Table of Fourier Transform

Function or Theorem Name	Time Domain $f(t) = \mathcal{F}^{-1}\{F(\omega)\}$	Frequency Domain (Fourier) $F(\omega) = \mathcal{F}\{f(t)\}$
Definition	$\mathcal{F}\{f\}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$	$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{F}(\omega)e^{j\omega t}dt$
Dirac Delta	$\delta(t)$	1
Shifted Delta	$\delta(t-a)$	$e^{-i\omega a}$
Constant	1	$2\pi\delta(\omega)$
Unit Step (Heaviside)	$u(t)$	$\dfrac{1}{i\omega} + \pi\delta(\omega)$
Exponential (real)	$e^{-at}u(t),\ a>0$	$\dfrac{1}{a+i\omega}$
Positive Exponential	$e^{-a\|t\|},\ a>0$	$\dfrac{2a}{a^2+\omega^2}$
Complex Exponential	$e^{i\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
Sine	$\sin(\omega_0 t)$	$\dfrac{\pi}{i}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$
Cosine	$\cos(\omega_0 t)$	$\pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right]$
Sine with Exponential	$e^{-at}\sin(\omega_0 t)\,u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
Cosine with Exponential	$e^{-at}\cos(\omega_0 t)\,u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
Sine Combination	$t^n e^{-at}\sin(\omega_0 t)u(t),\ a>0$	$\frac{n!}{2j}\left[\frac{1}{\left(a+j(\omega-\omega_0)\right)^{n+1}} - \frac{1}{\left(a+j(\omega+\omega_0)\right)^{n+1}}\right]$
Cosine Combination	$t^n e^{-at}\cos(\omega_0 t)u(t),\ a>0$	$\frac{n!}{2}\left[\frac{1}{\left(a+j(\omega-\omega_0)\right)^{n+1}} + \frac{1}{\left(a+j(\omega+\omega_0)\right)^{n+1}}\right]$
Rectangular Pulse	$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$
Triangular Pulse	$\Delta\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}^2\left(\frac{\omega T}{2\pi}\right)$
Sinc Function	$\mathrm{sinc}\left(\frac{t}{T}\right)$	$T\mathrm{rect}\left(\frac{\omega T}{2\pi}\right)$
Sinc Function Squared	$\mathrm{sinc}^2\left(\frac{t}{T}\right)$	$T\Delta\left(\frac{\omega T}{2\pi}\right)$
Gaussian	e^{-t^2}	$\sqrt{\pi}\,e^{-\omega^2/4}$
Conjugate	$\overline{f(t)}$	$\overline{F(-\omega)}$
Duality	$F(t)$	$2\pi f(-\omega)$
Derivative (1st)	$f'(t)$	$i\omega F(\omega)$
Derivative (n-th)	$f^{(n)}(t)$	$(i\omega)^n F(\omega)$
Integrator	$\int_{-\infty}^{t} f(\tau)d\tau$	$\pi F(0)\delta(\omega) + \frac{F(\omega)}{j\omega}$
Multiplication by t	$t\,f(t)$	$i\,\dfrac{d}{d\omega}F(\omega)$
Time Shift	$f(t-\tau)$	$e^{-i\omega\tau}F(\omega)$
Scaling (time scaling)	$f(at)$	$\frac{1}{\|a\|}F\left(\frac{\omega}{a}\right)$
Frequency Shift (Modulation)	$e^{i\omega_0 t}f(t)$	$F(\omega-\omega_0)$
Convolution in Time	$(f*g)(t)$	$F(\omega)G(\omega)$
Convolution in Frequency	$f(t)g(t)$	$\dfrac{1}{2\pi}\left(F(\omega)*G(\omega)\right)$
Parseval / Energy Theorem	$\displaystyle\int_{-\infty}^{\infty}\|f(t)\|^2dt$	$\displaystyle\frac{1}{2\pi}\int_{-\infty}^{\infty}\|F(\omega)\|^2d\omega$

Table of Laplace Transform

Function or Theorem	Time Domain $f(t)$	Laplace Transform $F(s) = \mathcal{L}\{f(t)\}$	ROC
Unit Impulse	$\delta(t)$	1	all s
Shifted Dirac Delta	$\delta(t-\tau)$	$e^{-\tau s}$	all s
Step Function	$u(t-\tau)$	$\frac{e^{-\tau s}}{s}$	$\mathrm{Re}(s) > 0$
Ramp Function	$tu(t)$	$\frac{1}{s^2}$	$\mathrm{Re}(s) > 0$
Shifted Ramp	$(t-a)u(t-a)$	$\frac{e^{-as}}{s^2}$	$\mathrm{Re}(s) > 0$
Exponential	$e^{at}u(t)$	$\frac{1}{s-a}$	$\mathrm{Re}(s) > a$
n-th Power	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}(s) > 0$
Shifted Power	$(t-a)^n u(t-a)$	$\frac{n! e^{-as}}{s^{n+1}}$	$\mathrm{Re}(s) > 0$
Half-Integer Power	$t^{n-\frac{1}{2}} u(t)$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$	$\mathrm{Re}(s) > 0$
Real Power	$t^p u(t)$, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$	$\mathrm{Re}(s) > 0$
Sine	$\sin(\omega t)u(t)$	$\frac{\omega}{s^2+\omega^2}$	$\mathrm{Re}(s) > 0$
Cosine	$\cos(\omega t)u(t)$	$\frac{s}{s^2+\omega^2}$	$\mathrm{Re}(s) > 0$
Sine with Phase Shift	$\sin(\omega t+\gamma)u(t)$	$\frac{s\sin\gamma + \omega\cos\gamma}{s^2+\omega^2}$	$\mathrm{Re}(s) > 0$
Cosine with Phase Shift	$\cos(\omega t+\gamma)u(t)$	$\frac{s\cos\gamma - \omega\sin\gamma}{s^2+\omega^2}$	$\mathrm{Re}(s) > 0$
Hyperbolic Sine	$\sinh(\alpha t)u(t)$	$\frac{\alpha}{s^2-\alpha^2}$	$\mathrm{Re}(s) > \lvert\alpha\rvert$
Hyperbolic Cosine	$\cosh(\alpha t)u(t)$	$\frac{s}{s^2-\alpha^2}$	$\mathrm{Re}(s) > \lvert\alpha\rvert$
First Shifting Theorem	$e^{\alpha t} f(t)$	$F(s-\alpha)$	shift ROC right by α: $\mathrm{Re}(s) > \sigma_0 + \alpha$
Time Scaling ($a > 0$)	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$	$\mathrm{Re}(s/a) > \sigma_0$
Exponential Decay Sine	$e^{-\alpha t}\sin(\omega t)u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$	$\mathrm{Re}(s) > -\alpha$
Exponential Decay Cosine	$e^{-\alpha t}\cos(\omega t)u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$	$\mathrm{Re}(s) > -\alpha$
Power with Exponential	$\frac{r}{(k-1)!} t^{k-1} e^{\lambda t}$	$\frac{r}{(s-\lambda)^k}$	$\mathrm{Re}(s) > \mathrm{Re}(\lambda)$
Second Shifting Theorem	$u(t-a)f(t-a)$	$e^{-as}F(s)$	same ROC as $F(s)$
Modified Second Shifting	$u(t-a)f(t)$	$e^{-as}\mathcal{L}\{f(t+a)\}$	shift ROC left by a
Multiplication by t	$tf(t)$	$-\frac{d}{ds}F(s)$	same ROC as $F(s)$
Multiplication by t^n	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}F(s)$	same ROC as $F(s)$
Integration in Time	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$	$\mathrm{Re}(s) > \sigma_0$
First Derivative	$f'(t)$	$sF(s) - f(0)$	same ROC as $F(s)$
Second Derivative	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$	same ROC as $F(s)$
n-th Derivative	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	same ROC as $F(s)$
Convolution	$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$	intersection of the two ROCs
Initial Value Theorem	$\lim_{t\to 0^+} f(t)$	$\lim_{s\to\infty} sF(s)$	requires no poles in $\mathrm{Re}(s) > 0$
Final Value Theorem	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$	requires poles only in $\mathrm{Re}(s) < 0$