EE102

Lecture 6

ABET Learning Outcomes

✔ Understand the concept of a signal and a system, plot continuous-time signals, evaluate the periodicity of a signal.

✔ Identify properties of continuous-time systems such as linearity, time-invariance, and causality.

✔ Calculate with the Dirac delta function.

Compute convolution of continuous-time functions.

Understand the concept of the impulse response function of a linear system, and its use to describe the input/output relationship.

Compute the Laplace transform of a continuous function, identify its domain of convergence, and be familiar with its basic properties, including the initial and final value theorems.

Find the inverse Laplace transform by partial fractions.

Use the Laplace transform to solve constant-coefficient differential equations with initial conditions

Use the Laplace transform to evaluate the transfer function of linear time-invariant systems.

Understand Parseval's relation in Fourier series, and its interpretation in terms of decomposing the signal's energy between its harmonics

Evaluate the response of a linear time-invariant system to periodic inputs.

Evaluate the Fourier transform of a continuous function, and be familiar with its basic properties. Relate it to the Laplace transform.

Evaluate and plot the frequency responses (magnitude and phase) of linear time-invariant systems, and apply it to filtering of input signals.

Understand conditions under which a band-limited function can be recovered from its samples

EE102 Announcements

- **Second Homework due tomorrow 10/17**
- **Practice Midterm out**
- **Midterm is 10/30**

Slide Credits: This lecture adapted from Prof. Jonathan Kao (UCLA) and recursive credits apply to Prof. Cabric (UCLA), Prof. Yu (Carnegie Mellon), Prof. Pauly (Stanford), and Prof. Fragouli (UCLA)

CYU: Stability Review

Is the following system stable?

$$y(t) = x(3t) + cos(\omega t)$$

$$|x(t)| \le B \quad \forall t$$

$$\therefore \quad x(3t) + \cos(\omega t) \le B + 1$$

$y(t)$ is BIBO Stable

CYU: Causality Review

Is the following system causal?

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau + cos(3\omega t)$$

Causal b/c it goes up to t

Still causal b/c this is not x(t).

Review of Last Lecture

Last Lecture Introduced a few concepts:

- CYU: How is the Impulse Response Defined?

$$x \rightarrow \boxed{H} \rightarrow y$$

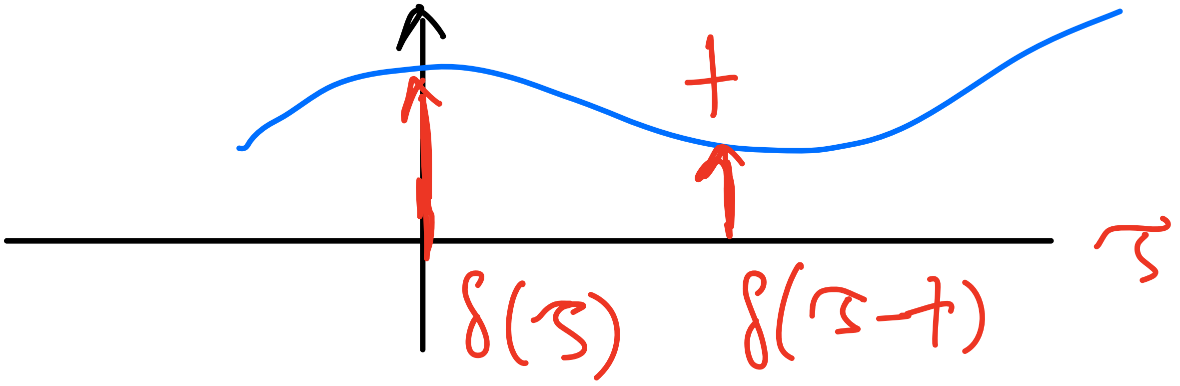
$$x = \delta(t)$$

- CYU: Why is the Impulse Response Useful?

IR lets you predict the response/output of the system to ANY input

[Assuming LTI System]

Derivation of this fact



Calculate $x(0)$ using δ's: $x(\tau)\,\delta(\tau) = \underline{x(0)\,\delta(\tau)}$

$\hookrightarrow$ Integrate $\Rightarrow x(0)$

Calculate $x(t)$ using δ's: $x(\tau)\,\delta(\tau-t) = \underline{x(t)\,\delta(\tau-t)}$

$\hookrightarrow$ Integrate $\Rightarrow x(t)$

The Convolution Integral

$$\int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t)\,d\tau = \int_{-\infty}^{\infty} x(t)\,\delta(\tau-t)\,d\tau$$

$$= x(t)\underbrace{\int_{-\infty}^{\infty} \delta(\tau-t)\,d\tau}_{=1}$$

$$= x(t)$$

// Convolution Integral

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t)\,d\tau$$

$$= \boxed{\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$

Canonical Form of Convolution Integral

The Convolution Integral (Cont'd)

$h(t) \triangleq H[\delta(t)]$

① If we have $h(t)$, then given $x(t)$ we can predict $y(t)$.

$$x(t) \longrightarrow \underset{LTI}{\boxed{H}} \longrightarrow y(t)$$

Goal: Given x, h we try to get y.

$$y(t) = H[x(t)]$$

Linearity

$$= H\left[\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right]$$

$$= \int_{-\infty}^{\infty} x(\tau)\,H[\delta(t-\tau)]\,d\tau$$

T.I.

$$= \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

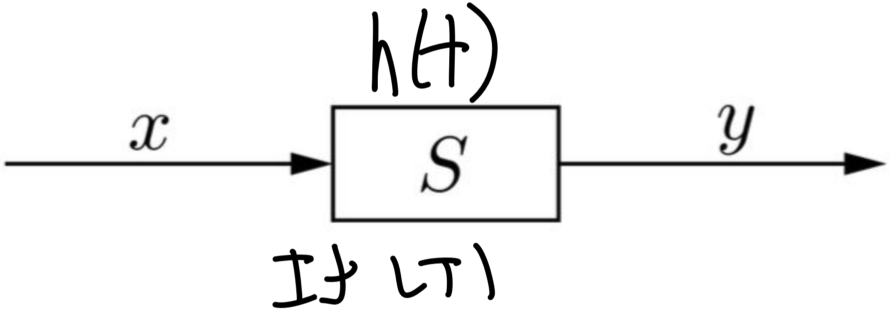
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$\Downarrow$$

$$y(t) = x(t) \underset{\text{Convolution}}{\underbrace{*}} h(t)$$

Convolution is what adds structure to the Black Box

A system transforms an *input signal*, $x(t)$, into an output system, $y(t)$.

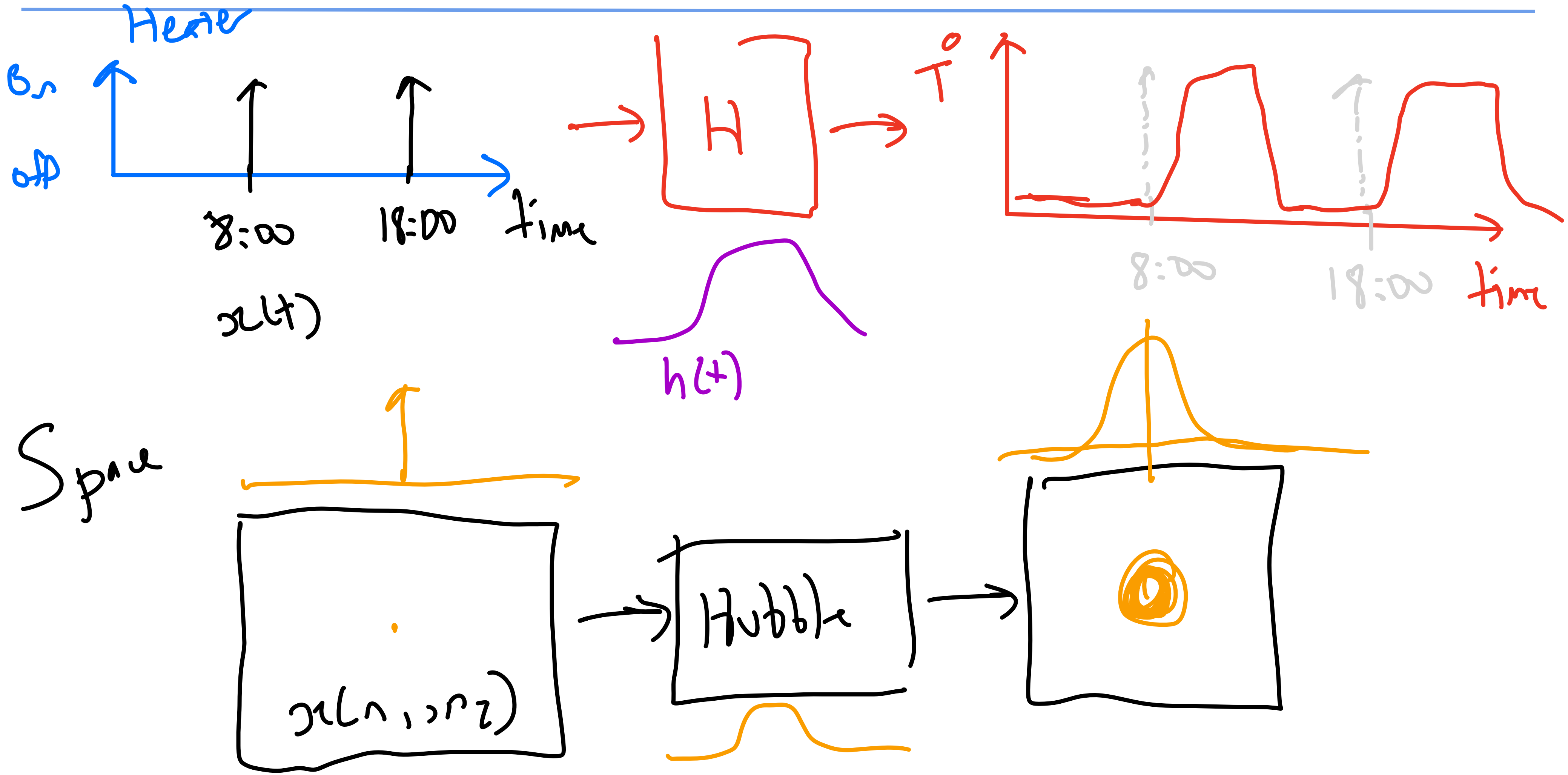


Then convolution is the op that maps x to y

$$y = x * h$$

Why does this work? (The Convolution Integral)

Physical "Gist" of convolution - smearing



Examples of Computing the Impulse Response

A

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

LTI System "Integrator"

$x \to \boxed{I} \to y$

$y = x * h$

What is the impulse response of the system?

1. Set $x(t) = \delta(t)$
2. Calculate the output

$$h(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

QYV: Show that A and B are equal.

$$y(t) = x(t) * h(t) = x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\, d\tau$$

B

CYU: Impulse Response and Convolutions

Calculate the impulse response and and check by writing y in terms of the convolution integral.

1/ $y(t) = x(t - s')$

① Ans. $h(t) = \delta(t - s')$

② Check. $y(t) = x * h = \int_{-\infty}^{\infty} x(s)h(t-s)\,ds = \underbrace{\int_{-\infty}^{\infty} x(s)\,\delta(t-s'-s)\,ds}_{\text{SIFTING PROP.}}$

$= x(t - s')$ ✓

Notation of Convolution

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

- x input
- h impulse response
- y output

Notation

Most Rigorous $y(t) = (x * h)(t)$

In Reality: $y = x * h$

$$y(t) = x(t) * h(t)$$

Block Notation

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y = x * h$$