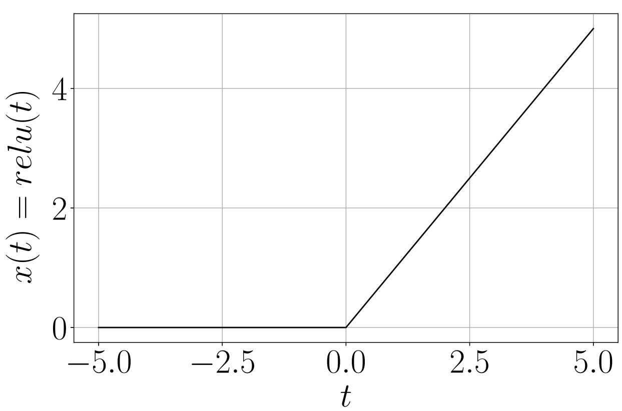
ECE102, Fall 2025 **Homework #1**
Signals & Systems Prof. Achuta Kadambi
University of California, Los Angeles; Department of ECE TAs: A. Vilesov, H. Zhang

Due Friday, 10 October 2025, by 11:59pm to Gradescope.
Covers material up to Lecture 2.
100 points total.

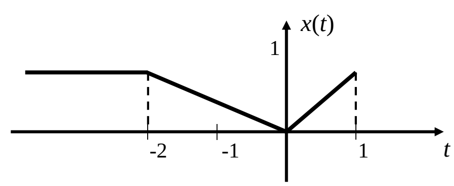
1. (10 points) **Even and odd parts.**
Sketch and write the even and odd components of the following signal:

$$x(t) = relu(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$



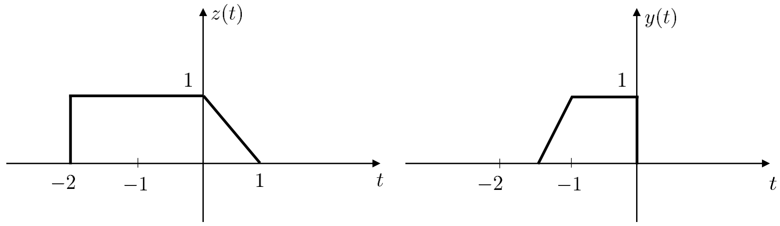
2. (15 points) **Time scaling and shifting.**

(a) (10 points) Consider the following signal.



Sketch the following:

i. $x(\frac{1}{2}t - 1)$
ii. $x(-3t - 2)$



(b) (5 points) The figure below shows two signals: $z(t)$ and $y(t)$. Can you express $y(t)$ in terms of $z(t)$?

3. (22 points) **Periodic signals.**

(a) (14 points) For each of the following signals, determine whether it is periodic or not. If the signal is periodic, determine the fundamental period and frequency.

i. $x_1(t) = \sin(2t + \pi/3)$

ii. $x_2(t) = \cos(\sqrt{2}\pi t)$

iii. $x_3(t) = \sin^2(3\pi t + 3)$

iv. $x_4(t) = x_1(t) + x_2(t)$

v. $x_5(t) = x_1(\pi t) + x_3(t)$

vi. $x_6(t) = e^{-t} x_1(t)$

vii. $x_7(t) = e^{j(\pi t+1)} x_2(t)$

(b) (4 points) Assume that the signal $x(t)$ is periodic with period T_0, and that $x(t)$ is odd (i.e. $x(t) = -x(-t)$). What is the value of $x(T_0)$?

(c) (4 points) If $x(t)$ is periodic, are the even and odd components of $x(t)$ also periodic?

4. (21 points) **Energy and power signals.**

(a) (15 points) Determine whether the following signals are energy or power signals. If the signal is an energy signal, determine its energy. If the signal is a power signal, determine its power.

i. $x(t) = e^{-|t|}$

ii. $x(t) = \begin{cases} \frac{1}{\sqrt{t}}, & \text{if } t \geq 1 \\ 0, & \text{otherwise} \end{cases}$

iii. $x(t) = \begin{cases} 1 + e^{-t}, & \text{if } t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

(b) (6 points) Show the following two properties:

- If $x(t)$ is an even signal and $y(t)$ is an odd signal, then $x(t)y(t)$ is an odd signal;
- If $z(t)$ is an odd signal, then for any $\tau > 0$ we have:

$$\int_{-\tau}^{\tau} z(t)dt = 0$$

Use these two properties to show that the energy of $x(t)$ is the sum of the energy of its even component $x_e(t)$ and the energy of its odd component $x_o(t)$, i.e.,

$$E_x = E_{x_e} + E_{x_o}$$

Assume $x(t)$ is a real signal.

5. (17 points) **Euler's identity and complex numbers.**

   (a) (9 points) Use Euler's formula to prove the following identities:

      i. $\cos^2(\theta) + \sin^2(\theta) = 1$

      ii. $\cos(\theta + \psi) = \cos(\theta)\cos(\psi) - \sin(\theta)\sin(\psi)$

   (b) (4 points) Show that $e^{j\theta} = 2sin(\frac{\theta}{2})e^{j[(\theta+\pi)/2]} + 1$.

   (c) (4 points) $x(t) = (5 + \sqrt{2}j)e^{j(t+2)}$ and $y(t) = 1/(2 - j)$.

      i. Compute the real and imaginary parts of $x(t)$ and $y(t)$.

      ii. Compute the magnitude and phase of $x(t)$ and $y(t)$.

6. (15 points) **Python tasks**
   For this question, please complete the included Jupyter Notebook from the zip file.

   Include all relevant code and plots as a pdf of the Jupyter Notebook appended to the end of the homework. You do not need to submit the actual ".ipynb" file, simply a pdf of the notebook will be fine.

   If you would like to complete the assignment in another programming language, you are welcome to, but you will have to translate the skeleton code from the provided notebook to the preferred language yourself.