ECE102, Fall 2025 | **Practice Final Exam**

Signals & Systems | Prof. A. Kadambi

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Name: ____________________

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Problem 6	_____	/ 15
Total	_____	/ 115 points

1. **Signal and Systems Basics** (21 points)

(a) (12 points) **System properties.** For each of the following systems, determine (with reasoning) if they are linear, time invariant, causal and stable.

i. (4 points) $y(t) = x(3t + 2) + 5$

Solution: Non-linear, not time invariant, not causal, stable

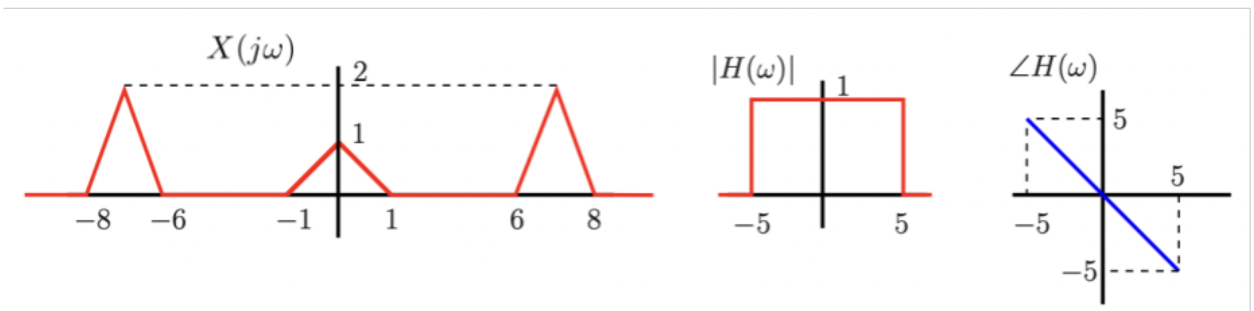
ii. (4 points) $y(t) = sin(\frac{dx(t)}{dt})$

Solution: Non-linear, time invariant, causal, stable

iii. (4 points) $y(t) = e^{x^2(t)}$

Solution: Non-linear, time invariant, causal, stable

(b) (9 points) **LTI System Analysis.** Consider an LTI system with input $x(t)$, output $y(t)$ and impulse response $h(t)$. The Fourier transforms $X(j\omega)$ and $H(j\omega)$ are as shown below.



Evaluate $y(t)$.

Solution: From the given information, $Y(j\omega)$ is such that,

$$|Y(j\omega)| = \begin{cases} 1 - |\omega|, & |\omega| < 1 \\ 0 & \text{otherwise.} \end{cases}$$

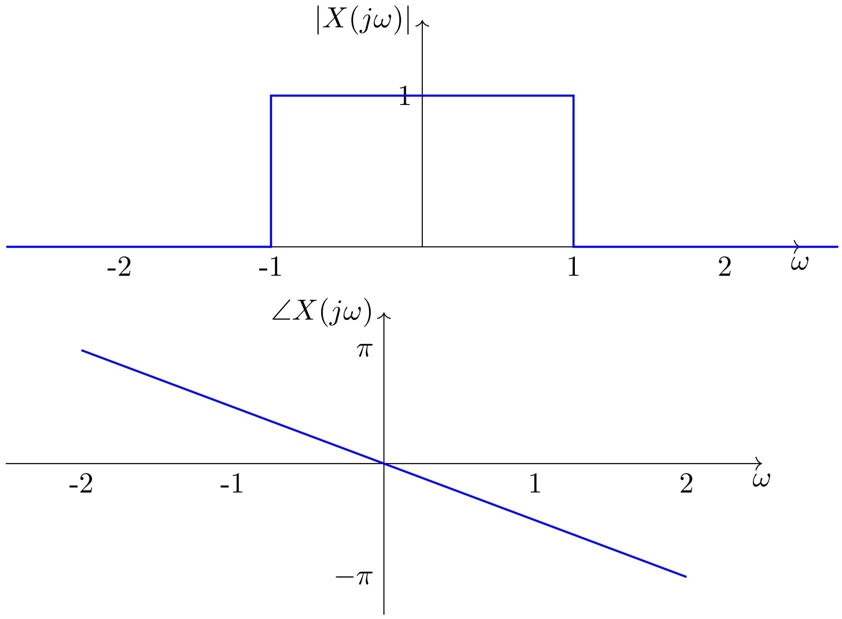
and,

$$\arg Y(j\omega) = -\omega.$$

Using Fourier transform and duality properties, and the time shift property, $y(t) = \frac{1}{2\pi} sinc^2\left(\frac{t-1}{2\pi}\right)$.

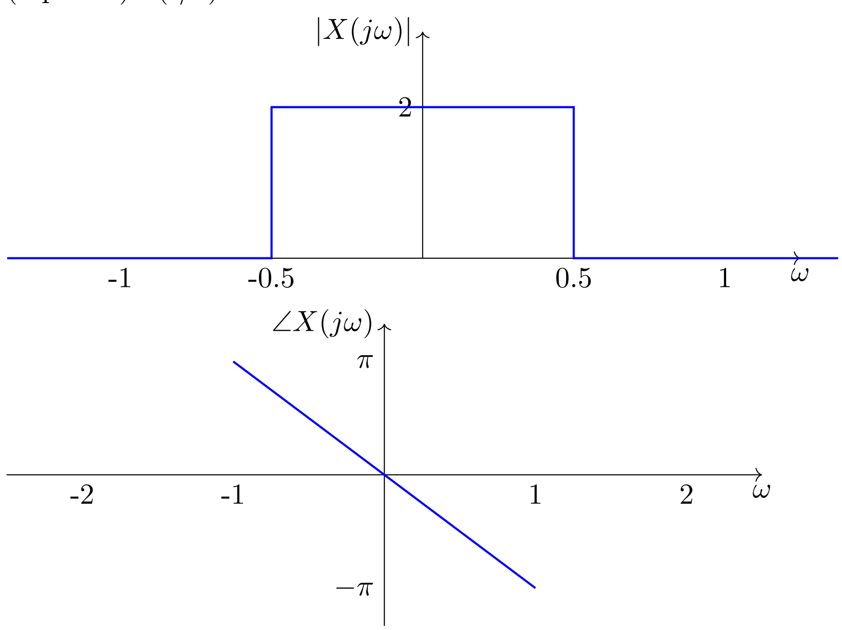
2. **Fourier transform** (29 points)

(a) (12 points) A signal x(t) has has the following the Fourier Transform.

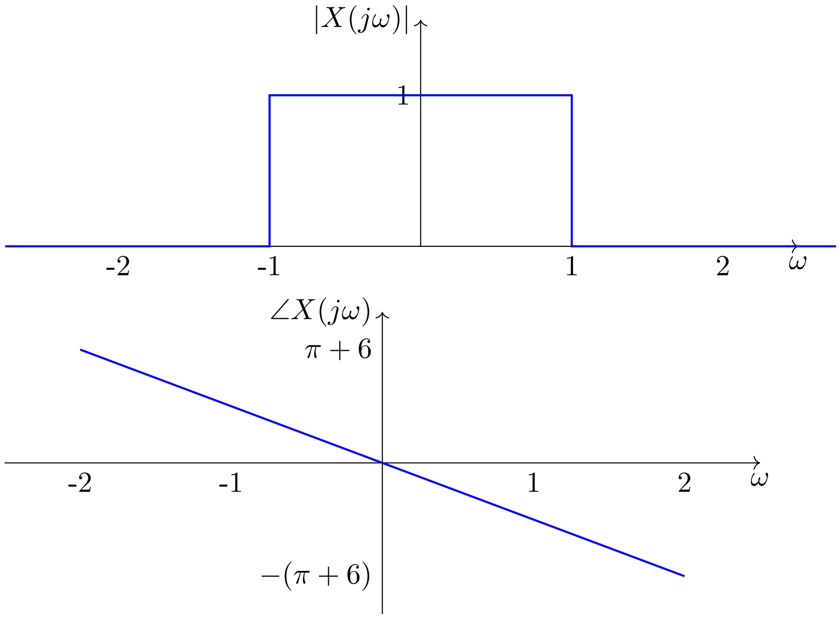


Plot the magnitude and phase plots for the Fourier Transform of the following signals:

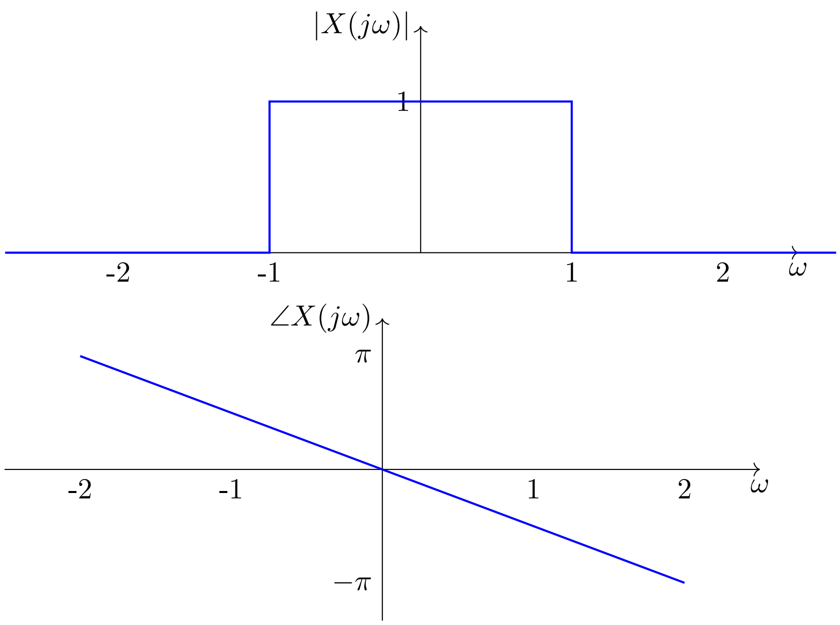
i. (4 points) $x(t/2)$



ii. (4 points) $x(t-3)$



iii. (4 points) $Re(x(t))$



(b) (9 points) Evaluate the Fourier Transforms of the following signals:

i. (4 points) $x(t) = e^{-2|t-1|}$

Solution: $\frac{4e^{-j\omega}}{4+\omega^2}$.

ii. (5 points) $x(t) = te^{-at}cos(\omega_0 t)u(t)$, $a > 0$

Solution: $\frac{(a+j\omega)^2-\omega_0^2}{((a+j\omega)^2+\omega_0^2)^2}$.

(c) (8 points) Evaluate the the time domain signals corresponding to the following Fourier transforms:

i. (4 points)

$$X(j\omega) = \begin{cases} 1 - |\omega|, & |\omega| < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

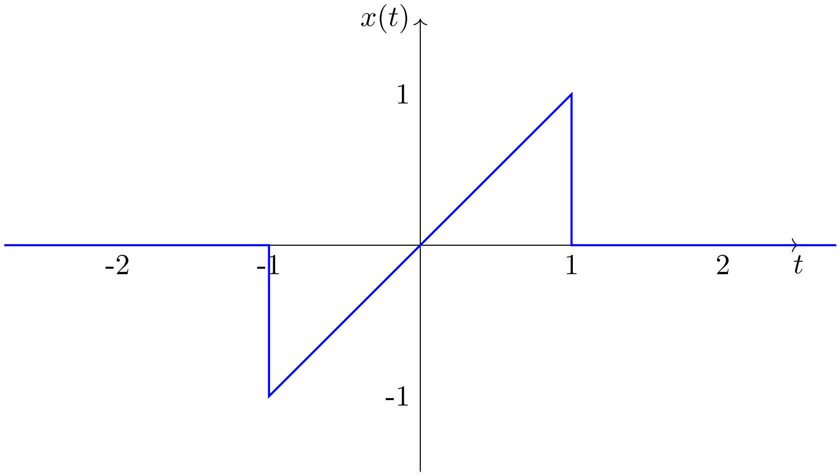
Solution: $\frac{1}{2\pi} sinc^2 \left(\frac{t}{2\pi}\right)$.

ii. (4 points) $X(j\omega) = cos(2\omega + \frac{\pi}{6})$

Solution: $\frac{1}{2} e^{-j\frac{\pi}{12}t}(\delta(t-2) + \delta(t+2))$ OR $\frac{1}{2}(e^{-j\frac{\pi}{6}}\delta(t-2) + e^{j\frac{\pi}{6}}\delta(t+2))$.

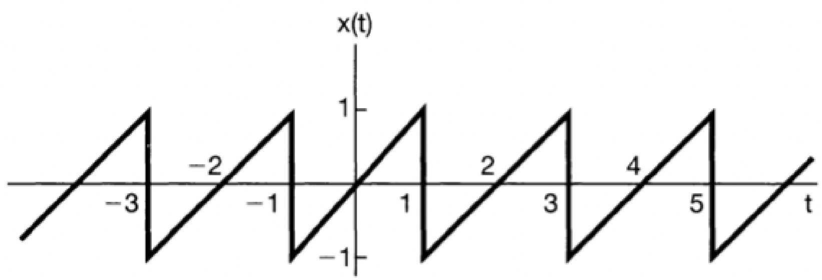
3. **Fourier Series** (15 points)

(a) (5 points) Evaluate the Fourier Transform of the following signal $x(t)$.



Solution: $\frac{2j}{\omega}(cos(\omega) - \frac{sin(\omega)}{\omega})$.

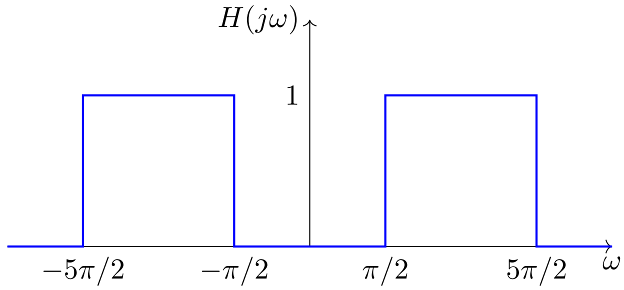
(b) (5 points) Using your solution from part (a), evaluate the fourier series of the following signal $\tilde{x}(t)$.



Solution:

$$c_k = \begin{cases} 0, & k = 0 \\ \frac{j}{k\pi}(-1)^k & \text{otherwise.} \end{cases} \tag{2}$$

(c) (5 points) Consider a system whose frequency response $H(j\omega)$ as follows:



What is the output when $\tilde{x}(t)$ is passed through this system?

Solution: $\frac{2}{\pi}sin(\pi t) - \frac{1}{\pi}sin(2\pi t)$.

4. **Frequency domain understanding** (20 points)

(a) Identify if the following statements are 'True' or 'False' with appropriately detailed reasoning.

i. (1 points) Sampling at a frequency greater than Nyquist rate is a necessary condition for perfect reconstruction, for every signal.

Solution: False. This is a sufficient condition for bandlimited signals.

ii. (2 points) If we have two bandlimited signals, $x_1(t)$ with a bandwidth B_1 and $x_2(t)$ with a bandwidth B_2, the signal $y(t) = x_1(t)x_2(t)$ has a bandwidth $max\{B_1, B_2\}$. ($max\{a, b\}$ is equal to the maximum value among a and b)

Solution: False. Bandwidth is $B_1 + B_2$.

iii. (2 points) Consider a periodic function $x(t)$ with a fundamental period T. If $x(t)$ is an odd function, the sum of all its Fourier series coefficients ($\Sigma_{k=-\infty}^{\infty} c_k$) is zero for any odd $x(t)$.

Solution: True, if exponential Fourier series used, false if sinusoid Fourier series used.

(b) Let $F(j\omega) = j2\pi\omega e^{-2|\omega|}$. Without computing $f(t)$ answer the following questions with appropriate reasoning.

i. (2 points) Is $f(t)$ real/imaginary/complex?

Solution: Real, since the Fourier transform is imaginary and odd.

ii. (2 points) Is $f(t)$ odd/even/neither?

Solution: Odd, since the Fourier transform is imaginary and odd.

iii. (1 points) What is $f(0)$?

Solution: $f(0) = 0$, since the signal is odd.

(c) Evaluate the following.

i. (7 points) Let $x(t) = \frac{4}{4+t^2}$. Evaluate the Fourier transform $X(j\omega)$. (Hint: use the duality property)

Solution: Using the duality property on the Fourier transform of $e^{-a|t|}$ ($\frac{2a}{a^2+\omega^2}$), $X(j\omega) = 2\pi e^{-2|\omega|}$.

ii. (3 points) Using the Fourier transform from the previous part, evaluate the energy of $x(t)$.

Solution: From Parseval's Theorem, energy$=\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$. Energy$=\pi$.

5. **Sampling** (15 points)

(a) (4 points) The sampling theorem says that for a bandlimited signal, a signal must be sampled at a frequency greater than the Nyquist rate to guarantee perfect reconstruction. Identify the minimum sampling rate, f_s (Hz), needed to accurately capture a signal without aliasing:

i. (2 points) $x(t) = cos(3000\pi t) - sin(2000\pi t)$

Solution: 3000 Hz.

ii. (2 points) $x(t) = \frac{sin(2000\pi t)}{\pi t}$

Solution: 2000 Hz.

(b) (6 points) Consider a signal $x(t)$ with a Nyquist rate, ω_0. Determine the Nyquist rate for the following signals:

i. (2 points) $x^2(t)$

Solution: $2\omega_0$.

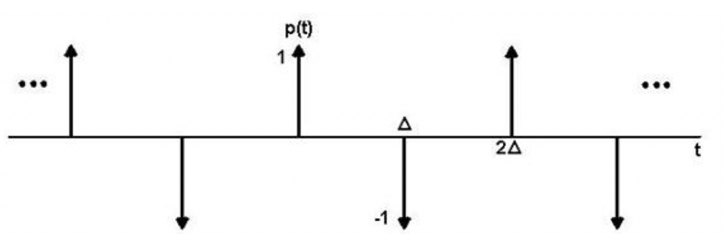
ii. (2 points) $x(t)cos(\omega_1 t)$

Solution: $2|\omega_1| + \omega_0$.

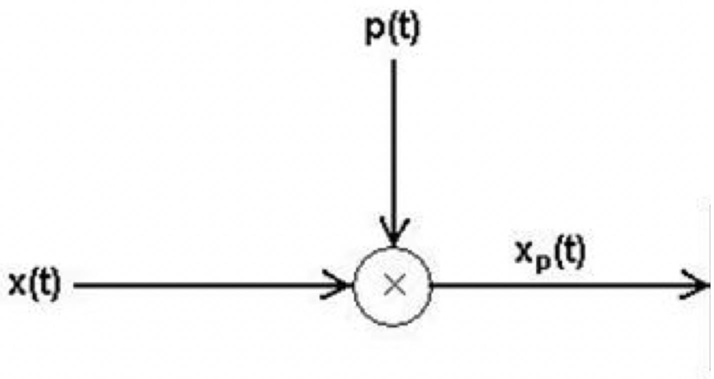
iii. (2 points) $\frac{dx(t)}{dt}$

Solution: ω_0.

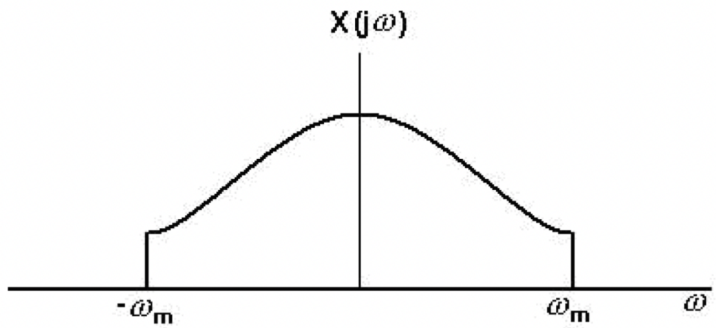
(c) (5 points) We know that ideal sampling is carried out by multiplying the time domain analog signal with an impulse train. Consider a modified sampling regime, where we multiply with the following signal $p(t)$:



The sampling process for a signal $x(t)$ is shown in the following figure:

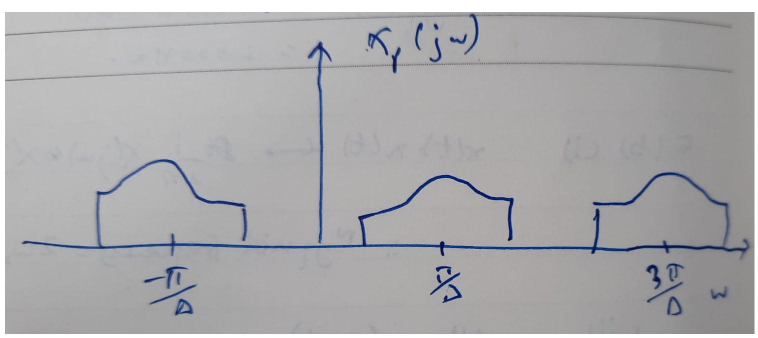


Let $x(t)$ be bandlimited with a one-sided bandwidth of ω_m, with the following fourier transform:



i. (2 points) If $\Delta < \frac{\pi}{\omega_m}$, draw the Fourier transform of $x_p(t)$.

Solution:



ii. (3 points) If $\Delta < \frac{\pi}{\omega_m}$, determine a system to recover $x(t)$ from $x_p(t)$.

Solution: Multiply by $cos\left(\frac{\pi}{\Delta}t\right)$, and then apply an appropriate low pass filter.

6. **Laplace transform (** (15 points)

A casual LTI system can be described by the following differential equation:

$$y''(t) + 4y'(t) + 4y(t) = 4x(t) + 1x''(t).$$

You may assume resting initial conditions (y(0)= 0, y'(0) =0, y"(0) = 0)

(a) (8 points) Find the transfer function $H(s)$.

Solution:

$$H(s) = \frac{4 + s^2}{s^2 + 4s + 4}.$$

(b) (7 points) What is the impulse response $h(t)$ of this system?

Solution: $h(t) = \delta(t) - 4e^{-2t}u(t) + 8te^{-2t}u(t).$