ECE102, Fall 2025 — **Homework #1**

Signals & Systems — Prof. A. Kadambi

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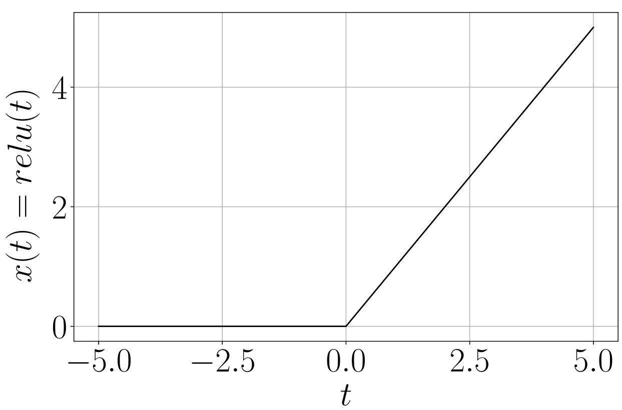
Due Friday, 10 October 2025, by 11:59pm to Gradescope.
Covers material up to Lecture 2.
100 points total.

1. (10 points) **Even and odd parts.**
Sketch and write the even and odd components of the following signal:

$$x(t) = relu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

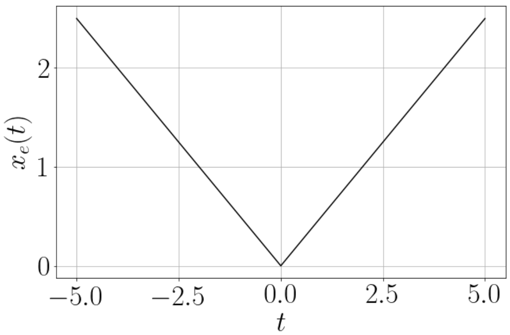


Solutions:
Using the expressions of the even and odd parts,

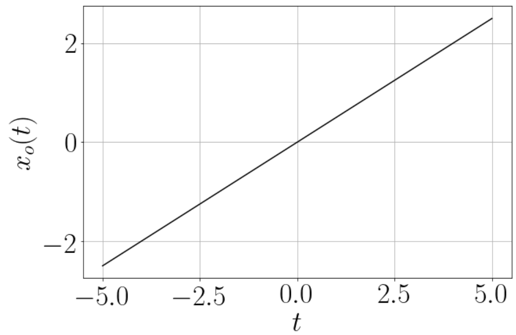
$$x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$$
$$x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right)$$

we can construct the even and odd components of $x(t)$:

$$x_e(t) = \frac{1}{2}|t|$$
$$x_0(t) = \frac{1}{2}t$$



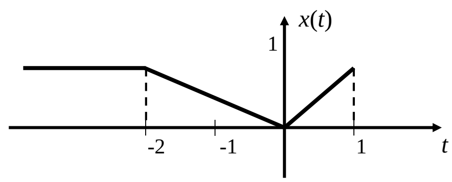
Even component of relu(t)



Odd component of relu(t)

2. (15 points) **Time scaling and shifting.**

   (a) (10 points) Consider the following signal.



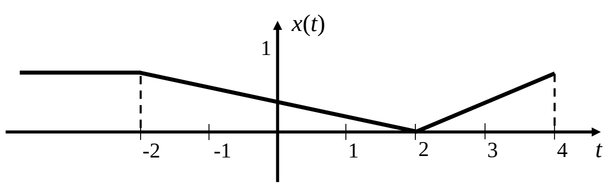
Sketch the following:

i. $x(\frac{1}{2}t - 1)$

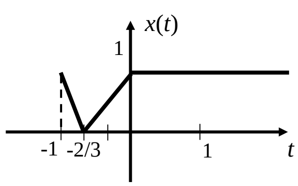
ii. $x(-3t - 2)$

Solutions:

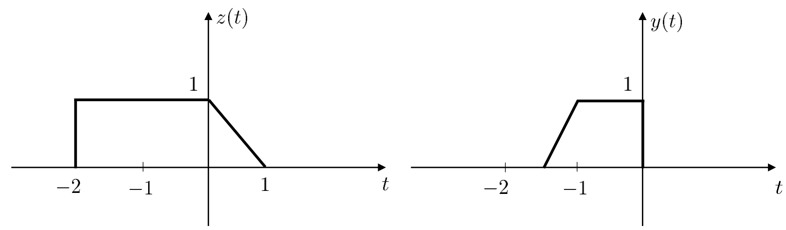
i. This is shifted right by 1, and then expanded by a factor of two.



ii. This is shifted right by 2, and then reversed and compressed by a factor of three.



(b) (5 points) The figure below shows two signals: $z(t)$ and $y(t)$. Can you express $y(t)$ in terms of $z(t)$?



Solutions:
$y(t) = z(-2t - 2)$

3. (X points) **Periodic signals.**

(a) (X points) For each of the following signals, determine whether it is periodic or not. If the signal is periodic, determine the fundamental period and frequency.

i. $x_1(t) = \sin(2t + \pi/3)$
ii. $x_2(t) = \cos(\sqrt{2}\pi t)$
iii. $x_3(t) = \sin^2(3\pi t + 3)$
iv. $x_4(t) = x_1(t) + x_2(t)$
v. $x_5(t) = x_1(\pi t) + x_3(t)$
vi. $x_6(t) = e^{-t}x_1(t)$
vii. $x_7(t) = e^{j(\pi t+1)}x_2(t)$

Solutions:

i. The signal is periodic with period $2\pi/(2) = \pi$ sec and the frequency is $1/\pi$ Hz

ii. The signal is periodic with period $2\pi/(\sqrt{2}\pi) = 2/\sqrt{2}$ sec and the frequency is $\sqrt{2}/2$ Hz.

iii. $x_3(t) = sin^2(3\pi t + 3) = \frac{1-cos(6\pi t+6)}{2}$. So the signal has period $2\pi/(6\pi) = 1/3$ sec and frequency 3 Hz.

iv. $x_4(t) = x_1(t) + x_2(t)$: let T_1 denote the period of $x_1(t)$ and T_2 the period of T_2. If we can find integers m and n such that $mT_1 = nT_2$, $x_3(t)$ will then be periodic with period $T_4 = mT_1 = nT_2$. In other words, the ratio

$$\frac{T_1}{T_2} = \frac{n}{m}$$

need to be rational for $x_4(t)$ to be periodic. However, we have from part (i) $T_1 = \pi$ and from part (ii) $T_2 = 2/\sqrt{2}$, so that

$$\frac{T_1}{T_2} = \sqrt{2}\pi/2$$

The ratio is not rational. Hence, $x_4(t)$ is not periodic.

v. First, we find what the period of $x_1(t)$ is. The signal becomes $x_1(t) = sin(2\pi t + \pi/3)$. The period becomes $2\pi/2\pi = 1$. Following the same logic as $x_4(t)$, $\frac{T_1}{T_3} = \frac{1}{1/3} = 3$. Therefore the period becomes 1 sec and the fundamental frequency is 1 Hz.

vi. $x_6(t) = e^{-t}x_1(t)$: this signal is not periodic since its magnitude decreases exponentially.

vii. $x_7(t) = e^{j(\pi t+1)}x_2(t) = e^{j(\pi t+1)} \times (\cos(\sqrt{2}\pi t)) = e^{j(\pi t+1)} \times (\frac{1}{2}\left(e^{j\sqrt{2}\pi t} + e^{-j\sqrt{2}\pi t}\right))$. Therefore, $x_7(t)$ can be equivalently written as:

$$x_7(t) = \frac{1}{2}e^j\left(e^{j(1+\sqrt{2})\pi t} + e^{j(1-\sqrt{2})\pi t}\right)$$

. The period of the first term is $\frac{2\pi}{1+\sqrt{2}}$. The period of the second term is $\frac{2\pi}{1-\sqrt{2}}$. Since the ratio of these terms is irrational, this signal is not periodic.

(b) (4 points) Assume that the signal $x(t)$ is periodic with period T_0, and that $x(t)$ is odd (i.e. $x(t) = -x(-t)$). What is the value of $x(T_0)$?
Solutions:
Since $x(t)$ is periodic, we know that $x(t) = x(t + T_0)$. So $x(T_0) = x(0)$. Since $x(t)$ is odd, we know that $x(0) = -x(0)$. The only value that can satisfy this is $x(T_0) = 0$.

(c) (4 points) If $x(t)$ is periodic, are the even and odd components of $x(t)$ also periodic?
Solutions:
The equation for the even and odd components of $x(t)$ is $x_e(t) = \frac{1}{2}(x(t) + x(-t))$ and $x_o(t) = \frac{1}{2}(x(t) - x(-t))$. Since $x(t)$ is periodic, and we know that the sum of two signals with the same period also has the same fundamental period, we know that the even and odd components of $x(t)$ are indeed periodic.

4. (X points) **Energy and power signals.**

(a) (X points) Determine whether the following signals are energy or power signals. If the signal is an energy signal, determine its energy. If the signal is a power signal, determine its power.

i. $x(t) = e^{-|t|}$

ii. $x(t) = \begin{cases} \frac{1}{\sqrt{t}}, & \text{if } t \geq 1 \\ 0, & \text{otherwise} \end{cases}$

iii. $x(t) = \begin{cases} 1 + e^{-t}, & \text{if } t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

Solutions:

i. $x(t) = e^{-|t|}$

The energy is given by:

$$
\begin{aligned}
E &= \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} \left| e^{-|t|} \right|^2 dt = \int_{-\infty}^{\infty} e^{-2|t|} dt \\
&= \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt = -2 * \frac{1}{2} * e^{-2t}|_0^{\infty} \\
&= 1
\end{aligned}
$$

ii. $x(t) = \begin{cases} \frac{1}{\sqrt{t}}, & \text{if } t \geq 1 \\ 0, & \text{otherwise} \end{cases}$

The energy is given by:

$$
E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{1}^{\infty} \left| 1/\sqrt{t} \right|^2 dt = \int_{1}^{\infty} \frac{1}{t} dt = ln(t)|_{t=1}^{\infty} = \infty
$$

Therefore it's not an energy signal.

On the other hand the power is given by:

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} \frac{1}{t} dt$$
$$= \lim_{T\to\infty} \frac{1}{2T} ln(t)\Big|_{t=1}^{\infty} = 0.$$

Therefore, this signal is actually not an energy or a power signal.

iii. $x(t) = \begin{cases} 1 + e^{-t}, & \text{if } t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The energy is given by:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{0}^{\infty} \left|1 + e^{-t}\right|^2 dt = \int_{0}^{\infty} 1 + 2e^{-t} + e^{-2t} dt = \infty$$

Therefore it's not a energy signal.

On the other hand the power is given by:

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} (1 + e^{-t})^2 dt$$
$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} \left(1 + 2e^{-t} + e^{-2t}\right) dt$$
$$= \lim_{T\to\infty} \frac{1}{2T} \left(T - 2(e^{-T} + 1) - \frac{1}{2}(e^{-2T} - 1)\right)$$
$$= \lim_{T\to\infty} \frac{1}{2T} T = \frac{1}{2}$$

(b) (X points) Show the following two properties:

- If $x(t)$ is an even signal and $y(t)$ is an odd signal, then $x(t)y(t)$ is an odd signal;
- If $z(t)$ is an odd signal, then for any $\tau > 0$ we have:

$$\int_{-\tau}^{\tau} z(t)dt = 0$$

Use these two properties to show that the energy of $x(t)$ is the sum of the energy of its even component $x_e(t)$ and the energy of its odd component $x_o(t)$, i.e.,

$$E_x = E_{x_e} + E_{x_o}$$

Assume $x(t)$ is a real signal.

Solutions:

First property: $x(-t)y(-t) = x(t)(-y(t)) = -x(t)y(t)$, therefore it's odd.

Second property:

$$\int_{-\tau}^{\tau} z(t)dt = \int_{-\tau}^{0} z(t)dt + \int_{0}^{\tau} z(t)dt$$

We apply to the first integral the following variable change: $t = -\lambda$.

$$\int_{-\tau}^{\tau} z(t)dt = -\int_{\tau}^{0} z(-\lambda)d\lambda + \int_{0}^{\tau} z(t)dt$$

We then change the order of the limits of the first integral:

$$\int_{-\tau}^{\tau} z(t)dt = \int_{0}^{\tau} z(-\lambda)d\lambda + \int_{0}^{\tau} z(t)dt$$

Since $z(t)$ is an odd signal, we then have $z(-\lambda) = -z(\lambda)$. Thus,

$$\int_{-\tau}^{\tau} z(t)dt = -\int_{0}^{\tau} z(\lambda)d\lambda + \int_{0}^{\tau} z(t)dt = 0$$

The energy of signal $x(t)$ is given by:

$$\begin{aligned} E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt &= \int_{-\infty}^{\infty} |x_e(t) + x_o(t)|^2 \, dt \\ &= \int_{-\infty}^{\infty} \left(x_e^2(t) + x_o^2(t) + 2x_e(t)x_o(t)\right) dt \\ &= \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt = E_e + E_o \end{aligned}$$

This is because $2x_e(t)x_o(t)$ is odd, therefore its integral is zero (according to the second property).

5. (17 points) **Euler's identity and complex numbers.**

   (a) (9 points) Use Euler's formula to prove the following identities:

      i. $\cos^2(\theta) + \sin^2(\theta) = 1$

      ii. $\cos(\theta + \psi) = \cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi)$

   Solutions:

      i. $e^{j\theta} = cos(\theta) + j\sin(\theta)$ and $e^{-j\theta} = cos(\theta) - j\sin(\theta)$.
      Thus, $(cos(\theta) + j\sin(\theta))(cos(\theta) - j\sin(\theta)) = \cos^2(\theta) + \sin^2(\theta) = e^{j\theta} \times e^{-j\theta} = 1$

      ii. $\cos(\theta) = (e^{j\theta} + e^{-j\theta})/2$
      $\sin(\theta) = (e^{j\theta} - e^{-j\theta})/2j$
      $\cos(\theta) \times \cos(\psi) = (e^{j(\theta+\psi)} + e^{-j(\theta+\psi)} + e^{j(\theta-\psi)} + e^{j(\psi-\theta)})/4$
      $\sin(\theta) \times \sin(\psi) = (-e^{j(\theta+\psi)} - e^{-j(\theta+\psi)} + e^{j(\theta-\psi)} + e^{j(\psi-\theta)})/(-4)$
      $Thus, \cos(\theta) \times \cos(\psi) - \sin(\theta) \times \sin(\psi) = (e^{j(\theta+\psi)} + e^{-j(\theta+\psi)})/2 = \cos(\theta + \psi)$

   (b) (8 points) Show that $e^{j\theta} = 2sin(\frac{\theta}{2})e^{j[(\theta+\pi)/2]} + 1$.
   Solutions:

$$\begin{aligned} e^{j\theta} &= 2sin(\frac{\theta}{2})e^{j[(\theta+\pi)/2]} + 1 \\ &= 2(\frac{e^{j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}}}{2j})e^{j\frac{\theta}{2}}e^{j\frac{\pi}{2}} + 1 \\ &= (e^{j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}})e^{j\frac{\theta}{2}} + 1 \\ &= e^{j\theta} \end{aligned}$$

(c) (4 points) $x(t) = (5 + \sqrt{2}j)e^{j(t+2)}$ and $y(t) = 1/(2 - j)$.

i. Compute the real and imaginary parts of $x(t)$ and $y(t)$.

ii. Compute the magnitude and phase of $x(t)$ and $y(t)$.

Solutions:

i. $x(t) = (5 + \sqrt{2}j)e^{j(t+2)} = (5 + \sqrt{2}j)(\cos(t + 2) + j\sin(t + 2)) = 5\cos(t + 2) - \sqrt{2}\sin(t + 2) + j(\sqrt{2}\cos(t + 2) + 5\sin(t + 2))$.
Therefore, the real part is: $5\cos(t + 2) - \sqrt{2}\sin(t + 2)$. The imaginary part is:$\sqrt{2}\cos(t + 2) + 5\sin(t + 2)$.

$y(t) = 1/(2 - j) = \frac{2+j}{(2-j)(2+j)} = 2/5 + 1/5j,$. Therefore, the real part is: $\frac{2}{5}$. The imaginary part is: $\frac{1}{5}$

ii. $x(t) = (5+\sqrt{2}j)e^{j(t+2)}$. First, we find the radius of the complex exponential representing $5+\sqrt{2}j)$, which is $3\sqrt{3}$. The angle of the complex exponential is $\tan^{-1}(\frac{\sqrt{2}}{5})$. Thus, we get $x(t) = 3\sqrt{3}e^{j\tan^{-1}(\frac{\sqrt{2}}{5})}e^{j(t+2)} = 3\sqrt{3}e^{j(t+2+\tan^{-1}(\frac{\sqrt{2}}{5}))}$. Therefore the magnitude is: $3\sqrt{3}$ and the phase is $(t + 2 + \tan^{-1}(\frac{\sqrt{2}}{5}))$ rad.

$y(t) = \frac{2}{5} + \frac{1}{5}j = \frac{1}{\sqrt{5}}e^{j\tan^{-1}(\frac{1}{2})}$, yielding a magnitude of $1/\sqrt{5}$, and a phase of $\tan^{-1}(\frac{1}{2})$ rad

6. (15 points) **Python tasks**
For this question, please check the provided jupyter notebook solution file.