ECE102, Fall 2025
Signals & Systems
University of California, Los Angeles; Department of ECE

Homework #4
Prof. A. Kadambi
TAs: H. Zhang, A. Vilesov

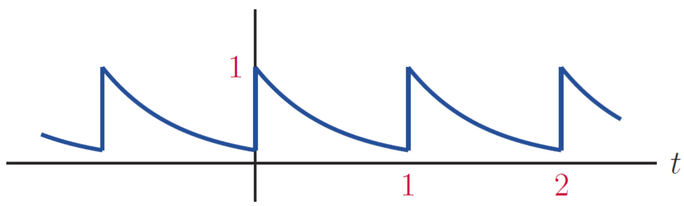
Due Friday, 14 November 2025, by 11:59pm to Gradescope.
100 points total.

1. (28 points) **Fourier Series**

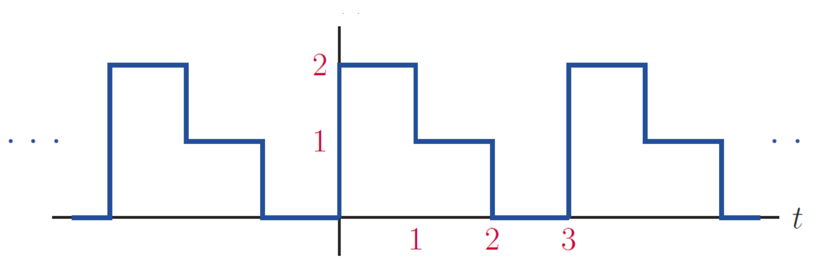
   (a) (18 points) Find the Fourier series coefficients for each of the following periodic signals:

      i. $f(t) = \cos(3\pi t) + \frac{1}{2}\sin(4\pi t)$

      ii. $f(t)$ is a periodic signal with period $T = 1$ s, where one period of the signal is defined as e^{-2t} for $0 < t < 1$ s, as shown below.



      iii. $f(t)$ is the periodic signal shown below:



   (b) (10 points) Suppose you have two periodic signals $x(t)$ and $y(t)$, of periods T_1 and T_2 respectively. Let x_k and y_k be the Fourier series coefficients of $x(t)$ and $y(t)$.

      i. If $T_1 = T_2$, express the Fourier series coefficients of $z(t) = x(t) + y(t)$ in terms of x_k and y_k.

      ii. If $T_1 = 2T_2$, express the Fourier series coefficients of $w(t) = x(t) + y(t)$ in terms of x_k and y_k.

2. (20 points) **Fourier series of transformation of signals**

   Suppose that $f(t)$ is a periodic signal with period T_0, with the following Fourier series:

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

   Determine the period of each of the following signals, then express its Fourier series in terms of c_k:

(a) $g(t) = f(t) + 1$

(b) $g(t) = f(-t)$

(c) $g(t) = f(at)$, where a is positive real number

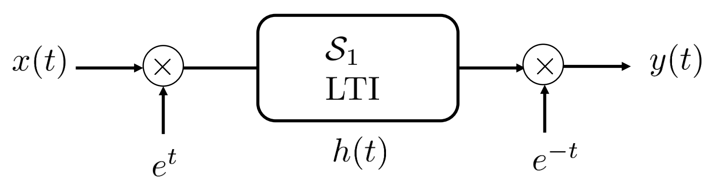
3. (10 points) **Eigenfunctions and LTI systems**

   (a) (5 points) Show that $f(t) = \cos(\omega_0 t)$ is not an eigenfunction of an LTI system.

   (b) (5 points) Show that $f(t) = t$ is not an eigenfunction of an LTI system.

4. (29 points) **LTI systems**

   Consider the following system:



   The system takes as input $x(t)$, it first multiplies the input with e^t, then sends it through an LTI system. The output of the LTI system gets multiplied by e^{-t} to form the output $y(t)$.

   (a) Show that we can write $y(t)$ as follows:

$$y(t) = \left[\left(e^t x(t)\right) * h(t)\right] e^{-t} \tag{1}$$

   (b) Use the definition of convolution to show that (1) can be equivalently written as:

$$y(t) = \int_{-\infty}^{\infty} h'(\tau) x(t-\tau) d\tau \tag{2}$$

   where $h'(t)$ is a function to define in terms of $h(t)$.

   (c) Equation (2) represents a description of the equivalent system that maps $x(t)$ to $y(t)$. Show using (2) that the equivalent system is LTI and determine its impulse response $h_{eq}(t)$ in terms of $h(t)$.

   (d) Suppose that system $\mathcal{S}_1$ is given by its step response $s(t) = r(t-1)$. Find the impulse response $h(t)$ of $\mathcal{S}_1$. What can you say about the causality and stability of system $\mathcal{S}_1$? What can you say about the causality and stability of the overall equivalent system?

5. (13 points) **Python tasks**

   For this question, please complete the included Jupyter Notebook from the zip file.

   Include all relevant code and plots as a pdf of the Jupyter Notebook appended to the end of the homework. You do not need to submit the actual ".ipynb" file, simply a pdf of the notebook will be fine.

   If you would like to complete the assignment in another programming language, you are welcome to, but you will have to translate the skeleton code from the provided notebook to the preferred language yourself.