

1. Trigonometric Identities

- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\sin^2 x + \cos^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$
- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x$
 $= 2 \cos^2 x - 1$
 $= 1 - 2 \sin^2 x$
- $\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
- $\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$
- $\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
- $\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$

2. Complex Numbers

- $z = x + jy = re^{j\phi}$
- $r = |z| = \sqrt{x^2 + y^2}$
- $\phi = \arctan \frac{y}{x}$
- $x = r \cos \phi, y = r \sin \phi$
- $e^{j\phi} = \cos \phi + j \sin \phi$
- $z^* = x - jy = re^{-j\phi}$
- $|z|^2 = z z^*$
- $|z_1 z_2| = |z_1| |z_2|, z_1, z_2 \in \mathbb{C}$
- $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}, z_1, z_2 \in \mathbb{C}$

3. Signal Energy and Power

- Energy $E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$
- Power $P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$

4. Standard Signals

- Step function $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$
- Unit rectangle $\text{rect}(t) = \begin{cases} 1 & |t| \leq 1/2 \\ 0 & \text{else} \end{cases}$

- Unit ramp $r(t) = \begin{cases} t & t \geq 0 \\ 0 & \text{else} \end{cases}$
- Unit triangle $\Delta(t) = \begin{cases} 1 - |t| & |t| \leq 1 \\ 0 & \text{else} \end{cases}$
- Sinc function $\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$
- Impulse function $\delta(t)$ satisfies,
 $\delta(t) = 0 \forall t \neq 0, \int_{-\infty}^{\infty} \delta(t) dt = 1$
 $f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)$
 $\int_{-\infty}^{\infty} f(t)\delta(t-\tau) dt = f(\tau)$
- Impulse train: $\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$

5. System Properties

S is the system, $x(t)$ is input, $y(t)$ is output.

- BIBO stability: Given $|x(t)| < \infty \forall t$, $y(t)$ should satisfy $|y(t)| < \infty \forall t$.
- Causality: $y(t)$ only depends on past and present values of input.
- Time invariance: True if $x(t - \tau)$ has output $y(t - \tau) \forall \tau$.
- Linearity: $ax_1(t) + bx_2(t)$ has output $ay_1(t) + by_2(t)$, given $x_1(t)$ and $x_2(t)$ have respective outputs $y_1(t)$ and $y_2(t)$.
- Memory: If output depends on past and/or future input values.
- Invertibility: If input can be uniquely recovered from output.

6. Signal Properties

- Odd signal: $x(-t) = -x(t)$
- Even signal: $x(-t) = x(t)$
- Real signal: $x^*(t) = x(t)$
- Purely imaginary signal: $x^*(t) = -x(t)$

7. Convolution

$x(t)$ is input, $y(t)$ is output, $h(t)$ is impulse response.

- $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$
 $= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau = (x * h)(t)$
- $(x * h)(t) = (h * x)(t)$
- $(f * (g * h))(t) = ((f * g) * h)(t)$
- $f * (\alpha g + \beta h) = \alpha(f * g) + \beta(f * h)$
- Time invariant

8. Fourier Series

- Fourier coeff.: $c_k = \frac{1}{T_0} \int_{\tau}^{\tau+T_0} f(t) e^{-jk\omega_0 t} dt$, $\omega_0 = \frac{2\pi}{T_0}$, $k \in \mathbb{Z}$
- Fourier series: $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$
- c_0 : average of signal
- Parseval's Theorem: Power = $\sum_{k=-\infty}^{\infty} |c_k|^2$

9. Fourier Transform

- $F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$
- $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$
- Linearity: $\mathcal{F}[af_1(t) + bf_2(t)] = a\mathcal{F}[f_1(t)] + b\mathcal{F}[f_2(t)]$
- Time scaling: $\mathcal{F}[f(at)] = \frac{1}{|a|} F\left(j\frac{\omega}{a}\right)$
- Time reversal: $\mathcal{F}[f(-t)] = F(-j\omega)$
- Complex conjugate: $f^*(t) \iff F^*(-j\omega)$
- Duality: $F(t) \iff 2\pi f(-j\omega)$
- Time shifting: $\mathcal{F}[f(t - \tau)] = e^{-j\omega\tau} F(j\omega)$
- Derivative: $\mathcal{F}[f'(t)] = j\omega F(j\omega)$
- Convolution: $\mathcal{F}[(f_1 \star f_2)(t)] = F_1(j\omega)F_2(j\omega)$
- Parseval's theorem: Energy = $\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$
- Multiplication: $\mathcal{F}[f_1(t)f_2(t)] = \frac{1}{2\pi} (F_1 \star F_2)(j\omega)$
- Modulation: $\mathcal{F}[f(t)e^{j\omega_0 t}] = F(j(\omega - \omega_0))$
- Periodic signal: $f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \iff \sum_{k=-\infty}^{\infty} c_k 2\pi\delta(\omega - k\omega_0)$

10. Fourier Transform Pairs

- $\delta(t) \iff 1$
- $\delta(t - \tau) \iff e^{-j\omega\tau}$
- $1 \iff 2\pi\delta(\omega)$
- $u(t) \iff \pi\delta(\omega) + \frac{1}{j\omega}$
- $e^{j\omega_0 t} \iff 2\pi\delta(\omega - \omega_0)$
- $\cos(\omega_0 t) \iff \pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$
- $\sin(\omega_0 t) \iff j\pi(\delta(\omega + \omega_0) - \delta(\omega - \omega_0))$
- $\text{rect}(t/T) \iff T\text{sinc}(\omega T/2\pi)$
- $e^{-at}u(t) \iff \frac{1}{a+j\omega}$
- $e^{-a|t|} \iff \frac{2a}{a^2+\omega^2}$
- $\text{sinc}(t/2\pi) \iff 2\pi\text{rect}(\omega)$
- $\Delta(t) \iff \text{sinc}^2(\omega/2\pi)$

- $\text{sinc}^2(t) \iff \Delta(\omega/2\pi)$
- $\delta_T(t) \iff \omega_0\delta_{\omega_0}(\omega)$
- $\sum_{k=-\infty}^{\infty} \delta(t - kT) \iff \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi k}{T})$

11. Sampling Theorem: Given a bandlimited signal with bandwidth B Hz, for perfect recovery, the sampling rate $\frac{1}{T}$ must satisfy $\frac{1}{T} \geq 2B$.

12. Laplace Transform

- Unilateral Laplace transform: $F(s) = \int_0^{\infty} f(t) e^{-st} dt$. Remember, a Laplace transform is incomplete without its region of convergence (RoC).
- Linearity: $\mathcal{L}[af_1(t) + bf_2(t)] = a\mathcal{L}[f_1(t)] + b\mathcal{L}[f_2(t)]$
- Time scaling: $\mathcal{L}[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$
- Time shifting: $\mathcal{L}[f(t - \tau)] = e^{-s\tau} F(s)$
- Frequency shift: $\mathcal{L}[f(t)e^{s_0 t}] = F(s - s_0)$
- Convolution: $\mathcal{L}[(f_1 \star f_2)(t)] = F_1(s)F_2(s)$
- Integration: $\mathcal{L}[\int_0^t f(\tau) d\tau] = \frac{1}{s} F(s)$
- Derivative: $\mathcal{L}[f'(t)] = sF(s) - f(0)$
- Multiplication by t : $\mathcal{L}[tf(t)] = -F'(s)$

13. Unilateral Laplace Transform Pairs

- $\delta(t) \iff 1$, ROC: All s
- $u(t) \iff \frac{1}{s}$, ROC: $\text{Re}\{s\} > 0$
- $r(t) \iff \frac{1}{s^2}$, ROC: $\text{Re}\{s\} > 0$
- $e^{-at}u(t) \iff \frac{1}{a+s}$, ROC: $\text{Re}\{s\} > -\text{Re}\{a\}$
- $\cos(\omega_0 t) \iff \frac{s}{s^2+\omega_0^2}$, ROC: $\text{Re}\{s\} > 0$
- $\sin(\omega_0 t) \iff \frac{\omega_0}{s^2+\omega_0^2}$, ROC: $\text{Re}\{s\} > 0$
- $\frac{r}{(k-1)!} t^{k-1} e^{\lambda t} \iff \frac{r}{(s-\lambda)^k}$, ROC: $\text{Re}\{s\} > \text{Re}\{\lambda\}$