EE102

Lecture 5

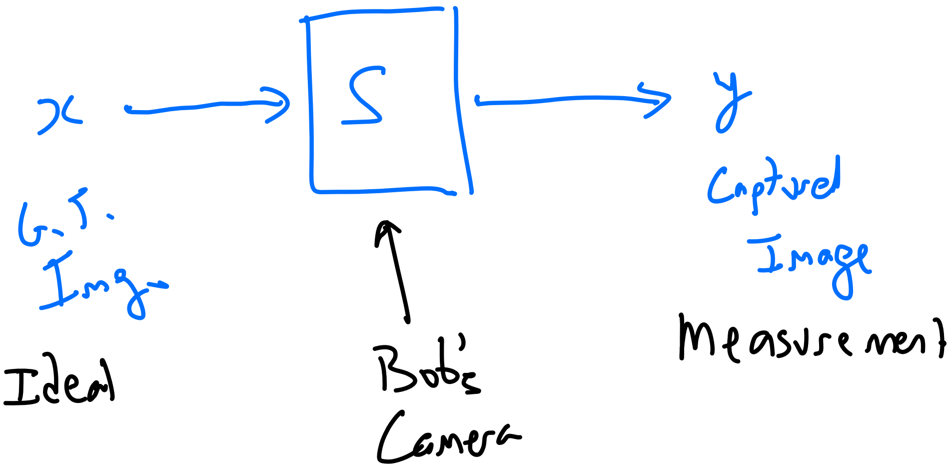
EE102 Announcements

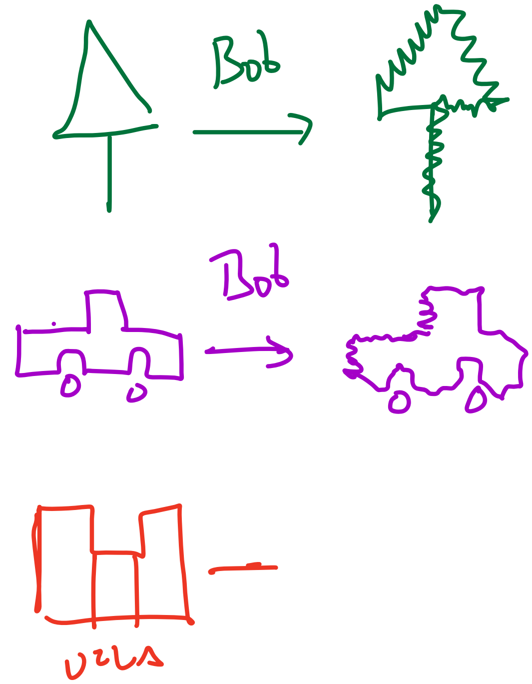
- **Syllabus link** is on BruinLearn
- Practice Midterm Released Early

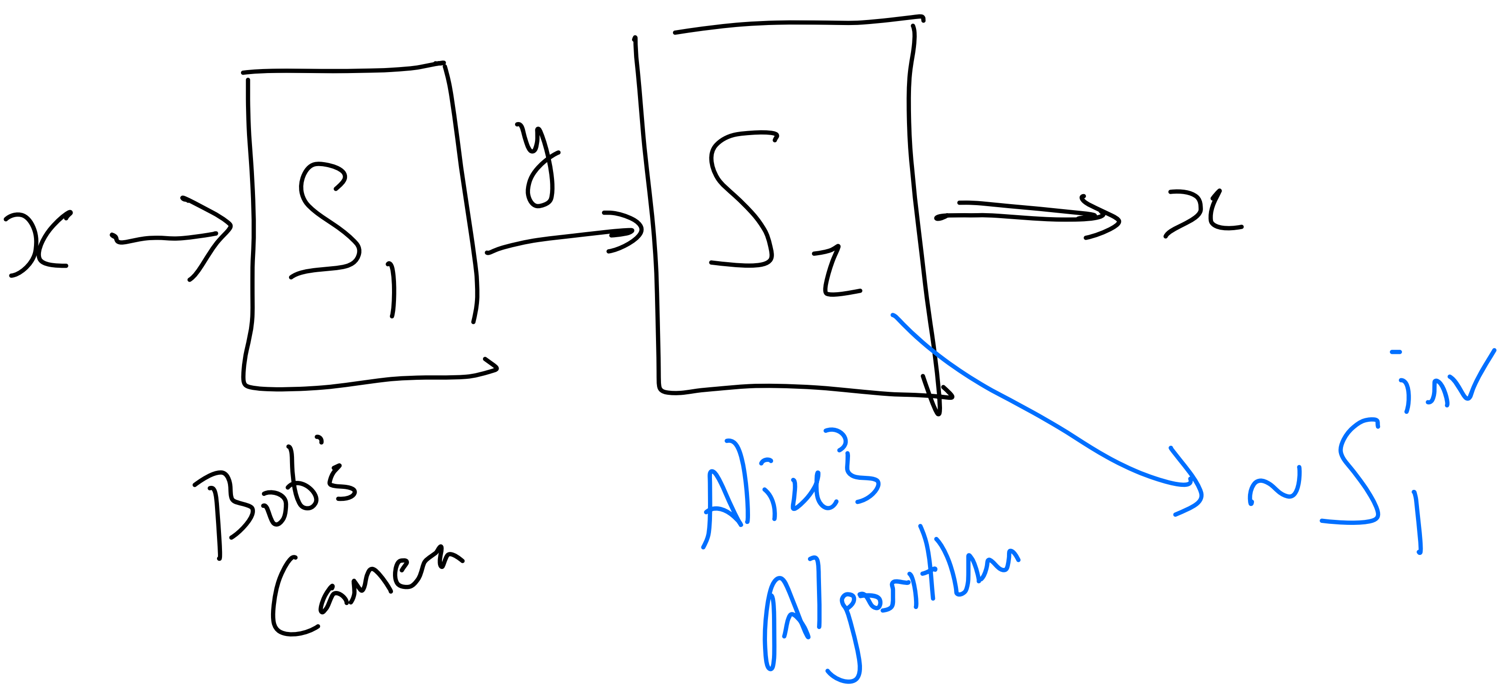
Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

Alice & Bob.

Image Corp







Impulse Response

System impulse response

This lecture introduces time-domain analysis of systems, including the impulse response. It also discusses linear time-invariant systems. Topics include:

- Impulse response definition
- Impulse response of LTI systems
- The impulse response as a sufficient characterization of an LTI system
- Impulse response and the convolution integral

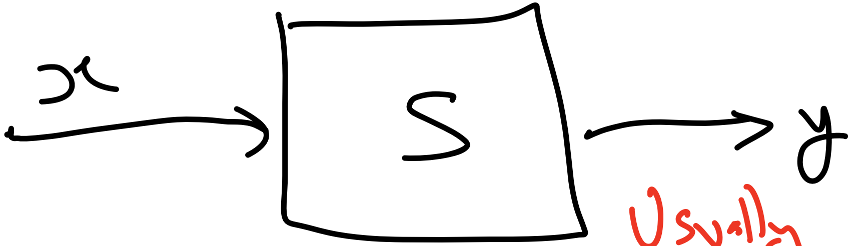
Impulse Response (cont.)

Why do we need the impulse response?

① E.g. $y(t) = [x(t)]^2$ This is a pset qst. NOT real life

② Real Life. We do not know much, if anything, about the functional form of S.

[Assuming S is LTI]



Usually S describes something complicated and not known

Goal: Given some input, specific input x, predict y.

Soln: If I know the systems response to $x = \delta(t)$, then I know response to ANY x.

Types of Responses $x \rightarrow \boxed{H} \rightarrow y$

Several systems are characterized by their functional response

- zero response: $H[0]$
- impulse response: $H[\delta(t)]$
- step response: $H[u(t)]$

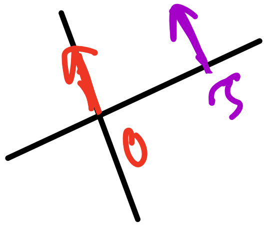
CYU: Say H is linear, what is its zero response?

Ans: $H(ax) = aH(x)$ $\forall$ a, x. Suppose $a = 0$

$$\boxed{H(0) = 0}$$

Impulse Response Definition

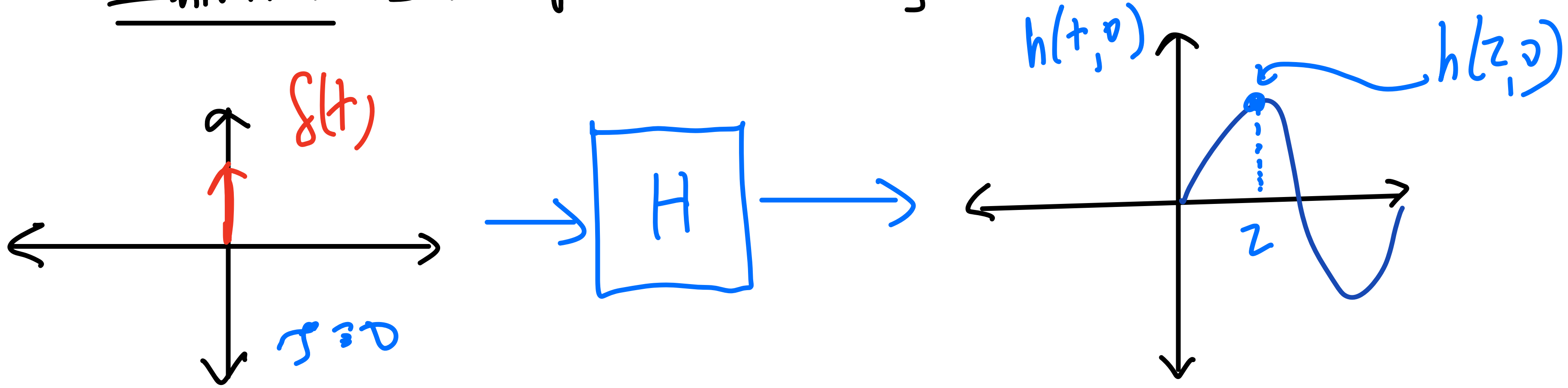
Impulse Response: Let H be a system and $y(t) = H[x(t)]$



The impulse response is

$$h(t,\tau) = H(\delta(t-\tau))$$

Intuition: Send impulse at time τ, receive output $h(t,\tau)$



$x(t)$ input $y(t)$ output $h(t)$ → typically reserved for IR.

$$h(t,\tau) = H(\delta(t-\tau))$$

There are important things to be careful of when looking at this equation.

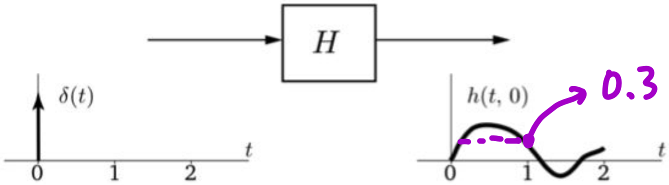
- The t on the left and right hand side of these equations *are not the same*!
- The t on the left hand side is the impulse response at a specific value of time.
- The t on the right hand side varies across all time.
- The output at the specific time t on the left will depend on the input at several times t on the right.

Notation on t

$$h(t, \tau) = H(\delta(t - \tau))$$

There are important things to be careful of when looking at this equation.

An example of these t's not being the same is shown below. In this example, let $\tau = 0$.



It may be tempting to write:

$$h(1, 0) = H(\delta(1))$$

This is wrong.

- On the left, $\delta(1) = 0$. We know if H is linear, then $H(0) = 0$, implying that $h(1, 0) = 0$.
- But in general, the impulse response can be non-zero, i.e., $h(1, 0) \neq 0$ in the above diagram, if the impulse response produces some non-zero response.

Assume $\tau = 0$

$h(t,0) = H[\delta(t)]$ } Do not confuse

Set $t=1$

$\underbrace{h(1,0)}_{\text{Non-zero } \approx 0.3} = \underbrace{H[\delta(1)]}_{\text{Zero.}}$

Time invariant Impulse Response

Time Invariant H

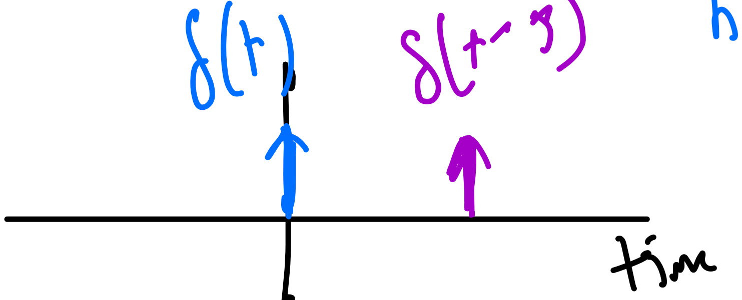
$$h(t,\tau) = H[\delta(t-\tau)]$$

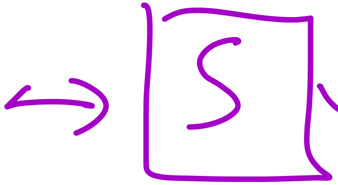
$$h(t,0) = H[\delta(t)]$$

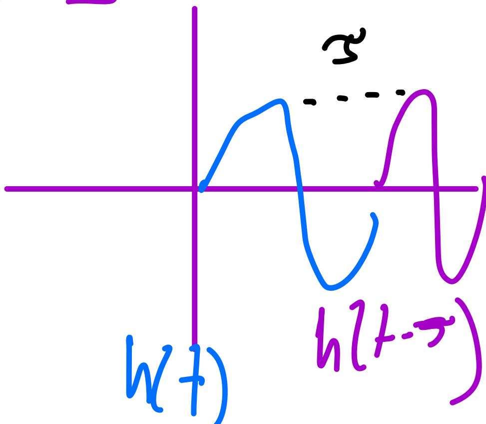
Suppose H is time invariant. $H[\delta(t-\tau)] = h(t-\tau, 0) = h(t,\tau)$

So I can simplify notation. $h(t) = H[\delta(t)]$

$$h(t-\tau) = H[\delta(t-\tau)]$$



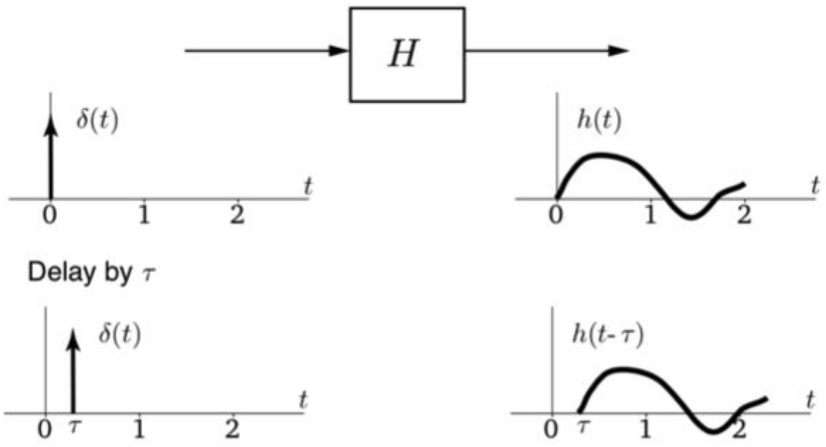




Time Invariant Impulse Response

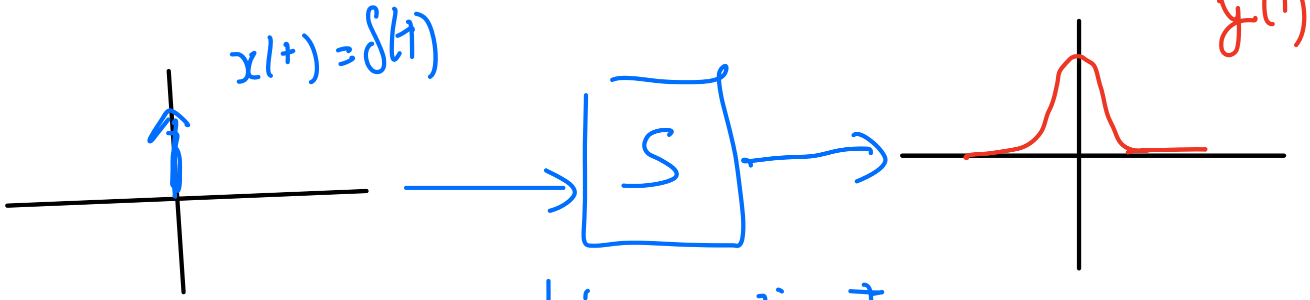
Impulse response of a time-invariant system (cont.)

This property of the impulse response for a time-invariant system is drawn below:



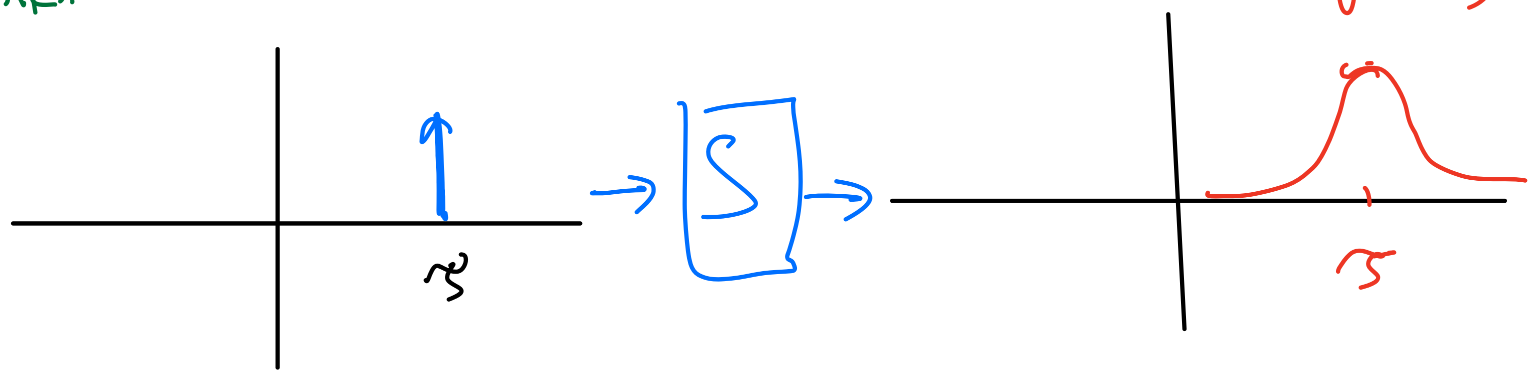
Input Examples

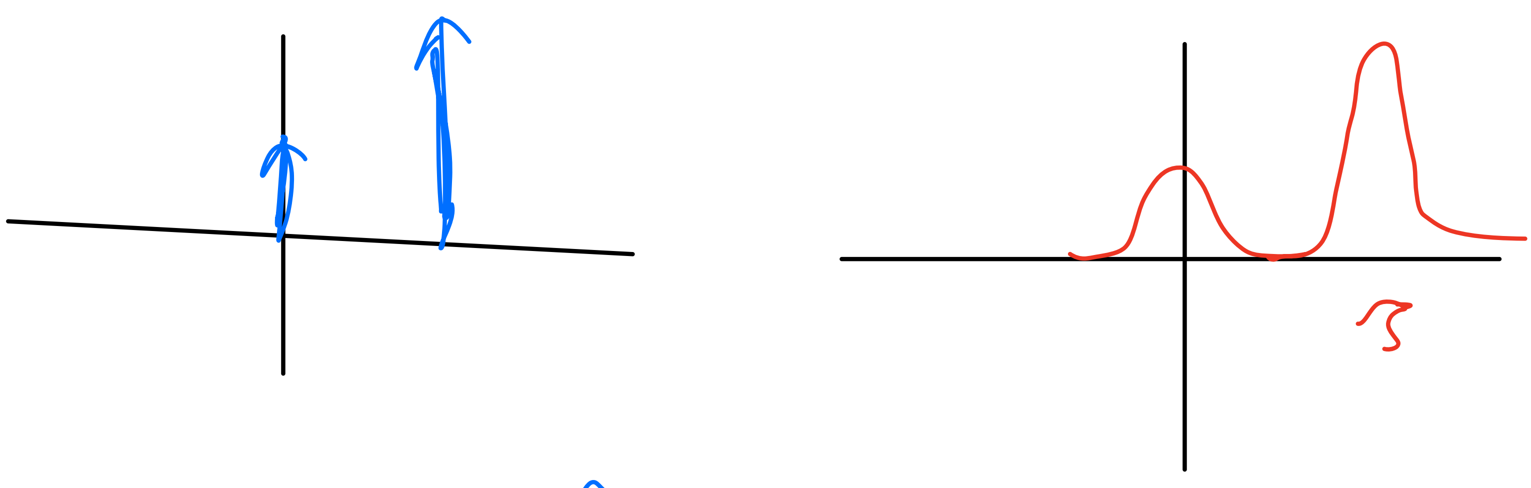
Outputs

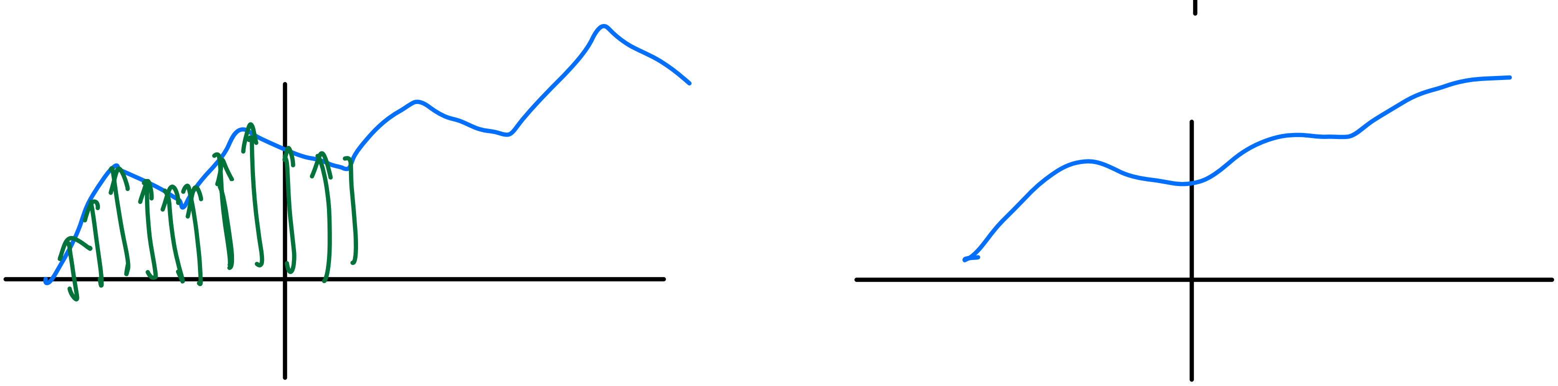


Known

Unknown







Important Fact about the Impulse Response

FACT: If H is an LTI (linear time-invariant system) with impulse response

$$h(t) = H(\delta(t))$$

then we can calculate $H(x(t))$ for ANY $x(t)$ **IF** we know $h(t)$.

This is a ***very important* result.**