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Digital Media Processing — Assignment I

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Linear time invariant systems, e.g., filters that damp or boost specific frequency ranges in audio processing, as well as, blurring filters and edge detectors in image processing, are often characterized by their behavior in the frequency domain. For this representation, often, complex coefficients are used, which allow for a compact representation. Further, as will be shown later in the lecture, such systems are further characterized by polynomial functions and their roots. The goal of this exercise is to (re-)train the required mathematical basics.

1 COMPLEX NUMBERS

Convert the following numbers to an expression using real and imaginary part. Exploit Euler's formula $e^{j\varphi} = \cos(\varphi) + j\sin(\varphi)$, where $\sqrt{-1} = j$.

1. $z = \sqrt{2}e^{j\pi/4}$
2. $z = \frac{1}{3}e^{-j\pi}$

Convert the following complex numbers z to an expression using magnitude and phase, i.e., $z = Ae^{j\varphi}$.

3. $z = 2 + 3j$
4. $z = 1 + e^{jq}$

2 Roots

Give all possible solutions for the following polynomials.

1. $(x-3)(x+4)^2(x-1)=0$
2. $2x^2+8=-10x$
3. $x^2-x+1=0$
4. $x^3=1$

3 Logarithm

Solve the following equations.

1. $4\left(1-2^{-2x}\right)=3$
2. $10\log_{10}(x)=10$
3. $\log(\sqrt{x})+1.5\log(x)=\log(2x)$

4 Trigonometric Functions

Show that the following equation holds.

$$a\cos(\omega t)+b\sin(\omega t)=A\cos(\omega t-\phi) \tag{4.1}$$

Sketch of the solution:
Reformulate the expression in (4.1) using Euler's formula and simplify. You should end up with an expression similar to $ce^{j\omega t}+c^*e^{-j\omega t}$ where c^* denotes the conjugate complex of c. Convert the factors c to the amplitude and phase notation which should lead to the result.

1. What is the value of A?
2. What is the value of ϕ?