ECE102, Fall 2025 **Homework #2**
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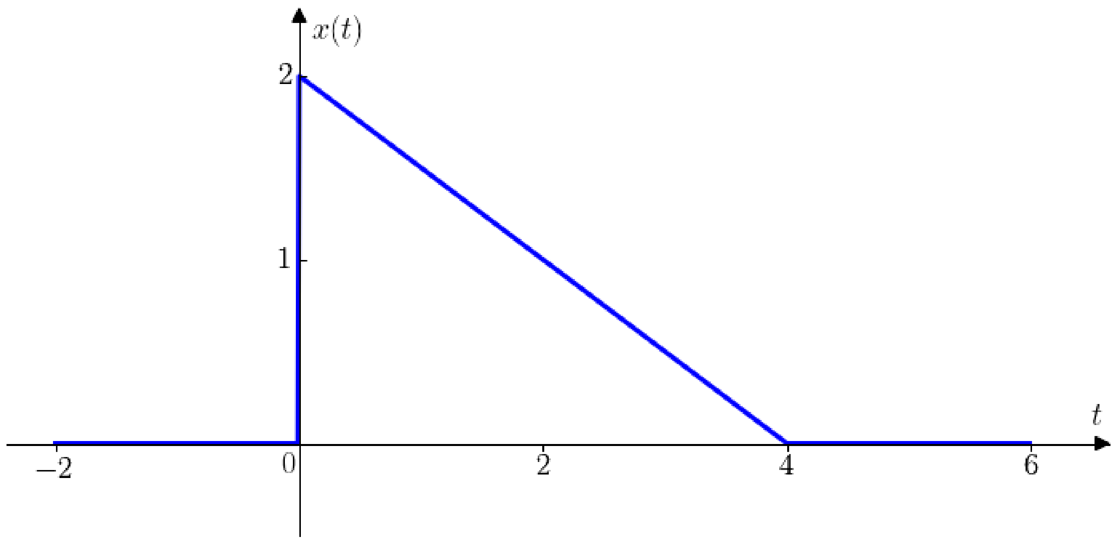
Due Friday, 17 October 2025, by 11:59pm to Gradescope.
Covers material up to Lecture 4.
100 points total.

1. (22 points) **Elementary signals.**

   (a) (9 points) Consider the signal $x(t)$ shown below. Sketch the following:



   i. $y(t) = x(t)\left(u(t-1) - u(2t-5)\right)$
   ii. $y(t) = \int_{-\infty}^{t} \delta(\tau - 2)x(\tau)d\tau$
   iii. $y(t) = x(t) - \left(2u(t) - \frac{1}{2}r(t) + \frac{1}{2}r(t-4)\right)$

   (b) (9 points) Evaluate these integrals:

   i. $\int_{-\infty}^{\infty} f(t+1)\delta(t+1)dt$
   ii. $\int_{t}^{\infty} e^{-2\tau}u(\tau - 1)d\tau$
   iii. $\int_{0}^{\infty} f(t)(\delta(t-1) + \delta(t+1))dt$

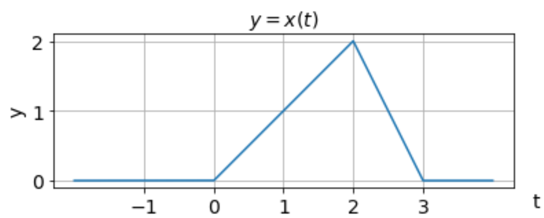
   (c) (4 points) Let b be a positive constant. Show the following property for the delta function:

$$\delta(bt) = \frac{1}{b}\delta(t)$$

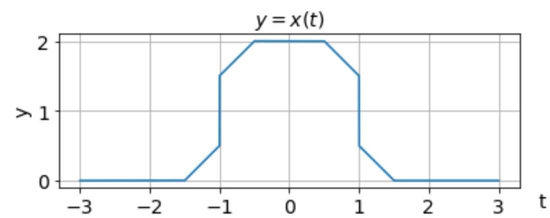
   Hint: what function is "delta-like"?

2. (23 points) **Expression for signals.**

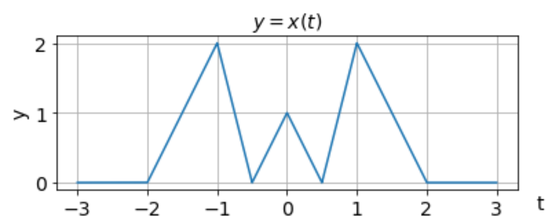
(a) (15 points) Write the following signals as a combination (sums or products) of unit triangles $\Delta(t)$ and unit rectangles $\mathrm{rect}(t)$.



a(i)

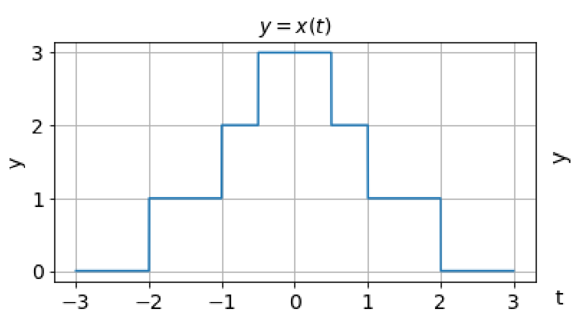


a(ii)

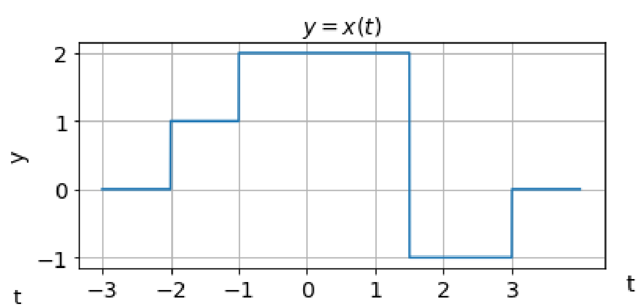


a(iii)

(b) (8 points) Express each of the signals shown below as sums of scaled and time shifted unit-step functions.



b(i)



b(ii)

3. (30 points) **System properties.**

(a) (20 points) A system with input $x(t)$ and output $y(t)$ can be time-invariant, causal or stable. Determine which of these properties hold for each of the following systems. Explain your answer.

i. $y(t) = |x(t)| + x(2t)$

ii. $y(t) = \int_{t-T}^{t+T} x(\lambda)d\lambda$, where T is positive and constant.

iii. $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)d\lambda$

iv. $y(t) = 1 + x(t)\cos(\omega t)$

v. $y(t) = \frac{1}{1+x^2(t)}$

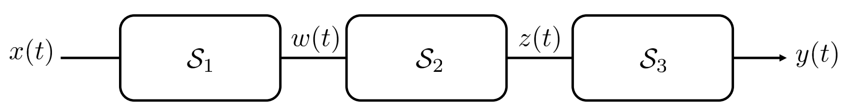
(b) (6 points) Consider the following three systems:

$$\mathcal{S}_1 : w(t) = x(t/2)$$

$$\mathcal{S}_2 : z(t) = \int_{-\infty}^{t} w(\tau)d\tau$$

$$\mathcal{S}_3 : y(t) = \mathcal{S}_3(z(t))$$

The three systems are connected in series as illustrated here:



Choose the third system $\mathcal{S}_3$, such that overall system is equivalent to the following system:

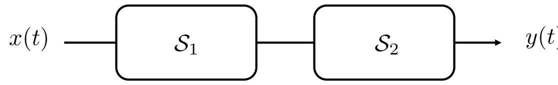
$$y(t) = \int_{-\infty}^{t-1} x(\tau)d\tau$$

(c) (4 points) In part (b), you saw an example of three systems connected in series. In general, systems can be interconnected in series or in parallel to form what we call cascaded systems. The figure below shows the difference between a series cascade and a parallel cascade. *Note that parts (b) and (c) are unrelated.*

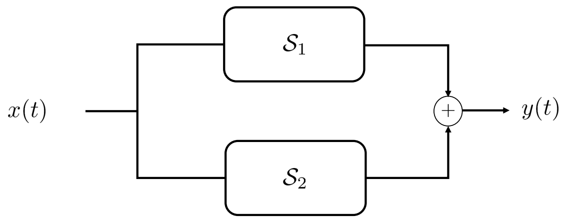
i. (2 points) Show that the series cascade of any two time-invariant systems is also time-invariant.

ii. (2 points) Show that the parallel cascade of any two time-invariant systems is also time-invariant.

iii. (*Optional*) Can you think of two **time-variant** systems, whose series cascade is **time-invariant**? Can you think of two **time-variant** systems, whose parallel cascade is **time-invariant**?



(a) Series Cascade



(b) Parallel Cascade

4. (10 points) **Power and energy of complex signals**

   (a) (5 points) Is $x(t) = Ae^{j\omega t} + Be^{-j\omega t}$ a power or energy signal? A and B are both real numbers, not necessarily equal. If it is an energy signal, compute its energy. If it is a power signal, compute its power. (*Hint: Use the fact that the square magnitude of a complex number v is: $|v|^2 = v^*v$, where v^* is the complex conjugate of the complex number v.*)

   (b) (5 points) Is $x(t) = e^{-(1+j\omega)t}u(t-1)$ an energy signal or power signal? Again, if it is an energy signal, compute its energy. If it is a power signal, compute its power.

5. (15 points) **Python tasks**
   For this question, please complete the included Jupyter Notebook from the zip file.

   Include all relevant code and plots as a pdf of the Jupyter Notebook appended to the end of the homework. You do not need to submit the actual ".ipynb" file, simply a pdf of the notebook will be fine.

   If you would like to complete the assignment in another programming language, you are welcome to, but you will have to translate the skeleton code from the provided notebook to the preferred language yourself.