UNIVERSITY OF HAMBURG, DEPARTMENT OF INFORMATICS, SIGNAL PROCESSING GROUP

Digital Media Signal Processing — Assignment VI

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1 DISCRETE-TIME FOURIER TRANSFORM (DTFT)

Compute the DTFT for the following signals. If a time series is given, the arrow indicates $\uparrow$ indicates $n = 0$.

1. $x(n) = \left\{ \underset{\uparrow}{\frac{1}{2}}, \frac{1}{2} \right\}$
2. $x(n) = \{2, 0, \underset{\uparrow}{0}, 0, -2\}$
3. $x(n) = u(n) - u(n-3)$
4. $x(n) = \left(\frac{1}{4}\right)^n u(n+2)$

For 1., 2., and 3., sketch the magnitude and the phase spectrum.

2 SPECIAL VALUES OF THE DISCRETE-TIME FOURIER TRANSFORM

Consider the signal

$$x(n) = \left\{ -1, 2, \underset{\uparrow}{-3}, 2, -1 \right\}$$

with Fourier transform $X(\omega)$. Compute the following quantities, without explicitly computing $X(\omega)$:

1. $X(0)$
2. $\int_{-\pi}^{\pi} X(\omega)\, d\omega$

3. $X(\pi)$
4. $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

3 Discrete-Time Fourier Series (DTFS)

A time-discrete, periodic signal $x(n)$ is defined as follows

$$x(n) = \begin{cases} A & 0 \le n \le L-1, \\ 0 & L \le n \le N-1. \end{cases}$$

Here, N is the period length, i.e., $x(n) = x(n+qN)$ with $q \in \mathbb{Z}$, and L is the pulse width, where $L \le N$. Figure 3.1 shows an example of $x(n)$ for $L = 4$ and $N = 8$.

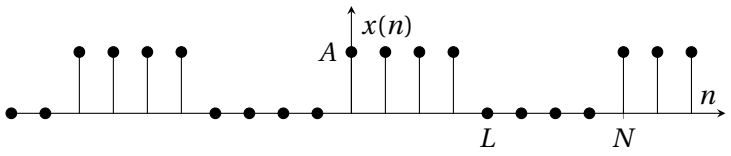

Figure 3.1: Example of $x(n)$ for $L = 4$ and $N = 8$

1. Show that the coefficients of the discrete-time Fourier series of $x(n)$ are given by

$$c_k = \frac{A}{N} e^{-j\frac{\pi(L-1)}{N}k} \frac{\sin\left(\frac{\pi L}{N}k\right)}{\sin\left(\frac{\pi}{N}k\right)}.$$

2. Sketch the magnitude and the phase of c_k for $L = 2$, $N = 4$, and $A = 2$.
3. Repeat the same for $L = 4$, $N = 4$, and $A = 2$.
4. Write a function in Python that computes discrete-time Fourier series for any given signal. Compare the derived result for $x(n)$ to the results of your implementation.
5. Use your implementation to compute the Fourier transform for the following signals.
   - $x(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 4, 5, 6, 7\}$
   - $x(n) = \cos(\pi/4n) + 2\sin(\pi/2n) + 3\cos(3\pi/4n)$ with period length $N = 8$