

# Mathematical Specification: Complete Model

Layered mathematical structure of hierarchical modeling

$$\begin{aligned} \beta &\sim \text{Normal}(0, 2.5) \\ \sigma_u, \sigma_v &\sim \text{Half-Cauchy}(0, 1) \\ u_i &\sim \text{Normal}(0, \sigma_u^2) \\ v_j &\sim \text{Normal}(0, \sigma_v^2) \end{aligned}$$

Bayesian Priors

$$\begin{aligned} \log(p_1/p_3) &= \alpha_1 + X_{ij_k} \beta_1 + u_i + v_j \\ \log(p_2/p_3) &= \alpha_2 + X_{ij_k} \beta_2 + u_i + v_j \end{aligned}$$

Linear Predictor

$$Y[ijk] \sim \text{Multinomial}(p_1, p_2, p_3)$$

Observation Model

Hierarchical Multinomial Model

$$\begin{aligned} i &= 1, \dots, 6 \text{ (monkeys)} \\ j &= 1, \dots, 88 \text{ (blocks)} \\ k &= 1, \dots, 1,443 \text{ (trials)} \\ p_1 + p_2 + p_3 &= 1 \end{aligned}$$

Fixed Effects:  $\beta_1, \beta_2, \beta_3, \beta_4$

Random Effects:  $u_i, v_j$

MCMC: 4 chains, 2000 iterations

Convergence:  $R_{\hat{\beta}_1} < 1.01$