

$$\begin{array}{ccc}
 o(G) = p & \xRightarrow{\text{Lagrange Cor. \#5}} & G \text{ is cyclic and has no} \\
 & & \text{non-trivial subgroups so} \\
 & & \text{it has no nontrivial} \\
 & & \text{normal subgroups} \\
 & & \implies G \text{ is simple} \\
 & & \implies G \text{ is abelian} \\
 & \downarrow & \\
 & \langle e \rangle &
 \end{array}$$