ECE102, Fall 2025 **Homework #3**
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Due Friday, 24 October 2025, by 11:59pm to Gradescope.
100 points total.

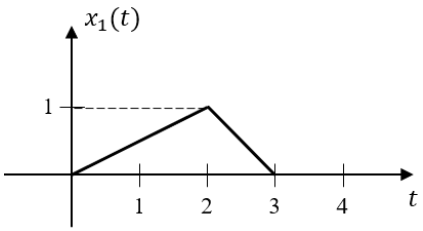
1. (20 points) **Linear systems**

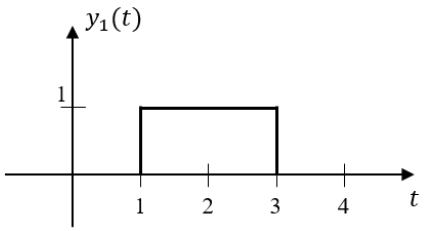
   Determine whether each of the following systems is linear or not. Explain your answer.

   (a) $y(t) = cos(t)x(t)$
   (b) $y(t) = \frac{d}{dt}(\frac{1}{2}x(t)^2)$
   (c) $y(t) = e^{x(t)}$
   (d) $y(t) = x(t) + 2u(t+1)$

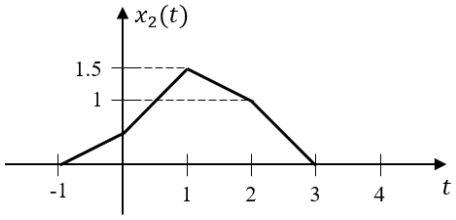
2. (13 points) **LTI systems**

   (a) (7 points) Consider an LTI (linear time-invariant) system whose response to $x_1(t)$ is $y_1(t)$, where $x_1(t)$ and $y_1(t)$ are illustrated as follows:





   Sketch the response of the system to the input $x_2(t)$.



   (a) $x_2(t)$

   (b) (6 points) Assume we have a linear system with the following input-output pairs:
   - the output is $y_1(t) = cos(t)u(t)$ when the input is $x_1(t) = u(t)$;
   - the output is $y_2(t) = cos(t)(u(t+1) - u(t))$ when the input is $x_2(t) = \text{rect}(t + \frac{1}{2})$.

   Is the system time-invariant?

3. (38 points) **Convolution**

(a) (10 points) For each pair of the signals given below, compute their convolution using the flip-and-drag technique.

i. $f(t) = \delta(t+1) + 2\delta(t-2), \qquad g(t) = e^{-t}u(t)$

ii. $f(t) = 2\,\mathrm{rect}(t-\frac{3}{2}), \qquad g(t) = 2\,r(t-1)\mathrm{rect}(t-\frac{3}{2})$

(b) (10 points) For each of the following, find a function $h(t)$ such that $y(t) = x(t) \star h(t)$.

i. $y(t) = \int_{t-T}^{t} x(\tau)d\tau$

ii. $y(t) = \sum_{n=-\infty}^{\infty} x(t - nT_s)$

(c) (10 points) Simplify the following expressions:

i. $[\delta(t-3) + \delta(t+2)] * \left[e^{3t}u(-t) + \delta(t+2) + 2\right]$

ii. $\frac{d}{dt}\left[(u(t) - u(t+1)) \star u(t-2)\right]$, *Hint: Show first that* $u(t) \star u(t) = r(t)$ *where* $r(t)$ *is the ramp function.*

(d) (8 points) Explain whether each of the following statements is true or false.

i. If $x(t)$ and $h(t)$ are both odd functions, and $y(t) = x(t) \star h(t)$, then $y(t)$ is an even function.

ii. If $y(t) = x(t) \star h(t)$, then $y(2t) = h(2t) \star x(2t)$.

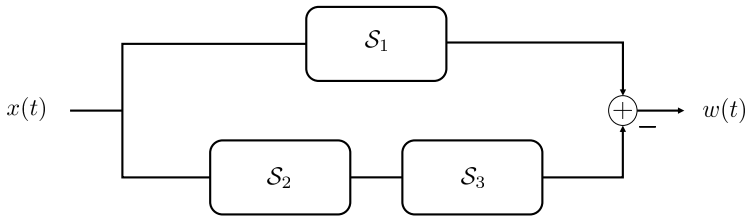
4. (12 points) **Impulse response and LTI systems**

Consider the following three LTI systems:

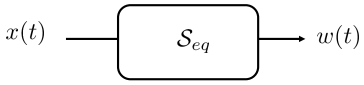
- The first system $\mathcal{S}_1$ is given by its input-output relationship: $y(t) = \int_{-\infty}^{t} x(\tau - t_0)d\tau$;
- The second system $\mathcal{S}_2$ is given by its impulse response: $h_2(t) = u(t-2)$;
- The third system $\mathcal{S}_3$ is given by its impulse response: $h_3(t) = u(t+3)$.

(a) (4 points) Compute the impulse responses $h_1(t)$ of system $\mathcal{S}_1$.

(b) (4 points) The three systems are interconnected as shown below.



Determine the impulse response $h_{eq}(t)$ of the equivalent system.



(c) (4 points) Determine the response of the overall system to the input $x(t) = \delta(t) + \delta(t-3)$.

5. (17 points) **Python tasks**
   For this question, please complete the included Jupyter Notebook from the zip file.

   Include all relevant code and plots as a pdf of the Jupyter Notebook appended to the end of the homework. You do not need to submit the actual ".ipynb" file, simply a pdf of the notebook will be fine.

   If you would like to complete the assignment in another programming language, you are welcome to, but you will have to translate the skeleton code from the provided notebook to the preferred language yourself.