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ECE102, Fall 2025
Signals & Systems
University of California, Los Angeles; Department of ECE

Practice Final Exam
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Name: ____________________

Signature: ____________________

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Total	_____	/ 115 points

1. **Signal and Systems Basics** (21 points)

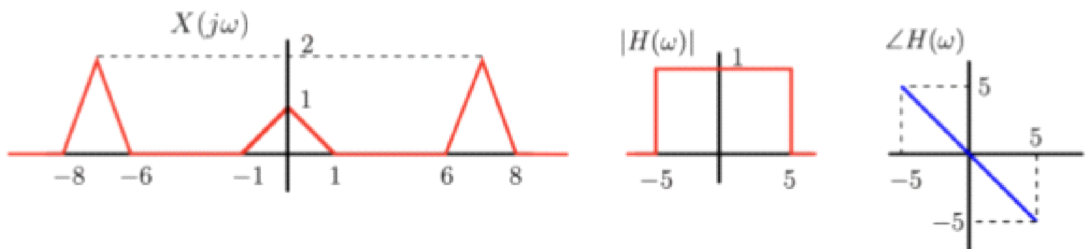
   (a) (12 points) **System properties.** For each of the following systems, determine (with reasoning) if they are linear, time invariant, causal and stable.

   i. (4 points) $y(t) = x(3t+2) + 5$ Not TI, Not Linear stable Not Causal

   ii. (4 points) $y(t) = sin(\frac{dx(t)}{dt})$ Not Linear, TI, stable causal

   iii. (4 points) $y(t) = y(t) = e^{x^2(t)}$ Not Linear, TI, stable, causal

   (b) (9 points) **LTI System Analysis.** Consider an LTI system with input $x(t)$, output $y(t)$ and impulse response $h(t)$. The Fourier transforms $X(j\omega)$ and $H(j\omega)$ are as shown below.



Evaluate $y(t)$.

$$Y(\omega) = X(\omega)\, H(\omega) \qquad |H(\omega)|\, e^{j\angle H(\omega)}$$

$$Y(\omega) = e^{-\omega j} \qquad -5 \le \omega \le 5$$

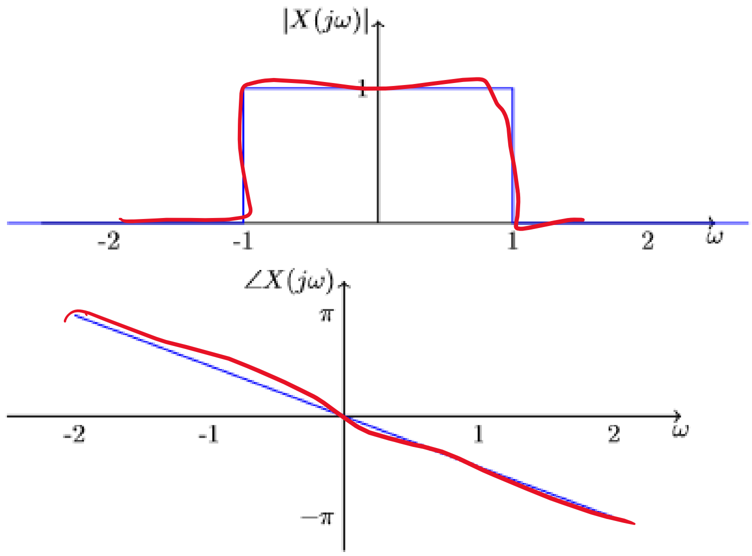
$$Y(\omega) = \Delta(\omega)\, e^{-\omega j} \qquad \Delta\left(\frac{\omega}{2\pi}\right) \qquad \frac{1}{2\pi} = a$$

$$Y(\omega) = \frac{1}{2\pi}\, 2\pi \Delta(\omega)$$

$$\boxed{y(t) = \frac{\mathrm{sinc}^2\left(\frac{1}{2\pi}(t-1)\right)}{2\pi}}$$

2. **Fourier transform** (29 points)

(a) (12 points) A signal x(t) has has the following the Fourier Transform.



$|e^{-\frac{\pi}{2}\omega}$

Plot the magnitude and phase plots for the Fourier Transform of the following signals:

i. (4 points) $x(t/2)$

ii. (4 points) $x(t-3)$

iii. (4 points) $re(x(t))$

(b) (12 points) Evaluate the Fourier Transforms of the following signals:

i. (4 points) $x(t) = e^{-2|t-1|}$

ii. (5 points) $x(t) = te^{-at}cos(\omega_0 t)u(t)$, $a > 0$

(c) (5 points) Evaluate the the time domain signals corresponding to the following Fourier transforms:

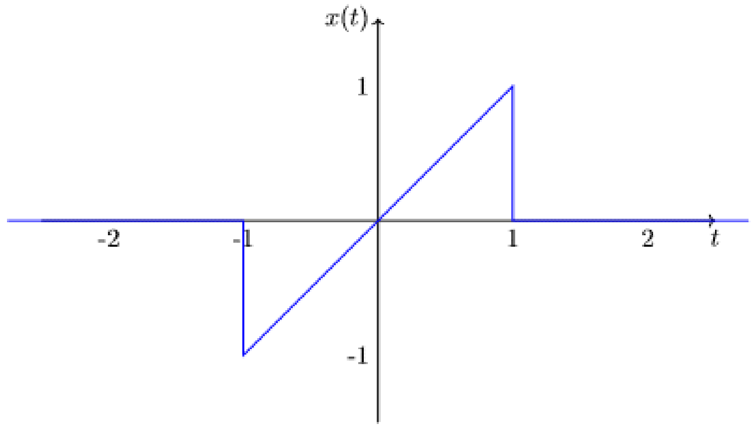
i. (3 points)

$$X(j\omega) = \begin{cases} 1 - |\omega|, & |\omega| < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

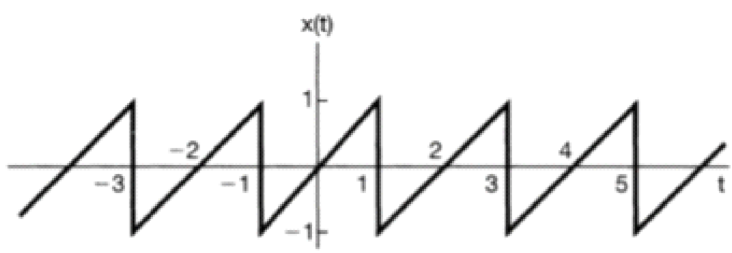
ii. (2 points) $X(j\omega) = cos(2\omega + \frac{\pi}{6})$

3. **Fourier Series** (15 points)

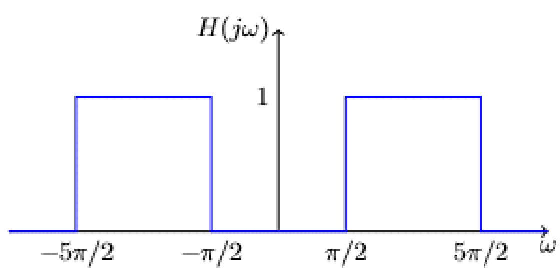
(a) (5 points) Evaluate the Fourier Transform of the following signal $x(t)$.



(b) (5 points) Using your solution from part (a), evaluate the fourier series of the following signal $\tilde{x}(t)$.



(c) (5 points) Consider a system whose frequency response $H(j\omega)$ as follows:



What is the output when $\tilde{x}(t)$ is passed through this system?

4. **Frequency domain understanding** (20 points)

(a) Identify if the following statements are 'True' or 'False' with appropriately detailed reasoning.

i. (1 points) Sampling at a frequency greater than Nyquist rate is a necessary condition for perfect reconstruction, for every signal.

ii. (2 points) If we have two bandlimited signals, $x_1(t)$ with a bandwidth B_1 and $x_2(t)$ with a bandwidth B_2, the signal $y(t) = x_1(t)x_2(t)$ has a bandwidth $max\{B_1, B_2\}$. ($max\{a, b\}$ is equal to the maximum value among a and b)

iii. (2 points) Consider a periodic function $x(t)$ with a fundamental period T. If $x(t)$ is an odd function, the sum of all its fourier series coefficients ($\Sigma_{k=-\infty}^{\infty} c_k$) is zero.

(b) Let $F(j\omega) = j2\pi\omega e^{-2|\omega|}$. Without computing $f(t)$ answer the following questions with appropriate reasoning.

i. (2 points) Is $f(t)$ real/imaginary/complex?

ii. (2 points) Is $f(t)$ odd/even/neither?

iii. (1 points) What is $f(0)$?

(c) Evaluate the following.

i. (7 points) Let $x(t) = \frac{4}{4+t^2}$. Evaluate the Fourier transform $X(j\omega)$. (Hint: use the duality property)

ii. (3 points) Using the Fourier transform from the previous part, evaluate the energy of $x(t)$.

5. **Sampling** (15 points)

(a) (4 points) The sampling theorem says that for a bandlimited signal, a signal must be sampled at a frequency greater than the Nyquist rate to guarantee perfect reconstruction. Identify the minimum sampling rate, f_s (Hz), needed to accurately capture a signal without aliasing:

i. (2 points) $x(t) = cos(3000\pi t) - sin(2000\pi t)$

ii. (2 points) $x(t) = \frac{sin(2000\pi t)}{\pi t}$

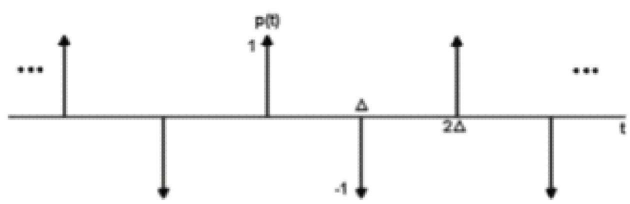
(b) (6 points) Consider a signal $x(t)$ with a Nyquist rate, ω_0. Determine the Nyquist rate for the following signals:

i. (2 points) $x^2(t)$

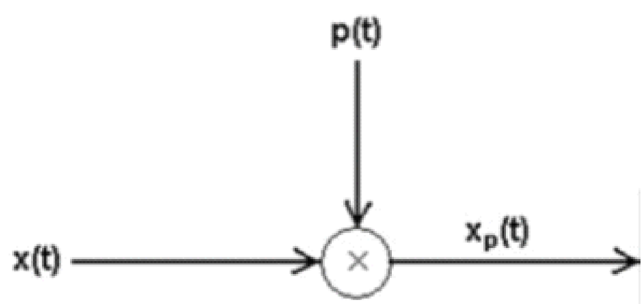
ii. (2 points) $x(t)cos(\omega_1 t)$

iii. (2 points) $\frac{dx(t)}{dt}$

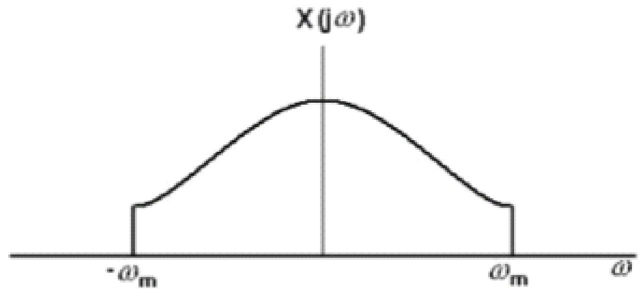
(c) (5 points) We know that ideal sampling is carried out by multiplying the time domain analog signal with an impulse train. Consider a modified sampling regime, where we multiply with the following signal $p(t)$:



The sampling process for a signal $x(t)$ is shown in the following figure:



Let $x(t)$ be bandlimited with a one-sided bandwidth of ω_{max}, with the following fourier transform:



i. (2 points) If $\Delta < \frac{\pi}{\omega_{max}}$, draw the Fourier transform of $x_p(t)$.
ii. (3 points) If $\Delta < \frac{\pi}{\omega_{max}}$, determine a system to recover $x(t)$ from $x_p(t)$.

6. **Laplace transform** (15 points)

A casual LTI system can be described by the following differential equation:

$$y''(t) + 4y'(t) + 4y(t) = 4x(t) + 1x''(t).$$

You may assume resting initial conditions (y(0)= 0, y'(0) =0, y"(0) = 0, x(0) = 0, x'(0) = 0, x"(0) = 0)

(a) (7 points) Find the transfer function $H(s)$.

(b) (8 points) What is the impulse response h(t) of this system?