UNIVERSITY OF HAMBURG, DEPARTMENT OF INFORMATICS, SIGNAL PROCESSING GROUP

Digital Media Signal Processing — Assignment VIII

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1 EIGENFUNCTIONS OF LTI SYSTEMS

An finite impulse response filter is described by the difference equation

$$y(n) = x(n) - x(n-6)$$

1. Compute and sketch its magnitude and phase response.
2. Determine its response to the inputs
   a) $x(n) = \cos\left(\frac{\pi}{10}n\right), \qquad -\infty < n < \infty$
   b) $x(n) = 5 + 6\cos\left(\frac{2\pi}{5}n + \frac{\pi}{2}\right), \qquad -\infty < n < \infty$

   Hint: Represent the $\sin(\cdot)$ and $\cos(\cdot)$ functions using the Euler function and exploit that these functions are eigenfunctions of LTI systems.

2 NON-LINEAR SYSTEMS

An LTI system cannot produce frequencies at its output that are different from those applied in its input. Thus if a system creates "new" frequencies, it must be nonlinear and/or time varying. For the system $y(n) = x^2(n)$, graphically determine the output spectrum when the input spectrum is

$$X(\omega) = \begin{cases} 1, & |\omega| \le \pi/4 \\ 0, & \pi/4 \le |\omega| \le \pi. \end{cases}$$

For this, exploit the properties of the Fourier transform.

- Is the system linear / non-linear?
- Is the system time-variant / time-invariant?

3 Scaling an Impulse Response to a Specific DC Gain

For a low pass filter, we desire a specific gain at frequency 0 (DC). Derive a way to scale an impulse response to have a specific target gain at DC.

4 Zero Phase IIR Filtering

If we have access to a whole signal, we can use an IIR filter as a zero phase filter. Show that filtering a signal with an IIR filter and then filtering the time-reversed result with the same filter, leads to a zero phase filter with the squared magnitude response of the actual filter. **Hints:**

- Solve this task partially in the time and Fourier domain
- Make use of Fourier properties, if necessary
- Don't forget to undo the time reversal after the second run through the filter

5 Python

In the given IPyton notebook we will look at the properties for typical IIR audio filters. Given are the functions to compute bell (or peaking) filters and low/ high shelving filters.

1. Write a function which plots the magnitude and phase response of a filter. Hint: use scipy's freqz function
2. Write a function that plots the poles and zeros of the filter. Hint: Either program it yourself for Biquads or use scipy to compute the locations
3. Write a function that lets you sweep over certain parameters, like frequency, gain, Q, while plotting the both above two functions results in one plot. We thereby want to gain a deeper understanding of the relationship between pole zero locations and the respective magnitude and phase response for different settings of the filters. Discuss your findings