Neural Networks for Computing

Convolution Networks

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Introduction

- Convolution networks or Convolution neural networks as a special kind of NN
- Usually used for data that are know to have a grid-like topology
- Examples are time-series data that can be thought of as a 1 dimensional grid of samples taken at regular intervals
- Or image data with can be considered as a 2-dimensional grid of pixels (or 3-dimensional volumes for color images)
- CNN have been very successful in practical applications
- The name "convolution" refers to an operation that is **similar** to the convolution operation used in physics and engineering.

Overview

- A convolution network usually has at least one convolution layer
- A convolution operation (slightly different from the usual definition) is done by multiplying
- Weights element-wise with a portion of the input
- The same operation with the same weights is repeated over all the input
- The set of weights are usually referred to as the **kernel**
- The result of the convolution is referred to as the **feature map**

Convolution operation

- The convolution of two functions f and g as used in physics and engineering is defined as

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

- The discrete version is written as

$$(f * g)[n] = \sum_{m=-\infty}^{\infty} f[m] g[n - m]$$

- For finite domains

$$(f * g)[n] = \sum_{m=-M}^{M} f[m] g[n - m]$$

Convolution in NN

- In neural networks we use a related operation, cross-correlation is an operation involving the input I and a kernel K is defined as

$$C[i,j] = (I * K)[i,j] = \sum_m \sum_n I[i+m, j+n]K[m,n]$$

- The range of the indices m and n depend on the kernel size.
- The equation above assumes a **stride** of size 1. Later we will deal with strides of different sizes.

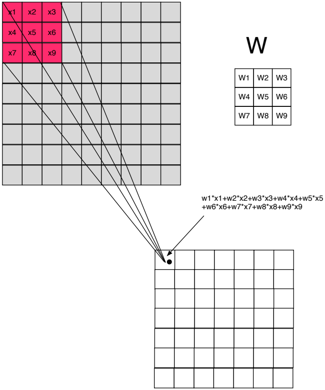
Motivation

- There three basic ideas behind convolution
  1. Sparse interaction
  2. Parameters sharing
  3. Equivariant representation
- The NN that we have dealt with so far use matrix multiplication of the input with the weights
- Each input unit interacts with each output unit.
- In convolution each output unit interacts with a portion(typically small) of the input
- This is done by making the kernel small compared with the input.
- This also leads to a smaller number of parameters.
- Parameters sharing: the same kernel (weights) are used for all the locations in the input.

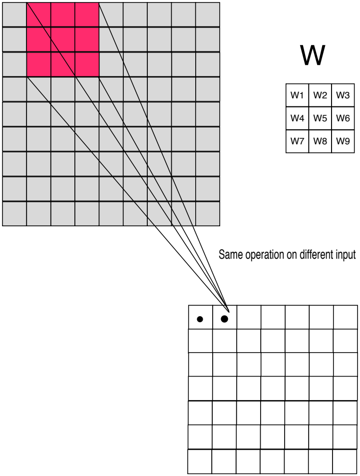
Translation equivariance

- First equivariance is NOT invariance. Let T be the translation operation and f the convolution operation.
- Let X be an input.
- Translation invariance means $f(T(X)) = f(X)$ a property which convolution does NOT possess.
- Translation equivariance means $f(T(X)) = T(f(X))$ a property which convolution does possess.
- What equivariance means is that if a feature is detected by the convolution at position x and reported at output y then when the input is shifted, say by d, then the feature will be detect in the output at $y + d$

How convolution works for grayscale images



How convolution works for grayscale images



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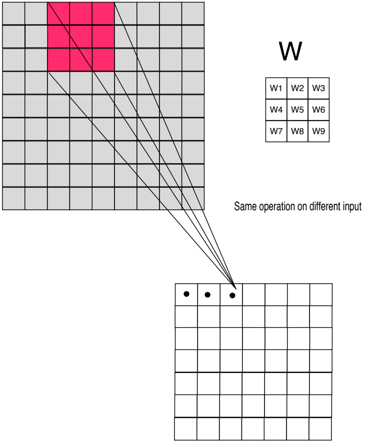

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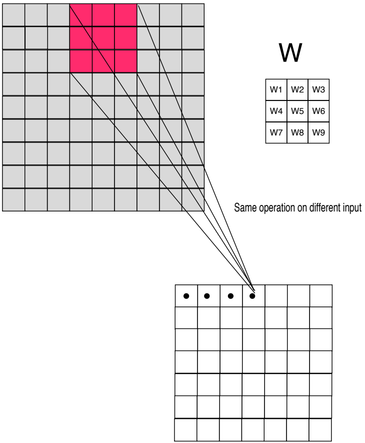

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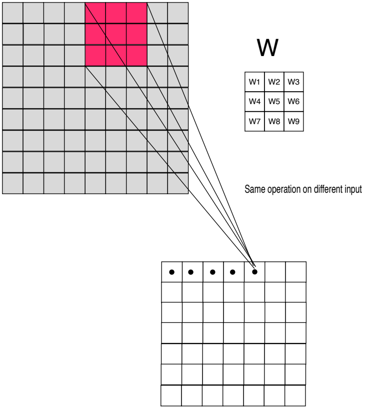

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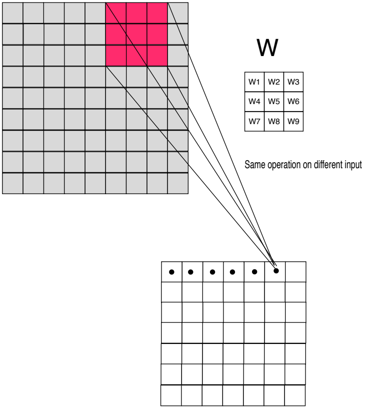

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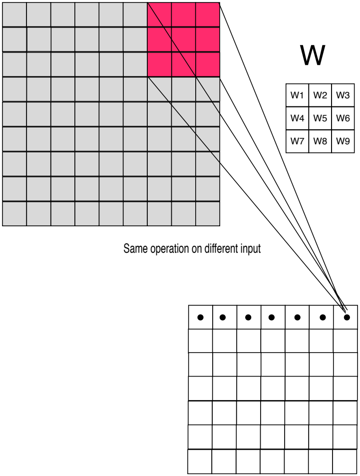

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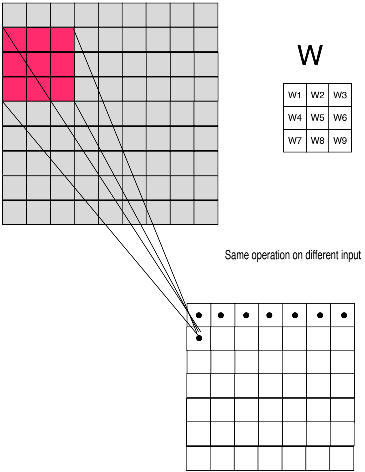
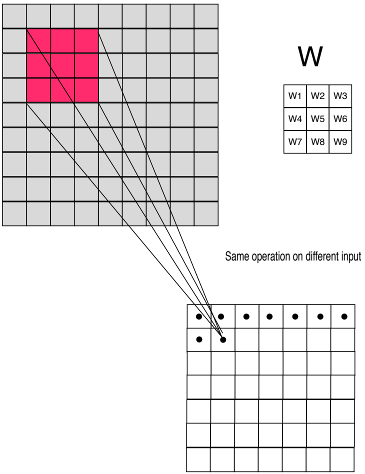


How convolution works for grayscale images



Example

Input

1	2	4	3
5	6	8	7
9	10	12	11
13	14	16	2

$\odot$

Kernel

1	-2
-3	4

=

6		

Example

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1	2	4	3
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$\odot$

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$\odot$

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-3	4

$=$

6	8	2
6	8	2
6	8	-50

What about color images?

- Typically each pixel in a color image is specified as RGB
- So in addition to the space position each pixel has a depth
- In this case the kernel will have depth 3 as well
- The convolution is done as before with the color an extra dimension
- Note that convolution networks are mostly used for images so basically we always have either 2 and 3 dimensions for grayscale and color images
- In our first example we had an input of 9x9 (grayscale image), a kernel with a receptive field of size 3x3
- The output was 3x3

Parameters

- In the second example we had an input of 4x4 (also grayscale), a kernel with a receptive field of size 2x2
- The output was also 3x3. Is the output always 3x3?
- No. If
  1. $W_1 \times H_1 \times D_1$ is the size of the input,
  2. $F \times F$ is the size of the receptive field,
  3. K the number of filters
- Then the output has size $W_2 \times H_2 \times K$ where

$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$
$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$

- Note that the depth of the output is equal to the number of filters.

numpy example

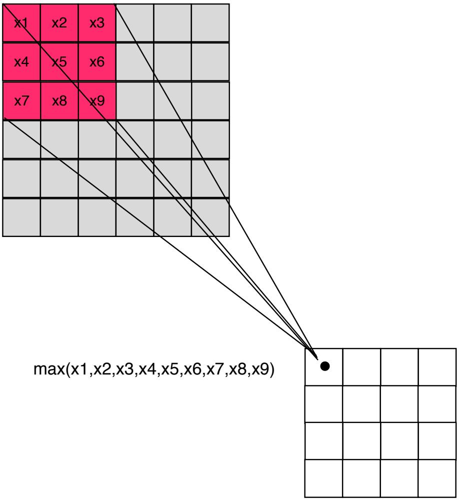
- Suppose the input X has size 4, we use 2 filters (W_0 and W_1) with size 2 and the stride is 1 (no padding). Then the output can be written (not all the values are shown)

```
Z[0,0,0]=np.sum(X[0:2,0:2,0]*W_0)
Z[0,1,0]=np.sum(X[0:2,1:3,0]*W_0)
Z[0,2,0]=np.sum(X[0:2,2:4,0]*W_0)
Z[1,0,0]=np.sum(X[1:3,0:2,0]*W_0)
...
Z[0,0,1]=np.sum(X[0:2,0:2,0]*W_1)
```

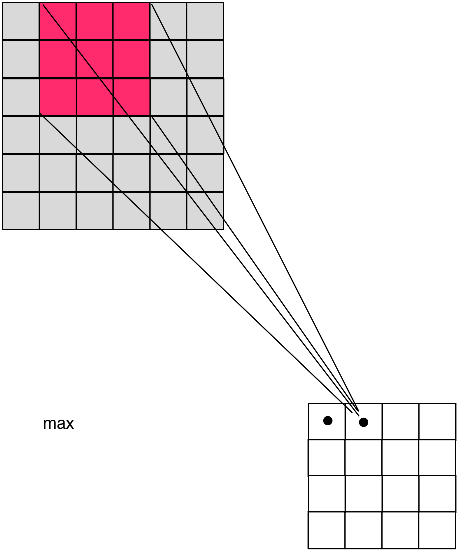
Pooling

- It is common to insert a pooling layer between successive convolution layers.
- The main functions of the pooling layer
  1. reduce the spatial size of the input
  2. reduce the amount of parameters
- Pooling operates independently on every depth-slice of the input and resizes it spatially.
- Usually, the pooling layer uses filters of size 2×2 applied with a stride of 2.
- The pooling is done by using the max operation on a receptive field.

Example pooling



Example pooling



Parameters

- As in the case of convolution if the input is of size $W_1 \times H_1 \times D$ and we choose the receptive field size F and a stride S then the output has dimensions $W_2 \times H_2 \times D_2$ where
  1. $D_2 = D_1$ since the pooling is done per depth slice
  2. $W_2 = \frac{W_1 - F}{S} + 1$
  3. $H_2 = \frac{H_1 - F}{S} + 1$
- It is NOT common to use padding in the pooling layer.
- The most common receptive field size and stride are $F = 2$ and $S = 2$.

Why pooling

- Pooling replace a region in the input with a summary statistic usually the max value when using max pooling.
- This makes the computation almost invariant to translation as can be seen in the example below

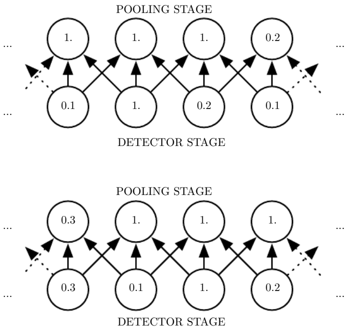


Figure : From Goodfellow et. al.

Architecture

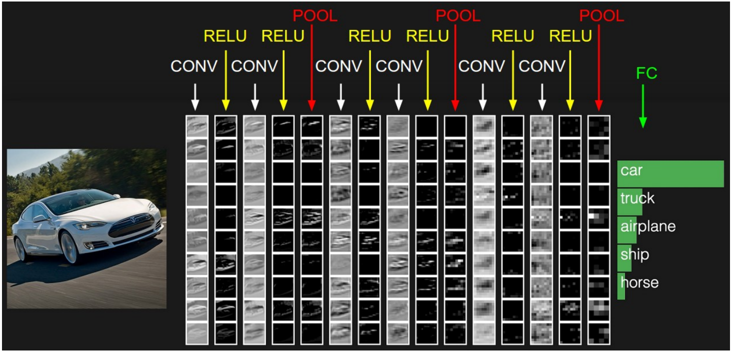
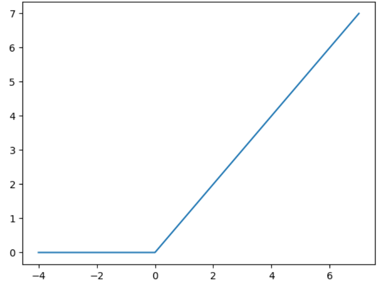


Figure : Andrej Karpathy

ReLU

- The ReLU (Rectified Linear Unit) function is used instead of the sigmoid or the softmax is defined by

$$ReLU(x) = \max(0, x)$$



Why ReLU

- The most important advantage of ReLU is the reduced likelihood of vanishing gradient
- For large values of the input the sigmoid and its derivative go to zero very quickly which makes learning hard
- In the case of the ReLU this does not happen, the derivative is constant
- As we discussed before this is very important especially when the architecture has many layers which makes learning with sigmoid almost impossible.
- Recall that $\sigma' = \sigma(1 - \sigma) \leq 0.25$ for all values