# EE102

Lecture 3

# EE102 Announcements

- **Syllabus link** is on BruinLearn
- **Results of Discussion Section Format**
    - **Majority voted for recorded discussion. We will upload discussion videos before 12 PM every Friday.**
- **First Homework (on lectures 1 and 2) due 10/10 11:59 pm**
- **Second Homework will be released 10/3 (on lectures 3 and 4)**
- **While people shuffle in, ponder about the notion of infinity?**

what is the nature of **infinity**?

is **infinity / infinity = 1?**

is **infinity + 1 > infinity?**

$\boxed{\text{cyu}}$ $x(t) = j^{13} e^{2+j3t}$

Find the Real and Imag Parts.

Ans: $j^{13} = j^4 j^4 j^4 j = j$

$$x(t) = je^2 e^{3tj}$$

$$= je^2 \left[\cos(3t) + j\sin(3t)\right]$$

$$= \underbrace{-e^2 \sin(3t)}_{a} + \underbrace{je^2 \cos(3t)}_{bj}$$

$= R = -e^2 \sin(3t)$

$I = e^2 \cos(3t)$

Signal power has units of Watts (Joules per time). Hence, to get the total energy of a signal, $x(t)$, across all time, we integrate the power.

$$\boxed{E_x = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt}$$

(We incorporate the absolute value, $|\cdot|$, in case $x(t)$ is a complex signal, reviewed in the next slides.) Like signal power, signal energy is usually not a *actual* energy.

We can also calculate the *average power* of the signal by calculating:

$$\boxed{P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt}$$

Can we simplify this expression to obtain the power of a periodic signal?

$E_x = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$ CYU $P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

## Finite Energy and Finite Power Signals

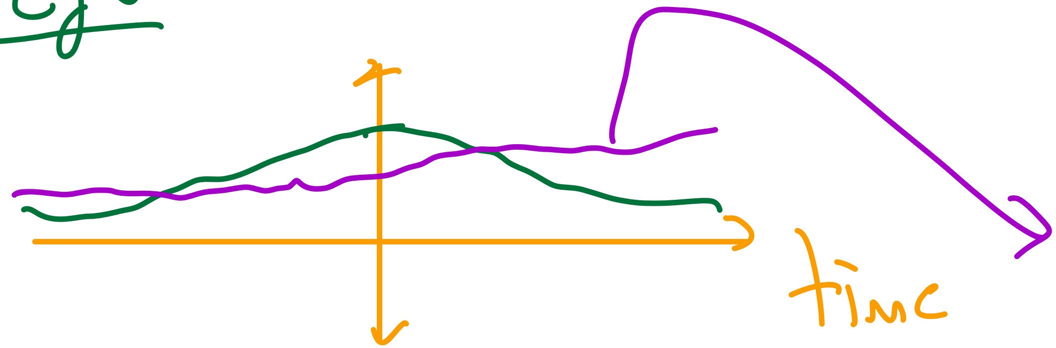
If $0 < E_x < \infty$, then $x$ is "energy signal"

If $0 < P_x < \infty$, then $x$ is "pwr signal"

CYU: If $E_x = C$, then what can you say of $P_x$? $P_x = 0$

CYU

Is green signal an energy or power signal?

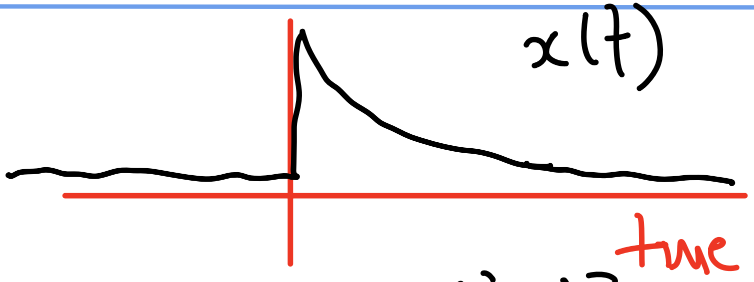



Power Signal

# CYU

## Finite Energy and Finite Power Signals

CYU: $x(t) = \begin{cases} Ae^{-at}, & t \geq 0 \quad a > 0 \\ 0, & \text{o.w.} \end{cases}$




Pls tell me if $x(t)$ is power signal or energy signal?

Ans: ① "Proof" by picture $E_x = C$

② Analysis $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{\infty} A^2 e^{-2at} dt$

$$= \frac{A^2}{-2a} e^{-2at} \Big|_0^{\infty}$$

$$= 0 - \frac{A^2}{-2a} = \frac{A^2}{2a}$$

$P_x = 0$

# Elementary Signal Models

## Real sinusoids

We previously discussed the real sinusoid, which we'll recap here for completeness of these notes. A cosine is defined by:

$$\begin{aligned} x(t) &= A\cos(\omega t - \theta) \\ &= A\cos(2\pi f t - \theta) \end{aligned}$$

with

- $A$ defining the amplitude of the signal (i.e., how large it gets).
- $\omega$ defining the ____ frequency of the signal (in units of radians per second). As $\omega$ gets larger, the sinusoid repeats more times in a given time interval.
- The natural frequency is related to the frequency, $f$, of the signal (in units of Hertz, or $s^{-1}$) through the relationship: $\omega = 2\pi f$. The frequency, $f$, is the inverse of the period, i.e.,

$$T_0 = \frac{1}{f} = \frac{2\pi}{\omega}$$

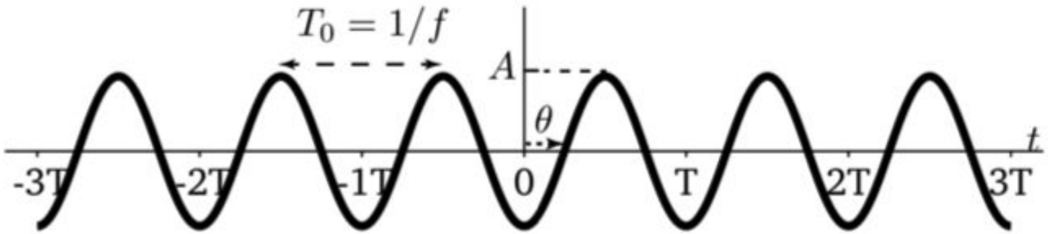
- $\theta$ is the phase of the signal in terms of radians, shifting the sinusoid.

$$\mathbb{R}\left\{ e^{j(\omega t - \theta)} \right\}$$

## Real sinusoids (cont.)

We illustrate a sinusoid signal below:

$$x(t) = A\cos(\omega t - \theta)$$



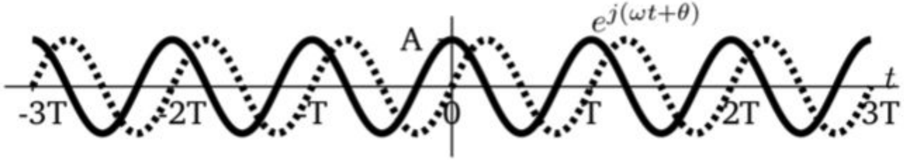
$e^{jx} = \cos(x) + j\sin(x)$ $\mathbb{R}[e^{jx}]$ $\mathbb{I}[e^{jx}]$

## Complex sinusoids

The complex sinusoid is given by:

$$Ae^{j(\omega t+\theta)} = A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)$$

We draw complex signals with dotted lines.




The real part of the complex sinusoid (solid line) is:

$$\Re\left(Ae^{j(\omega t+\theta)}\right) = A\cos(\omega t+\theta)$$

The imaginary part of the complex sinusoid (dotted line) is:

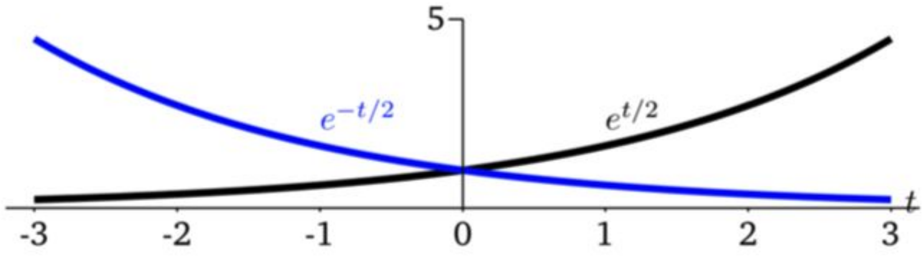
$$\Im\left(Ae^{j(\omega t+\theta)}\right) = A\sin(\omega t+\theta)$$

## Exponential

An exponential signal is given by

$$x(t) = e^{\sigma t}$$

- If $\sigma > 0$, this signal grows with increasing $t$ (black signal in plot below). This is called exponential growth.
- If $\sigma < 0$, this signal decays with increasing $t$ (blue signal in plot below). This is called exponential decay.



# Sidebar: Regarding Periodic Signals

The sum or product of a periodic signal is itself periodic if:

$x_1$: period $T_1$ $\rightsquigarrow$ $f(t+T_1) = f(t) \ \forall t,$

$x_2$: period $T_2$

cyu: $z = x_1 + x_2$ .... when is $z$ periodic and what period?

Ans. $z$ is periodic if $\exists\, T$ s.t. $z(t+T) = z(t) \ \forall t$

$T = k_1 T_1 = k_2 T_2$ | $\dfrac{T_1}{T_2} = \dfrac{k_2}{k_1}$

cyu: Can you find constants when this does not hold.

# CYU: Periodic Signals

Come up with two signals x1, x2 such that the sum is aperiodic

CYU: Come up w/ $x_1$ and $x_2$ s.t. sum is aperiodic

$$\frac{T_1}{T_2} = c\pi$$

$$\cos(\pi t) + \cos(2t) = z$$

Aperiodic

# CYU: Periodicity of a Complex Exponential

For the following signal, determine if it is periodic. If it is, what is its fundamental period:

$$x(t) = e^{j(\pi t+1)} cos(2\pi t)$$

Ans. $\underline{x(t)} = e^{j(\pi t+1)} \cdot \frac{1}{2}\left(e^{j2\pi t} + e^{-j2\pi t}\right)$

$= \frac{1}{2} e^{j} e^{j\pi t}\left(e^{j2\pi t} + e^{-j2\pi t}\right)$

$= \frac{1}{2} e^{j}\left(\underline{e^{j3\pi t}} + \underline{e^{-j\pi t}}\right)$

Period: 2

Period: 2/3

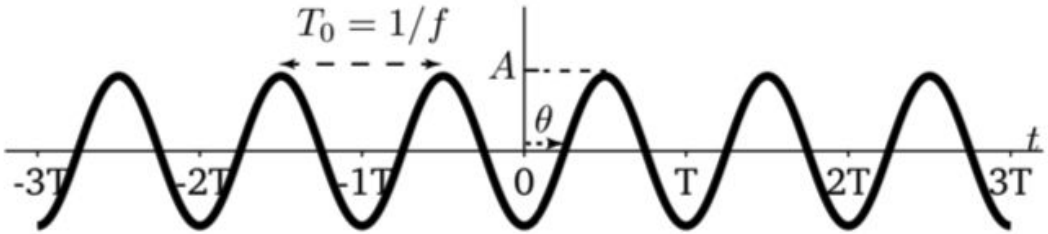
Period: 2

# Signal Models

## Real sinusoids (cont.)

We illustrate a sinusoid signal below:

$$x(t) = A\cos(\omega t - \theta)$$



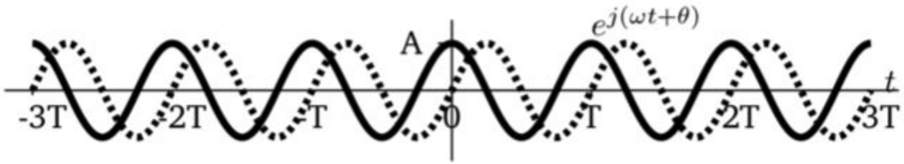
# Signal Models

## Complex sinusoids

The complex sinusoid is given by:

$$Ae^{j(\omega t+\theta)} = A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)$$

We draw complex signals with dotted lines.




The real part of the complex sinusoid (solid line) is:

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# Signal Models

## Exponential

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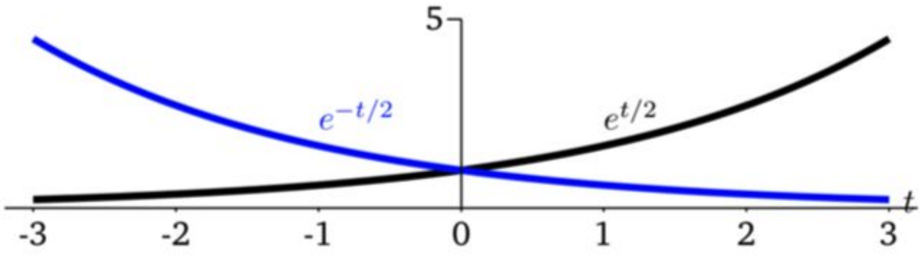


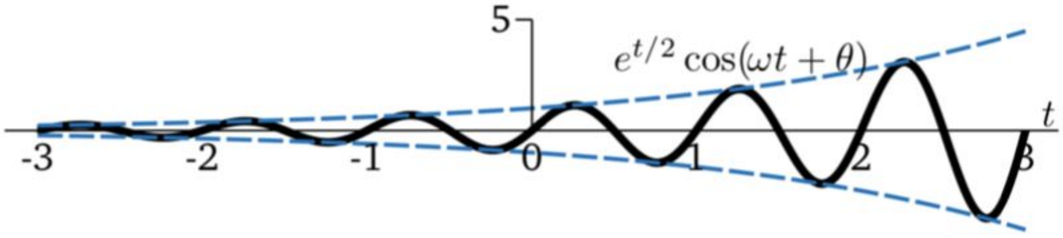
# Signal Models

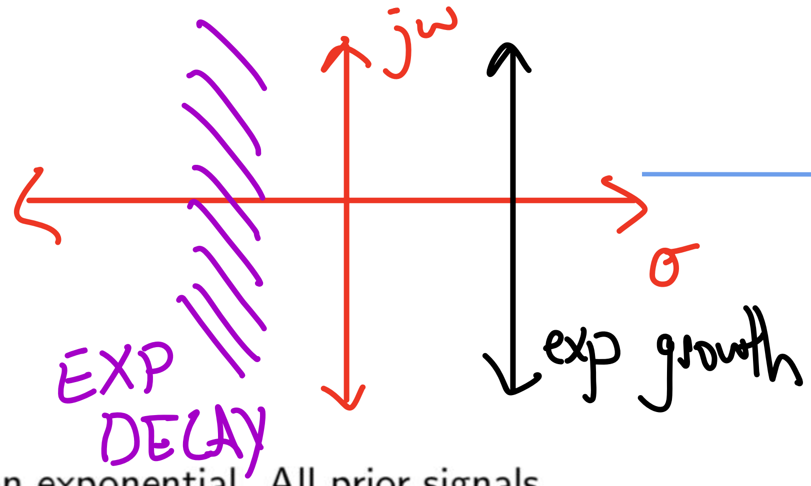
## Damped or growing sinusoids

A damped or growing sinusoid is denoted

$$x(t) = e^{\sigma t}\cos(\omega t + \theta)$$

The sinusoid will grow exponentially if $\sigma > 0$ and decay exponentially if $\sigma < 0$.



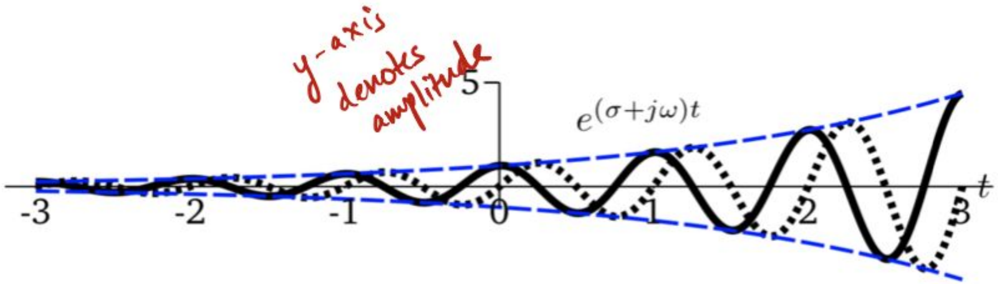



## Complex exponential

A complex sinusoid is denoted

$$x(t) = e^{(\sigma + j\omega)t}$$

It is a combination of the complex sinusoid and an exponential. All prior signals are special cases of the complex exponential signal.



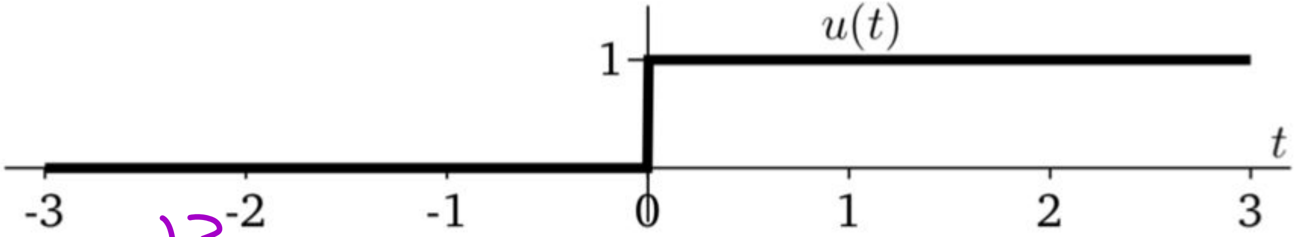

It is helpful to think of $\sigma$ and $j\omega$ in the complex plane. $\sigma$ is the x-axis and $j\omega$ is the y-axis. Then complex exponentials in the left complex plane are decreasing signals and those in the right are increasing signals.

# Heaviside Step Function

The unit step function, denoted $u(t)$ in this class, is given by

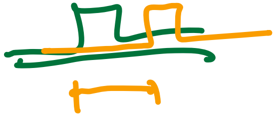
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

It is also called the Heaviside step function. Drawn below:




CYN: Is this causal?

Ans: Yes

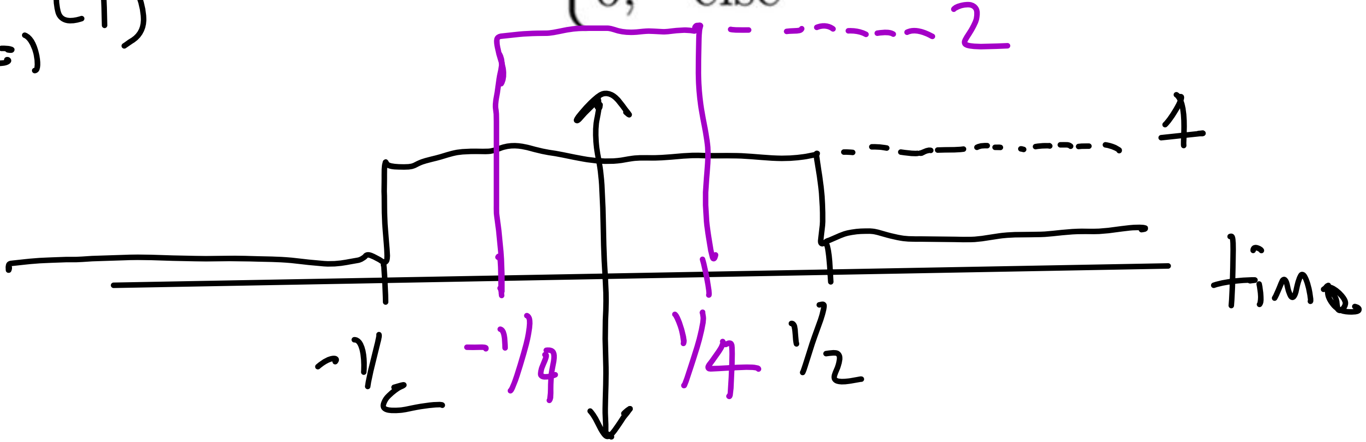
LIDAR 

# Unit Rectangle "Boxcar"

rect(t) : width/support of 1

$\text{rect}(t) = \text{rect}_{\Delta=1}(t)$

$$\text{rect}(t) = \begin{cases} 1, & |t| \le 1/2 \\ 0, & \text{else} \end{cases}$$



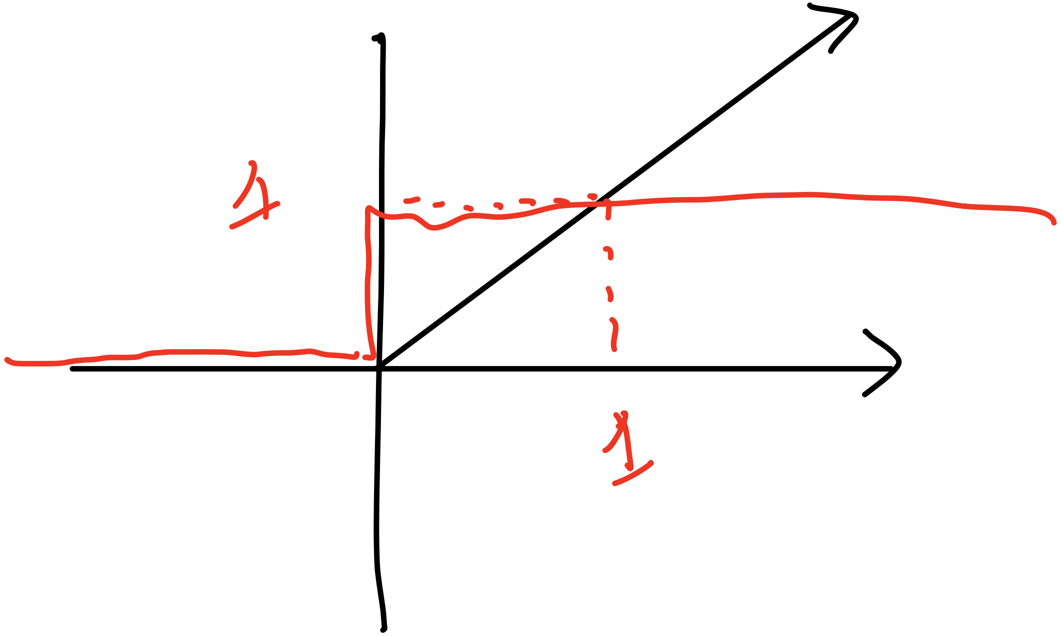

$$\text{rect}_{\Delta}(t) = \begin{cases} 1/\Delta, & |t| \le \Delta/2 \\ 0, & \text{o.w.} \end{cases}$$

$\text{rect}_{\Delta=0.5}(t) = ?$

ReLU

# Unit Ramp Function

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & \text{o.w.} \end{cases}$$




o.w. "otherwise"

# CYU: Unit Ramp Function

How do I express the unit ramp function in terms of the previous building blocks I've learned?

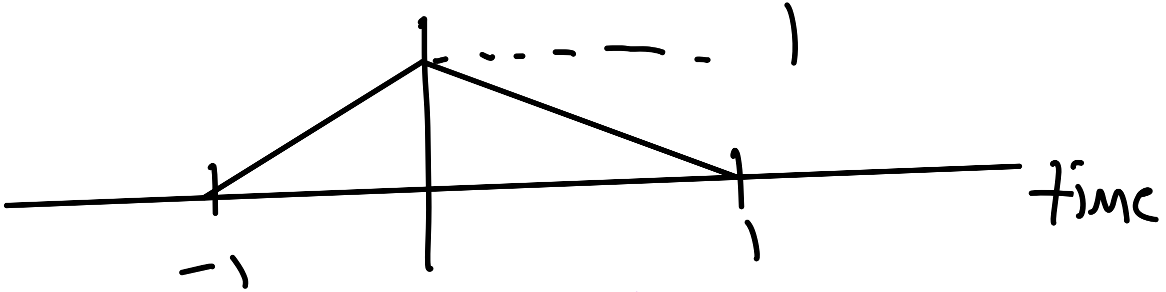
$$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$$

$\tau$ : dummy variable

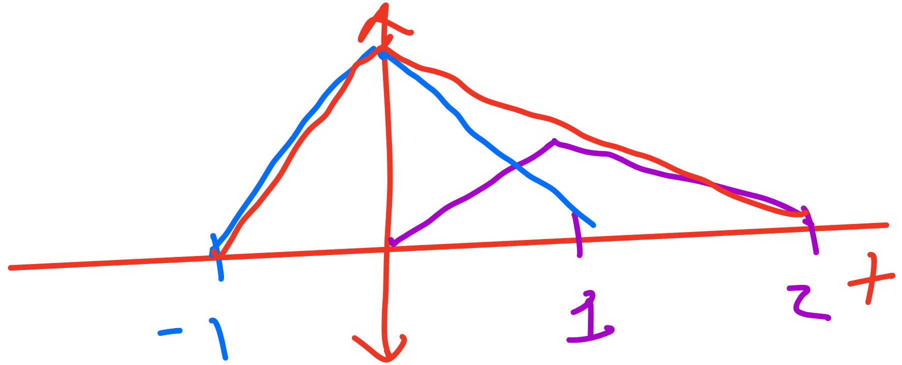
$$r(t) = t\,u(t)$$

# Unit Triangle

$$\Delta(t) = \begin{cases} 1-|t|, & |t| \leq 1 \\ 0, & \text{o.w.} \end{cases}$$




$$z = 2\Delta(t) + \Delta(t-1)$$



Dirac

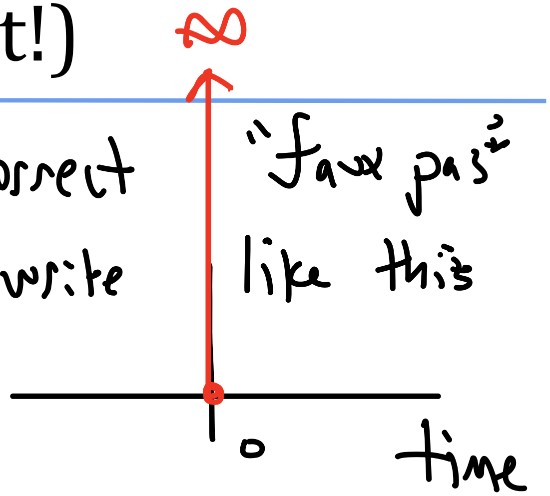
# Impulse Function (Important!)

$$\xcancel{\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & \text{o.w.} \end{cases}}$$

Incorrect to write

"faux pas"

like this

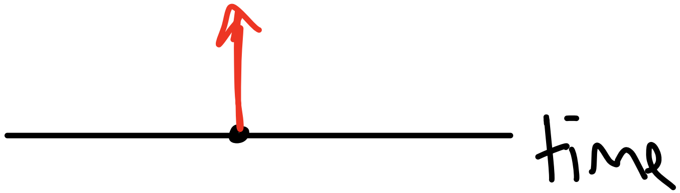



Properties of $\delta(t)$

① It's v. large at $t=0$

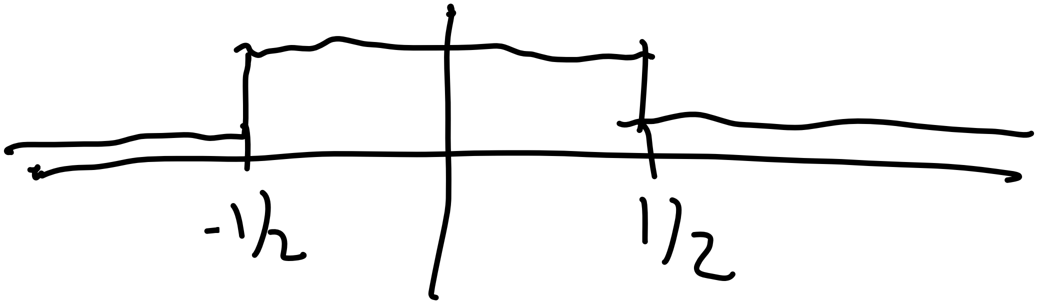
② It's zero everywhere else

③ The area under curve $=1$

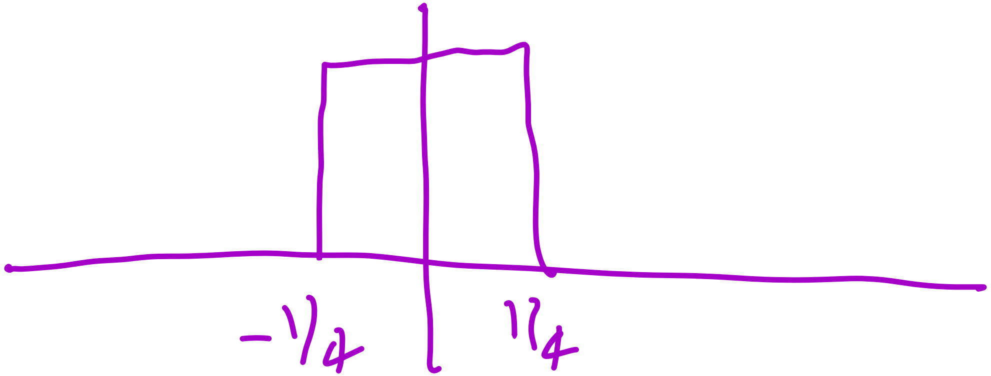


# Impulse Function (intuition)

$\text{rect}(t)$



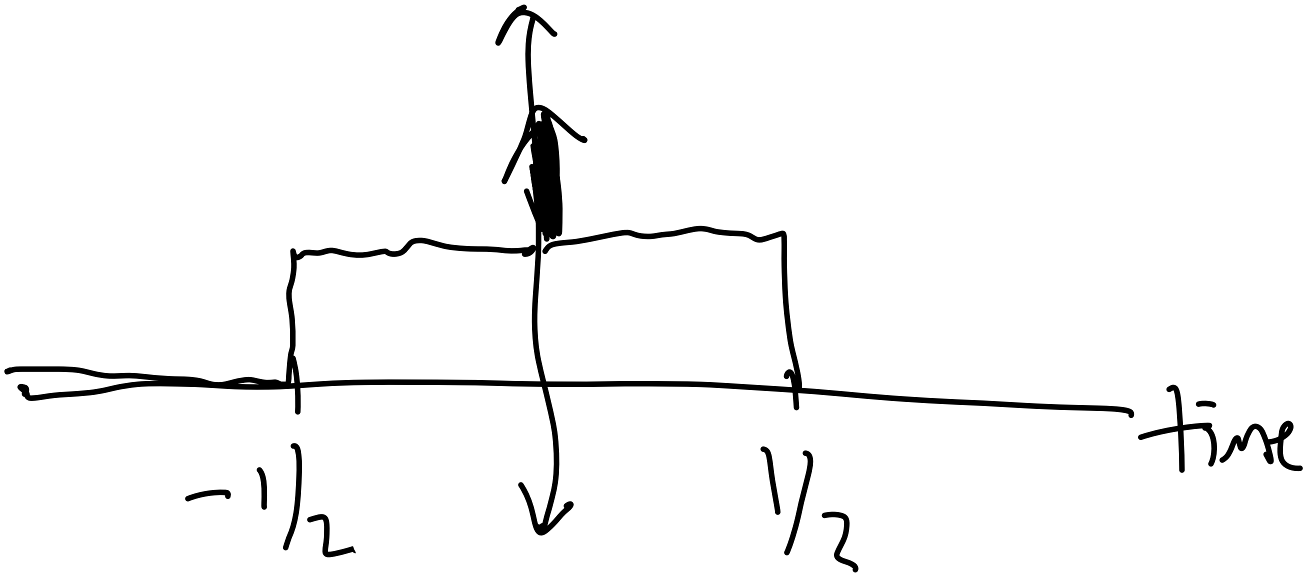

$\text{rect}_{1/2}(t)$




$\delta(t)$ is $\lim_{\Delta \to 0} \text{rect}_{\Delta}(t)$

# Impulse Function Intuition

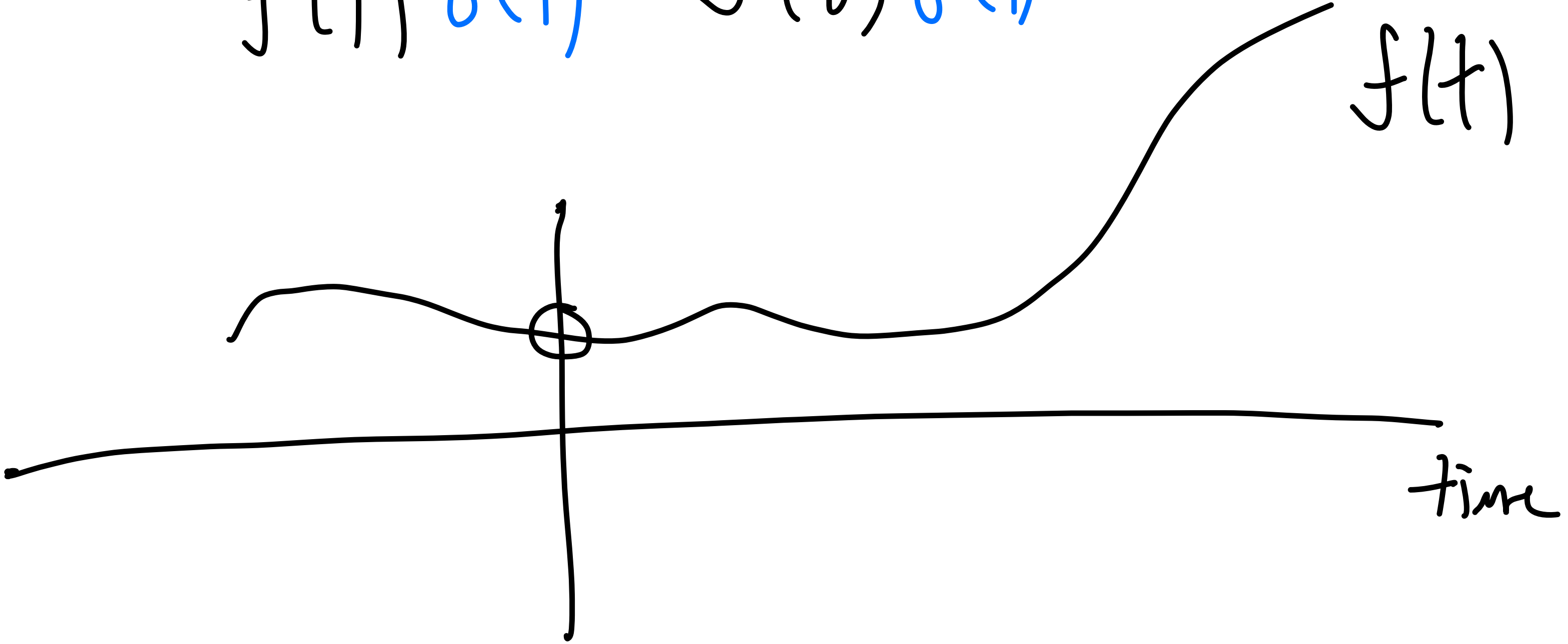
$$x(t) = \text{rect}(t) + \delta(t)$$



# Impulse Sampling Property

$f(t=0)$

$$f(t)\,\delta(t) = f(0)\,\delta(t)$$



# Impulse Sampling Property

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} f(0)\,\delta(t)\,dt$$

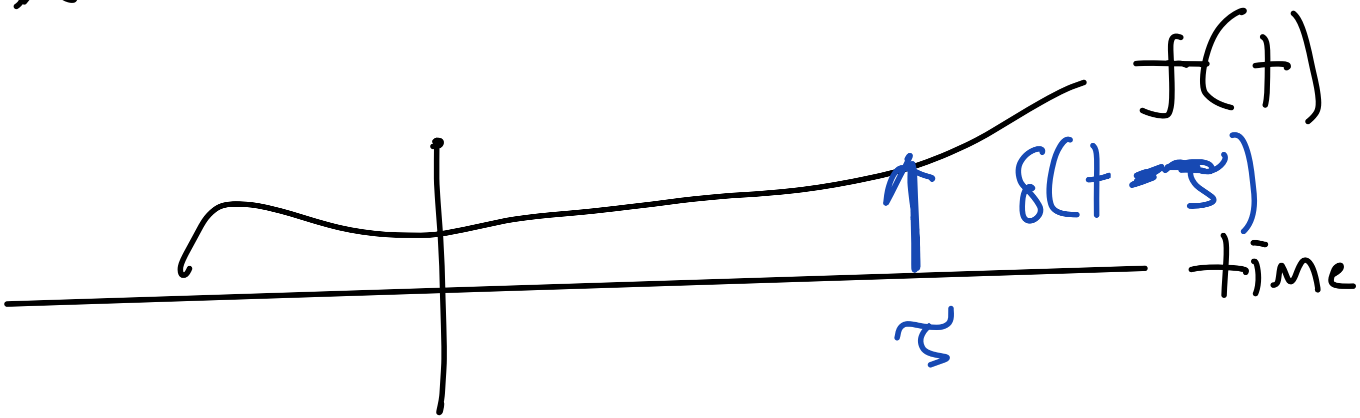
$$= f(0)\underbrace{\int_{-\infty}^{\infty} \delta(t)\,dt}_{=1}$$

$$= f(0)$$

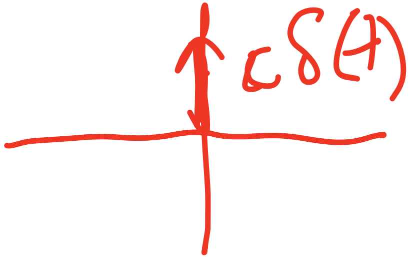
Dirac Delta Has Area Under Curve of 1.

# Impulse Sifting Property

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-\tau)\,dt = f(\tau)$$




// $c\,\delta(t)$ $\quad \int f(t)\,c\,\delta(t) = c\,f(0).$



# Impulse Sifting Property

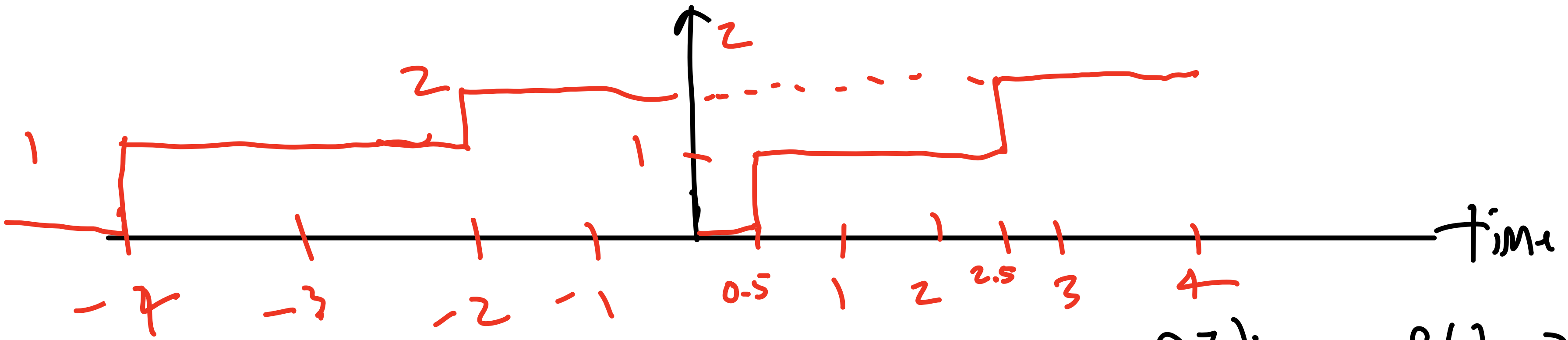
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\int_{-\infty}^{0^-} \delta(t)\, dt = 0$$

$$\int_{-\infty}^{0^+} \delta(t)\, dt = 1$$

# CYU: Calculate

$$\int_{-2}^{3+} f(t)\,[1 + \delta(t+1) - 3\delta(t-1) + 2\delta(t+3)]\,dt$$




$$\int_{-2}^{3+} f(t)\,dt + f(-1) - 3f(1) + 2\underbrace{\int_{-2}^{3+} f(t)\delta(t+3)\,dt}_{0}$$

$$7 + 2 - 3 + 0 = 6$$

# CYU: Integral of an Impulse

$$\int_{-\infty}^{t} \delta(\tau) d\tau =$$

# CYU: Visual

Suppose $x(t) = 1 + \delta(t-1) - 2\delta(t-2)$ then what is $y(t) = \int_0^t x(\tau)d\tau$