum_{k=1}^{4} r_{j,k} = 0; \quad j = 1, 2, 3 \tag{7}$$

The sum of squares of the residuals is:

$$\Sigma r^2 = \sum_{j=1}^{3}\sum_{k=1}^{4} r_{j,k}^2 \tag{8}$$

For $n = 3$ sets of directions to $t = 4$ targets the number of degrees of freedom is:

$$\nu = (3-1) \times (4-1) = 6 \tag{9}$$

and the experimental standard deviation s of a direction $x_{j,k}$ taken in one set observed in both face positions of the telescope amounts to:

$$s = \sqrt{\frac{\Sigma r^2}{\nu}} = \sqrt{\frac{\Sigma r^2}{6}} \tag{10}$$

5.3.2 Full test procedure

The evaluation of the measured values is an adjustment of observation equations. Within the i^{th} series of measurements, one direction is marked by $x_{j,k,\text{I}}$ or $x_{j,k,\text{II}}$, the index j being the number of the set and the index k being the target. I and II indicate the face position of the telescope. Each of the $m = 4$ series of measurements shall be evaluated separately.

First of all, the mean values

$$x_{j,k} = \frac{x_{j,k,\text{I}} + x_{j,k,\text{II}} \pm 180^\circ}{2} \left(= \frac{x_{j,k,\text{I}} + x_{j,k,\text{II}} \pm 200 \text{ gon}}{2} \right); \quad j = 1, 2, 3; \quad k = 1, \ldots, 5 \tag{11}$$

of the readings in both face positions I and II of the telescope are calculated. Reduction into the direction of the target No. 1 results in:

$$x'_{j,k} = x_{j,k} - x_{j,1}; \quad j = 1, 2, 3; \quad k = 1, \ldots, 5 \tag{12}$$

The mean values of the directions resulting from $n = 3$ sets to target No. k are:

$$\overline{x}_k = \frac{x'_{1,k} + x'_{2,k} + x'_{3,k}}{3}; \quad k = 1, \ldots, 5 \tag{13}$$

From the differences

$$d_{j,k} = \overline{x}_k - x'_{j,k}; \quad j = 1, 2, 3; \quad k = 1, \ldots, 5 \tag{14}$$

for each set of measurements, the arithmetic mean values result in:

$$\overline{d}_j = \frac{d_{j,1} + d_{j,2} + d_{j,3} + d_{j,4} + d_{j,5}}{5}; \quad j = 1, 2, 3 \tag{15}$$

from which the residuals result:

$$r_{j,k} = d_{j,k} - \overline{d}_j; \quad j = 1, 2, 3; \quad k = 1, \ldots, 5 \tag{16}$$

Except for rounding errors, each set must meet the condition:

$$\sum_{k=1}^{5} r_{j,k} = 0; \quad j = 1,\ 2,\ 3 \tag{17}$$

The sum of squares of the residuals of the i^{th} series of measurements is:

$$\Sigma r_i^2 = \sum_{j=1}^{3}\sum_{k=1}^{5} r_{j,k}^2 \tag{18}$$

For $n = 3$ sets of directions to $t = 5$ targets for each series the number of degrees of freedom is:

$$\nu_i = (3 - 1) \times (5 - 1) = 8 \tag{19}$$

and the experimental standard deviation s_i of a direction $x_{j,k}$ taken in one set observed in both face positions of the telescope, valid for the i^{th} series of measurements amounts to:

$$s_i = \sqrt{\frac{\Sigma r_i^2}{\nu_i}} = \sqrt{\frac{\Sigma r_i^2}{8}} \tag{20}$$

The experimental standard deviation, s, of a horizontal direction observed in one set (arithmetic mean of the readings in both face positions of the telescope) according to this part of ISO 17123, calculated from all $m = 4$ series of measurements at a degree of freedom of

$$\nu = 4 \times \nu_i = 32 \tag{21}$$

amounts to:

$$s = \sqrt{\frac{\sum_{i=1}^{4} \Sigma r_i^2}{\nu}} = \sqrt{\frac{\sum_{i=1}^{4} \Sigma r_i^2}{32}} = \sqrt{\frac{\sum_{i=1}^{4} s_i^2}{4}} \tag{22}$$

$$s_{\text{ISO-THEO-HZ}} = s \tag{23}$$

5.4 Statistical tests

5.4.1 General

Statistical tests are recommended for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using the experimental standard deviation, s, of a horizontal direction observed in one set in both face positions of the telescope in order to answer the following questions:

a) Is the calculated experimental standard deviation, s, smaller than the value, σ, stated by the manufacturer or smaller than another predetermined value, σ?

b) Do two experimental standard deviations, s and $\widetilde{s}$, as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same degree of freedom, ν?

The experimental standard deviations, s and $\widetilde{s}$, may be obtained from:

— two samples of measurements by the same instrument but different observers;

— two samples of measurements by the same instrument at different times;

— two samples of measurements by different instruments.

For the following tests, a confidence level of $1 - \alpha = 0{,}95$ and, according to the design of the measurements, a number of degrees of freedom of $\nu = 32$ is assumed.

Table 1 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s \leqslant \sigma$	$s > \sigma$
b)	$\sigma = \widetilde{\sigma}$	$\sigma \neq \widetilde{\sigma}$

5.4.2 Question a)

The null hypothesis stating that the experimental standard deviation, s, of a horizontal direction observed in both positions is smaller than or equal to a theoretical or a predetermined value, σ, is not rejected if the following condition is fulfilled:

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu)}{\nu}} \tag{24}$$

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{0,95}(32)}{32}} \tag{25}$$

$$\chi^2_{0,95}(32) = 46{,}19 \tag{26}$$

$$s \leqslant \sigma \times \sqrt{\frac{46{,}19}{32}} \tag{27}$$

$$s \leqslant \sigma \times 1{,}20 \tag{28}$$

Otherwise, the null hypothesis is rejected.

5.4.3 Question b)

In the case of two different samples, a test indicates whether the experimental standard deviations, s and $\widetilde{s}$, belong to the same population. The corresponding null hypothesis, $\sigma = \widetilde{\sigma}$, is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\nu,\nu)} \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant F_{1-\alpha/2}(\nu,\nu) \tag{29}$$

$$\frac{1}{F_{0,975}(32{,}32)} \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant F_{0,975}(32{,}32) \tag{30}$$

$$F_{0,975}(32{,}32) = 2{,}02 \tag{31}$$

$$0{,}49 \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant 2{,}02 \tag{32}$$

Otherwise, the null hypothesis is rejected.

The degree of freedom and, thus, the corresponding test values $\chi^2_{1-\alpha}(\nu)$ and $F_{1-\alpha/2}(\nu,\nu)$ (taken from reference books on statistics) change if a different number of measurements is analysed.

6 Measurement of vertical angles

6.1 Configuration of the test field

The theodolite shall be set up in a distance approximately 50 m from a high building. At this building, well defined points (parts of windows, corners of bricks, parts of antennas, etc.) or targets fixed at a wall shall be selected or set up to cover a range of the vertical angle of approximately 30° (see Figure 2).

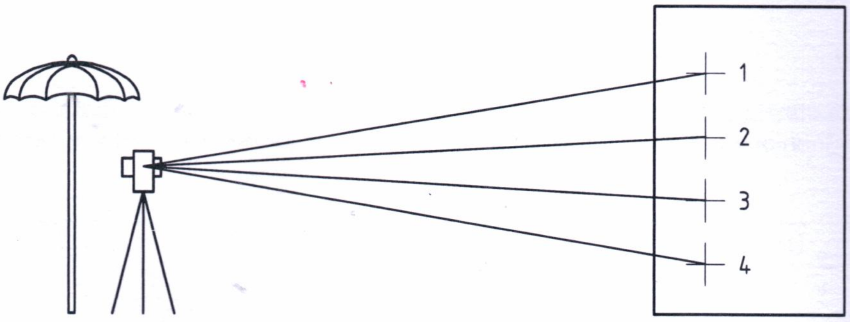

Figure 2 — Test configuration for measurement of vertical angles

6.2 Measurements

Before commencing the measurements, allow the instrument to acclimatize to the ambient temperature. The time required is about two minutes per degree Celsius temperature difference.

For the simplified test procedure, $m = 1$ series of measurements, $x_{j,k}$, shall be taken. This series of measurements shall consist of $n = 3$ sets (j) of directions to the $t = 4$ targets (k).

For the full test procedure, $m = 4$ series of measurements (i) shall be taken under various but not extreme weather conditions. Each series of measurements shall consist of $n = 3$ sets (j) of directions to the $t = 4$ targets (k).

The $t = 4$ targets shall be observed in each of the $n = 3$ sets in face position I of the telescope in the sequence from target No. 1 to target No. 4, and in the same set in face position II of the telescope in the sequence from target No. 4 to target No. 1.

6.3 Calculation

The evaluation of the measured values is a least squares adjustment of observation equations. Within the i^{th} series of measurements, one vertical angle (normally zenith angle) is marked by $x_{j,k,\text{I}}$ or $x_{j,k,\text{II}}$, the index k being the number of the target. I and II indicate the face positions of the telescope. In the full test procedure, each of the $m = 4$ series of measurements is evaluated separately.

First of all, the mean values

$$x'_{j,k} = \frac{x_{j,k,\text{I}} - x_{j,k,\text{II}} + 360^\circ}{2} \left(= \frac{x_{j,k,\text{I}} - x_{j,k,\text{II}} + 400\ \text{gon}}{2} \right); \quad j = 1, 2, 3; \quad k = 1, ..., 4 \qquad (33)$$

of the readings in both face positions I and II of the telescope are calculated. These values are not affected by the vertical index error, δ_i. The vertical index error, δ_i, shall be calculated for each series of measurements separately (recommended for the full test procedure only):

$$\delta_i = \frac{1}{n \times t} \sum_{j=1}^{3} \sum_{k=1}^{4} \frac{x_{j,k,\mathrm{I}} + x_{j,k,\mathrm{II}} - 360^\circ}{2} \left(= \frac{1}{n \times t} \sum_{j=1}^{3} \sum_{k=1}^{4} \frac{x_{j,k,\mathrm{I}} + x_{j,k,\mathrm{II}} - 400 \text{ gon}}{2} \right) \tag{34}$$

$$\delta = \frac{\sum_{i=1}^{4} \delta_i}{4}$$

The mean values of the vertical angles resulting from $n = 3$ sets to target No. k are:

$$\overline{x}_k \frac{x'_{1,k} + x'_{2,k} + x'_{3,k}}{3}; \quad k = 1, \ldots, 4 \tag{35}$$

The residuals result

$$r_{j,k} = x'_{j,k} - \overline{x}_k; \quad j = 1, 2, 3; \quad k = 1, \ldots, 4 \tag{36}$$

Except for rounding errors, the residuals of all sets shall meet the condition:

$$\sum_{j=1}^{3} \sum_{k=1}^{4} r_{j,k} = 0 \tag{37}$$

The sum of squares of the residuals of the i^{th} series of measurements is:

$$\Sigma r_i^2 = \sum_{j=1}^{3} \sum_{k=1}^{4} r_{j,k}^2 \tag{38}$$

For $n = 3$ sets of vertical angles to $t = 4$ targets, in each case the number of degrees of freedom is:

$$\nu_i = (3 - 1) \times 4 = 8 \tag{39}$$

and the experimental standard deviation, s_i, of a vertical angle, $x'_{j,k}$, observed in one set in both face positions of the telescope, valid for the i^{th} series of measurements amounts to:

$$s_i = \sqrt{\frac{\Sigma r_i^2}{\nu_i}} = \sqrt{\frac{\Sigma r_i^2}{8}} \tag{40}$$

The following equations (41) and (42) apply only to the simplified test procedure:

$$\nu = \nu_1 \tag{41}$$

$$s = s_1 \tag{42}$$

The following equations (43) to (59) apply only to the full test procedure:

For the experimental standard deviation, s, calculated from all $m = 4$ series of measurements, the number of degrees of freedom is:

$$\nu = 4 \times \nu_i = 32 \tag{43}$$

and the experimental standard deviation of a vertical angle observed in both face positions, calculated from all $m = 4$ series of measurements, is:

$$s = \sqrt{\frac{\sum\limits_{i=1}^{4} \Sigma r_i^2}{\nu}} = \sqrt{\frac{\sum\limits_{i=1}^{4} \Sigma r_i^2}{32}} = \sqrt{\frac{\sum\limits_{i=1}^{4} s_i^2}{4}} \tag{44}$$

$$s_{\text{ISO-THEO-V}} = s \tag{45}$$

6.4 Statistical tests

6.4.1 General

Statistical tests are recommended for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using

— the experimental standard deviation, s, of a vertical angle observed in both face positions, and

— the vertical index error, δ, (orientation of the vertical circle) and its experimental standard deviation, s_δ

in order to answer the following questions (see Table 2):

a) Is the calculated experimental standard deviation, s, smaller than a corresponding value, σ, stated by the manufacturer or smaller than another predetermined value, σ?

b) Do two experimental standard deviations, s and $\widetilde{s}$, as determined from two different samples of measurements, belong to the same population, assuming that both samples have the same number of degrees of freedom, ν?

   The experimental standard deviations, s and $\widetilde{s}$, may be obtained from:

   — two samples of measurements by the same instrument but different observers;

   — two samples of measurements by the same instrument at different times;

   — two samples of measurements by different instruments.

c) Is the vertical index error, δ, equal to zero?

For the following tests, a confidence level of $1 - \alpha = 0{,}95$ and, according to the design of the measurements, a number of degrees of freedom of $\nu = 32$ are assumed.

Table 2 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s \leqslant \sigma$	$s > \sigma$
b)	$\sigma = \widetilde{\sigma}$	$\sigma \neq \widetilde{\sigma}$
c)	$\delta = 0$	$\delta \neq 0$

6.4.2 Question a)

The null hypothesis stating that the experimental standard deviation, s, of a vertical angle observed in both face positions is smaller than or equal to a theoretical or a predetermined value, σ, is not rejected if the following condition is fulfilled:

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu)}{\nu}} \tag{46}$$

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{0,95}(32)}{32}} \tag{47}$$

$$\chi^2_{0,95}(32) = 46{,}19 \tag{48}$$

$$s \leqslant \sigma \times \sqrt{\frac{46{,}19}{32}} \tag{49}$$

$$s \leqslant \sigma \times 1{,}20 \tag{50}$$

Otherwise, the null hypothesis is rejected.

6.4.3 Question b)

In the case of two different samples, a test indicates whether the experimental standard deviations, s and $\widetilde{s}$, belong to the same population. The corresponding null hypothesis, $\sigma = \widetilde{\sigma}$, is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\nu,\nu)} \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant F_{1-\alpha/2}(\nu,\nu) \tag{51}$$

$$\frac{1}{F_{0,975}(32,32)} \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant F_{0,975}(32,32) \tag{52}$$

$$F_{0,975}(32,32) = 2{,}02 \tag{53}$$

$$0{,}49 \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant 2{,}02 \tag{54}$$

Otherwise, the null hypothesis is rejected.

6.4.4 Question c)

The hypothesis stating that the vertical index error, δ, is equal to zero is not rejected if the following condition is fulfilled:

$$|\delta| \leqslant s_\delta \times t_{1-\alpha/2}(\nu) \tag{55}$$

$$|\delta| \leqslant s_\delta \times t_{0,975}(32) \tag{56}$$

$$s_\delta = \frac{s}{\sqrt{12} \times \sqrt{4}} \tag{57}$$

$$t_{0,975}(32) = 2{,}04 \tag{58}$$

$$\begin{aligned} |\delta| &\leqslant \frac{s}{\sqrt{48}} \times 2{,}04 \\ &\leqslant s \times 0{,}3 \end{aligned} \tag{59}$$

Otherwise, the null hypothesis is rejected.

The number of degrees of freedom and, thus, the corresponding test values $\chi^2_{1-\alpha}(\nu)$, $F_{1-\alpha/2}(\nu,\nu)$ and $t_{1-\alpha/2}(\nu)$ (taken from reference books on statistics) change if a different number of measurements is analysed.

Annex A
(informative)

Example of the simplified test procedure (horizontal directions)

A.1 Measurements

Table A.1 contains in columns 1 to 4 the measured values $x_{j,k,\mathrm{I}}$ and $x_{j,k,\mathrm{II}}$.

Observer: S. Miller

Weather: sunny, + 10 °C

Instrument type and number: NN xxx 630401

Date: 1999-04-15

NOTE The circle of the instrument is divided in 400 gon (instead of 360°).

Table A.1 — Measurements and residuals

1 j	2 k	3 $x_{j,k,\mathrm{I}}$ gon	4 $x_{j,k,\mathrm{II}}$ gon	5 $x_{j,k}$ gon	6 $x'_{j,k}$ gon	7 $\overline{x}_k$ gon	8 $d_{j,k}$ mgon	9 $r_{j,k}$ mgon	10 $r^2_{j,k}$ mgon^2
1	1	310,475	110,470	310,4725	0,0000	0,0000	0,0	0,0	0,00
	2	6,131	206,126	6,1285	95,6560	95,6553	−0,7	−0,7	0,49
	3	130,481	330,477	130,4790	220,0065	220,0058	−0,7	−0,7	0,49
	4	208,878	8,872	208,8750	298,4025	298,4040	+1,5	+1,5	2,25
	Σ	655,965	655,945	655,9550	614,0650	614,0651	+0,1	+0,1	3,23
2	1	376,749	176,744	376,7465	0,0000		0,0	−0,5	0,25
	2	72,403	272,398	72,4005	95,6540		+1,3	+0,8	0,64
	3	196,753	396,749	196,7510	220,0045		+1,3	+0,8	0,64
	4	275,154	75,148	275,1510	298,4045		−0,5	−1,0	1,00
	Σ	921,059	921,039	921,0490	614,0630		+2,1	+0,1	2,53
3	1	42,049	242,044	42,0465	0,0000		0,0	+0,6	0,36
	2	137,705	337,700	137,7025	95,6560		−0,7	−0,1	0,01
	3	262,056	62,050	262,0530	220,0065		−0,7	−0,1	0,01
	4	340,454	140,449	340,4515	298,4050		−1,0	−0,4	0,16
	Σ	782,264	782,243	782,2535	614,0675		−2,4	0,0	0,54
									6,30[a]

[a] Value represents Σr^2.

A.2 Calculation

First, the values $x_{j,k}$ are calculated with the measurements $x_{j,k,\mathrm{I}}$ and $x_{j,k,\mathrm{II}}$. In equation (1), $\pm$ 180° was substituted by $\pm$ 200 gon (see column 5 in Table A.1).

Then the values $x_{j,k}$ are reduced into the direction $x_{j,1}$ of the target No. 1. These values $x'_{j,k}$ are calculated according to equation (2) (see column 6 in Table A.1).

Column 7 in Table A.1 contains the mean values $\overline{x}_k$ of the reduced directions $x'_{j,k}$ [see equation (3)].

The differences $d_{j,k}$ result from the values of $\overline{x}_k$ and $x'_{j,k}$, according to equation (4) (see columns 6 to 8 in Table A.1).

For each set of directions, the mean value $\overline{d}_j$ of $d_{j,k}$ is calculated according to equation (5) ($\sum_{k=1}^{4} d_{j,k} = 4\overline{d}_j$, see lines Σ in column 8 in Table A.1).

With the values $d_{j,k}$ and $\overline{d}_j$, the residuals $r_{j,k}$ are calculated according to equation (6) (see column 9 in Table A.1).

The sum $\Sigma r^2 = 6{,}30$ mgon^2 is then calculated with the values in column 10 in Table A.1 [according to equation (8)].

The experimental standard deviation of a direction $x_{j,k}$ measured in one set of measurements in both face positions I and II, according to equation (10), amounts to

$$s = \sqrt{\frac{6{,}30\ \text{mgon}^2}{6}} = 1{,}0\ \text{mgon}$$

As arithmetic checks for each set of directions ($j = 1,\ 2,\ 3$), the sums in the columns in Table A.1 have to fulfill the following conditions (except for rounding errors):

— the sum in column 3 plus the sum in column 4 shall be two times the sum in column 5 $\pm \mu \times$ 200 gon (μ is a suitable integer number):

$655{,}965 + 655{,}945 = 2 \times 655{,}955\,0$

$921{,}059 + 921{,}059 = 2 \times 921{,}949\,0$

$782{,}264 + 782{,}264 = 2 \times 782{,}253\,5$

— the sum in column 5 minus four times the value of the direction to target No. 1 shall be equal to the sum in column 6 $\pm \mu \times$ 400 gon (μ is a suitable integer number):

$655{,}955\,0 - 4 \times 310{,}472\,5 = 614{,}065 - 3 \times 400$

$921{,}049\,0 - 4 \times 376{,}746\,5 = 614{,}063 - 3 \times 400$

$782{,}253\,5 - 4 \times 42{,}046\,5 = 614{,}067\,5 + 0 \times 400$

— the difference between the sum in column 7 and the sum in column 6 shall be equal to the sum in column 8:

$614{,}065\,1 - 614{,}065 = +0{,}000\,1$

$614{,}065\,1 - 614{,}063 = +0{,}002\,1$

$614{,}065\,1 - 614{,}067\,5 = -0{,}002\,4$

— the sum in column 9 shall be equal to zero [see equation (7)];

— the sum of all twelve values in column 6 shall be equal to three times the sum of the four values in column 7:

$614{,}065 + 614{,}063 + 614{,}067\,5 \approx 3 \times 614{,}065\,1$

— the sum of all twelve values in column 8 shall be equal to zero:

$0{,}1 + 2{,}1 - 2{,}4 = -0{,}2; \approx 0{,}0$

Annex B
(informative)

Example of the full test procedure (horizontal directions)

B.1 Measurements

Table B.1 contains in columns 1 to 4 the measured values $x_{j,k,\text{I}}$ and $x_{j,k,\text{II}}$ of the series of measurements No. 1 (the series of measurements Nos. 2, 3 and 4 were not printed).

Observer:	S. Miller
Weather:	sunny, + 10 °C
Instrument type and number:	NN xxx 630401
Date:	1999-04-15

Table B.1 — Measurements and residuals of series No. 1

1	2	3			4			5			6			7			8	9	10
j	k	$x_{j,k,\text{I}}$			$x_{j,k,\text{II}}$			$x_{j,k}$			$x'_{j,k}$			$\overline{x}_k$			$d_{j,k}$	$r_{j,k}$	$r^2_{j,k}$
		°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	″	″	(″)^2
1	1	28	12	37	208	12	42	28	12	39,5	0	00	00,0	0	00	00,0	0,0	+0,1	0,01
	2	83	50	35	263	50	40	83	50	37,5	55	37	58,0	55	38	00,3	+2,3	+2,4	5,76
	3	141	45	30	321	45	35	141	45	32,5	113	32	53,0	113	32	50,8	−2,2	−2,1	4,41
	4	219	30	49	39	30	50	219	30	49,5	191	18	10,0	191	18	9,5	−0,5	−0,4	0,16
	5	308	26	31	128	26	33	308	26	32,0	280	13	52,5	280	13	52,5	0,0	+0,1	0,01
	Σ	781	46	02	961	46	20	781	46	11,0	640	42	53,5	640	42	53,1	−0,4	+0,1	10,35
2	1	87	48	51	267	48	55	87	48	53,0	0	00	00,0				0,0	−1,7	2,89
	2	143	26	52	323	26	51	143	26	51,5	55	37	58,5				+1,8	+0,1	0,01
	3	201	21	41	21	21	47	201	21	44,0	113	32	51,0				−0,2	−1,9	3,61
	4	279	07	01	99	06	59	279	07	00,0	191	18	07,0				+2,5	+0,8	0,64
	5	8	02	42	188	02	40	8	02	41,0	280	13	48,0				+4,5	+2,8	7,84
	Σ	719	47	07	899	47	12	719	47	09,5	640	42	44,5				+8,6	+0,1	14,99
3	1	147	08	13	327	08	08	147	08	10,5	0	00	00,0				0,0	+1,7	2,89
	2	202	46	17	22	46	13	202	46	15,0	55	38	04,5				−4,2	−2,5	6,25
	3	260	41	01	80	40	57	260	40	59,0	113	32	48,5				+2,3	+4,0	16,00
	4	338	26	24	158	26	20	338	26	22,0	191	18	11,5				−2,0	−0,3	0,09
	5	67	22	07	247	22	08	67	22	07,5	280	13	57,0				−4,5	−2,8	7,84
	Σ	1 016	24	02	836	23	46	1 016	23	54,0	640	43	01,5				−8,4	+0,1	33,07
																			58,41[a]

[a] Value represents Σr_1^2.

B.2 Calculation

First, the values $x_{j,k}$ are calculated with the measurements $x_{j,k,\mathrm{I}}$ and $x_{j,k,\mathrm{II}}$ according to equation (11) (see column 5 in Table B.1).

Then, the values $x_{j,k}$ are reduced into the direction $x_{j,1}$; of the target No. 1. These values $x'_{j,k}$ are calculated according to equation (12) (see column 6 in Table B.1).

Column 7 in Table B.1 contains the mean values $\overline{x}_k$ of the reduced directions $x'_{j,k}$ [see equation (13)].

The differences $d_{j,k}$ result from the values of $\overline{x}_k$ and $x'_{j,k}$, according to equation (14) (see columns 6 to 8 in Table B.1).

For each set of directions the mean value $\overline{d}_j$ of $d_{j,k}$ is calculated according to equation (15) ($\sum_{k=1}^{5} d_{j,k} = 5\overline{d}_j$, see lines Σ in column 8 in Table B.1).

With the values $d_{j,k}$ and $\overline{d}j$ the residuals $r_{j,k}$ are calculated according to equation (16) (see column 9 in Table B.1).

The sum $\Sigma r_1^2 = 58{,}41\ ('')^2$ is then calculated with the values in column 10 in Table B.1 [according to equation (18)].

The experimental standard deviation of a direction $x_{j,k}$ measured in one set of measurements in both face positions I and II, valid for the series No. 1, according to equation (20) amounts to

$$s_1 = \sqrt{\frac{58{,}41\ ('')^2}{8}} = 2{,}7''$$

As arithmetic checks for each set of directions ($j = 1, 2, 3$), the sums in the columns in Table B.1 have to fulfill the following conditions (except for rounding errors):

— the sum in column 3 plus the sum in column 4 shall be two times the sum in column 5 $\pm \mu \times 180°$ (μ is a suitable integer number):

$$781°46'02'' + 961°46'20'' = 2 \times (781°46'11'') + 1 \times 180°$$

$$719°47'07'' + 899°47'12'' = 2 \times (719°47'9{,}5'') + 1 \times 180°$$

$$1\,016°24'02'' + 836°23'46'' = 2 \times (1\,016°23'54'') - 1 \times 180°$$

— the sum in column 5 minus five times the value of the direction to target No. 1 shall be equal to the sum in column 5 $\pm\mu \times 360°$ (μ is a suitable integer number):

$$781°46'11'' - 5 \times (28°12'39{,}5'') = 640°42'53{,}5'' + 0 \times 360°$$

$$719°47'9{,}5'' - 5 \times (87°48'53'') = 640°42'44{,}5'' + 0 \times 360°$$

$$1\,016°23'54'' - 5 \times (147°08'10{,}5'') = 640°43'1{,}5'' + 0 \times 360°$$

— the difference between the sum in column 7 and the sum in column 6 shall be equal to the sum in column 8:

$$640°42'53{,}1'' - 640°42'53{,}5'' = -0{,}4''$$

$$640°42'53{,}1'' - 640°42'44{,}5'' = +8{,}6''$$

$$640°42'53{,}1'' - 640°43'1{,}5'' = -8{,}4''$$

— the sum in column 9 shall be equal to zero [see equation (17)];

— the sum of all fifteen values in column 6 shall be equal to three times the sum of the five values in column 7:

$$640°42'53{,}5'' + 640°42'44{,}5'' + 640°43'1{,}5'' \approx 3 \times (640°42'53{,}1'')$$

— the sum of all fifteen values in column 8 shall be equal to zero:

$$-0{,}4'' + 8{,}6'' - 8{,}4'' = -0{,}2'' \approx 0''$$

The results of the four series of measurements are:

$$s_1 = 2{,}7''$$

$$s_2 = 1{,}6''$$

$$s_3 = 2{,}0''$$

$$s_4 = 2{,}3''$$

The overall experimental standard deviation, s, and the number of degrees of freedom, ν, are calculated according to the equations (22) and (21):

$$s = \sqrt{\frac{19{,}14\,('')^2}{4}} = 2{,}2''$$

$$\nu = 32$$

$$s_{\text{ISO-THEO-HZ}} = 2{,}2''$$

B.3 Statistical tests

B.3.1 Statistical test according to question a)

$$\sigma = 2''$$

$$s = 2{,}2''$$

$$\nu = 32$$

$$\begin{aligned} 2{,}2'' &\leqslant 2'' \times 1{,}20 \\ &\leqslant 2{,}4'' \end{aligned}$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviation $s = 2{,}2''$ is smaller than or equal to the manufacturer's value $\sigma = 2''$ is not rejected at the confidence level of 95 %.

B.3.2 Statistical test according to question b)

$$s = 2{,}2''$$

$$\widetilde{s} = 1{,}6''$$

$$\nu = 32$$

$$0,49 \leqslant \frac{4,84\,('')^2}{2,56\,('')^2} \leqslant 2,02$$

$$0,49 \leqslant 1,89 \leqslant 2,02$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations $s = 2{,}7''$ and $\widetilde{s} = 1{,}6''$ belong to the same population is not rejected at the confidence level of 95 %.

Annex C
(informative)

Example of both test procedures (vertical angles)

C.1 Measurements

Table C.1 contains in columns 1 to 4 the measured vertical angles $x_{j,k,\mathrm{I}}$ and $x_{j,k,\mathrm{II}}$ for the simplified test procedure or for the series of measurements No. 1 of the full test procedure (the series of measurements Nos. 2, 3 and 4 were not printed).

Observer:	S. Miller
Weather:	sunny, $+$ 10 °C
Instrument type and number:	NN xxx 630401
Date:	1999-04-15

NOTE The circle of the instrument is divided in 400 gon (instead of 360°).

Table C.1 — Measurements and residuals

1 j	2 k	3 $x_{j,k,\mathrm{I}}$ gon	4 $x_{j,k,\mathrm{II}}$ gon	5 $\delta_{j,k}$ mgon	6 $x'_{j,k}$ gon	7 $\overline{x}_k$ gon	8 $r_{j,k}$ mgon	9 $r^2_{j,k}$ mgon^2
1	1	49,367 7	350,632 6	0,15	49,367 55	49,367 27	−0,28	0,078
	2	86,353 3	313,646 7	0,00	86,353 30	86,353 43	+0,13	0,017
	3	101,416 9	298,583 2	0,05	101,416 85	101,416 97	+0,12	0,014
	4	113,649 0	286,351 8	0,40	113,648 60	113,648 62	+0,02	0,000
	Σ	350,786 9	1249,214 3	0,60	350,786 30	350,786 29	−0,01	0,109
2	1	49,367 2	350,632 8	0,00	49,367 20		+0,07	0,005
	2	86,353 8	313,646 5	0,15	86,353 65		−0,22	0,048
	3	101,416 9	298,582 9	−0,10	101,417 00		−0,03	0,001
	4	113,648 7	286,351 7	0,20	113,648 50		+0,12	0,014
	Σ	350,786 6	1249,213 9	0,25	350,786 35		−0,06	0,068
3	1	49,367 5	350,633 4	0,45	49,367 05		+0,22	0,048
	2	86,353 2	313,646 5	−0,15	86,353 35		+0,08	0,006
	3	101,417 1	298,583 0	0,05	101,417 05		−0,08	0,006
	4	113,649 0	286,351 5	0,25	113,648 75		−0,13	0,017
	Σ	350,786 8	1249,214 4	0,60	350,786 20		+0,09	0,077
								0,254[a]

[a] Value represents Σr_1^2.

C.2 Calculation

First, the vertical error, δ_1, is calculated (for the full test procedure only). In equation (34), $-360°$ was substituted by -400 gon.

$$\delta_1 = \frac{0{,}60 + 0{,}25 + 0{,}60}{12} \text{ mgon} = 1{,}2 \text{ mgon}$$

Then, the values $x'_{j,k}$ are calculated with the original measurements $x_{j,k,\mathrm{I}}$ and $x_{j,k,\mathrm{II}}$. In equation (33), $+360°$ was substituted by $+400$ gon (see column 6 in Table C.1).

Column 7 in Table C.1 contains the mean values $\overline{x}_k$ of the vertical angles $x'_{j,k}$ [see equation (35)].

The residuals $r_{j,k}$ are the differences of the mean values of $\overline{x}_k$ and the angles $x'_{j,k}$ obtained according to equation (36) (see column 8 in Table C.1).

Then, the sum $\Sigma r_1^2 = 0{,}254 \text{ mgon}^2$ is calculated with the values in column 8 or 9 in Table C.1 [see equation (38)].

For the simplified test procedure, the experimental standard deviation of a vertical angle $x_{j,k}$ measured in one set of measurements in both face positions I and II equals, according to the equations (39) and (40):

$$s = \sqrt{\frac{0{,}254 \text{ mgon}^2}{8}} = 0{,}18 \text{ mgon}$$

This is the final result obtained by the simplified procedure.

For the full test procedure, the experimental standard deviation of a vertical angle $x_{j,k}$ measured in one set of measurements in both face positions I and II, valid for series of measurements No. 1, equals, according to the equations (39) and (40):

$$s_1 = \sqrt{\frac{0{,}254 \text{ mgon}^2}{8}} = 0{,}18 \text{ mgon}$$

As arithmetic checks for each set of vertical angles ($j = 1, 2, 3$), the sums in the columns in Table C.1 have to fulfill the following conditions (except for rounding errors):

— the sum in column 3 plus the sum in column 4 less four times 400 gon shall be two times the sum in column 5:

$$350{,}786\,9 + 1\,249{,}214\,3 - 4 \times 400 = 2 \times 0{,}000\,60$$

$$350{,}786\,6 + 1\,249{,}213\,9 - 4 \times 400 = 2 \times 0{,}000\,25$$

$$350{,}786\,8 + 1\,249{,}214\,4 - 4 \times 400 = 2 \times 0{,}000\,60$$

— the difference between the sum in column 3 and the sum in column 4 plus 1 600 gon shall be equal to two times the sum in column 6:

$$350{,}786\,9 - 1\,249{,}214\,3 + 4 \times 400 = 2 \times 350{,}786\,30$$

$$350{,}786\,6 - 1\,249{,}213\,9 + 4 \times 400 = 2 \times 350{,}786\,35$$

$$350{,}786\,8 - 1\,249{,}214\,4 + 4 \times 400 = 2 \times 350{,}786\,20$$

— the difference between the sum in column 7 and the sum in column 6 shall be equal to the sum in column 8:

$$350{,}786\,29 - 350{,}786\,30 = -0{,}000\,1$$

$$350{,}786\,29 - 350{,}786\,35 = -0{,}000\,6$$

$350{,}786\,29 - 350{,}786\,20 = +0{,}000\,9$

— the sum of all twelve values in column 8 is zero.

The results of the series of measurements are:

$$s_1 = 0{,}18 \text{ mgon}; \quad \delta_1 = 0{,}12 \text{ mgon}$$
$$s_2 = 0{,}12 \text{ mgon}; \quad \delta_2 = 0{,}70 \text{ mgon}$$
$$s_3 = 0{,}11 \text{ mgon}; \quad \delta_3 = 0{,}42 \text{ mgon}$$
$$s_4 = 0{,}21 \text{ mgon}; \quad \delta_4 = 0{,}59 \text{ mgon}$$
$$\delta = 0{,}46 \text{ mgon}$$

The overall standard deviation, s, and the number of degrees of freedom, ν, are calculated according to the equations (43), (44), and (45):

$$s = \sqrt{\frac{0{,}103 \text{ mgon}^2}{4}} = 0{,}16 \text{ mgon}$$

$$\nu = 32$$

$$s_{\text{ISO-THEO-V}} = 0{,}16 \text{ mgon}$$

C.3 Statistical tests

C.3.1 Statistical test according to question a)

$$\sigma = 0{,}1 \text{ mgon}$$
$$s = 0{,}16 \text{ mgon}$$
$$\nu = 32$$

$$0{,}16 \text{ mgon} \leqslant 0{,}1 \text{ mgon} \times 1{,}20$$
$$\leqslant 0{,}12 \text{ mgon}$$

Since the above condition is not fulfilled, the null hypothesis stating that the experimental standard deviation $s = 0{,}16$ mgon is smaller than or equal to the manufacturer's value $\sigma = 0{,}1$ mgon is rejected at the confidence level of 95 %.

C.3.2 Statistical test according to question b)

$$s = 0{,}16 \text{ mgon}$$

$$\widetilde{s} = 0{,}12 \text{ mgon}$$

$$\nu = 32$$

$$0{,}49 \leqslant \frac{0{,}025\,6 \text{ mgon}^2}{0{,}014\,4 \text{ mgon}^2} \leqslant 2{,}02$$

$$0{,}49 \leqslant 1{,}78 \leqslant 2{,}02$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations $s = 0{,}16$ mgon and $\widetilde{s} = 0{,}12$ mgon belong to the same population is not rejected at the confidence level of 95 %.

C.3.3 Statistical test according to question c)

$s = 0{,}16$ mgon

$\nu = 32$

$\delta = 0{,}46$ mgon

$s_\delta = 0{,}023$ mgon

$$0{,}46 \text{ mgon} \leqslant 0{,}023 \text{ mgon} \times 2{,}04$$
$$\leqslant 0{,}05 \text{ mgon}$$

Since the above condition is not fulfilled, the null hypothesis stating that the vertical index error is equal to zero is rejected at the confidence level of 95 %.

ICS 17.180.30

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