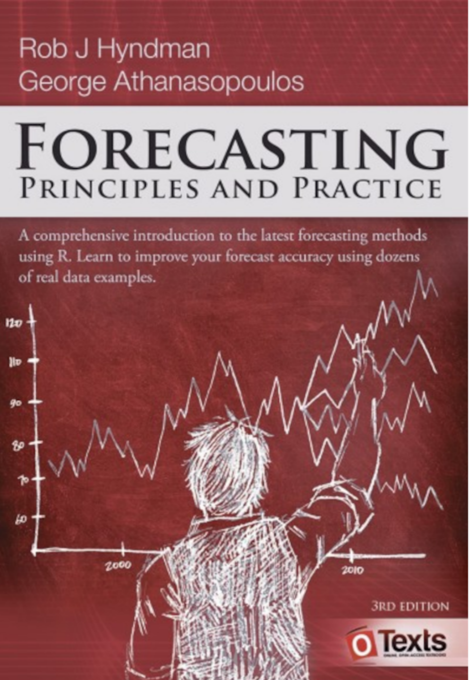


7. Time series regression models

7.4 Some useful predictors

OTexts.org/fpp3/

Trend

Linear trend

$$x_t = t$$

- $t = 1, 2, \ldots, T$
- Strong assumption that trend will continue.

Nonlinear trend

Piecewise linear trend with bend at τ

$$x_{1,t} = t$$

$$x_{2,t} = \begin{cases} 0 & t < \tau \\ (t - \tau) & t \geq \tau \end{cases}$$

Nonlinear trend

Piecewise linear trend with bend at τ

$$x_{1,t} = t$$
$$x_{2,t} = \begin{cases} 0 & t < \tau \\ (t-\tau) & t \geq \tau \end{cases}$$

Quadratic or higher order trend

$$x_{1,t} = t, \quad x_{2,t} = t^2, \quad \dots$$

Nonlinear trend

Piecewise linear trend with bend at τ

$$x_{1,t} = t$$
$$x_{2,t} = \begin{cases} 0 & t < \tau \\ (t - \tau) & t \geq \tau \end{cases}$$

Quadratic or higher order trend

$$x_{1,t} = t, \quad x_{2,t} = t^2, \quad \ldots$$

Dummy variables

If a categorical variable takes only two values (e.g., ‘Yes’ or ‘No’), then an equivalent numerical variable can be constructed taking value 1 if yes and 0 if no. This is called a **dummy variable**.

	A	B
1	Yes	1
2	Yes	1
3	No	0
4	Yes	1
5	No	0
6	No	0
7	Yes	1
8	Yes	1
9	No	0
10	No	0
11	No	0
12	No	0
13	Yes	1
14	No	0

Dummy variables

If there are more than two categories, then the variable can be coded using several dummy variables (one fewer than the total number of categories).

	A	B	C	D	E
1	Monday	1	0	0	0
2	Tuesday	0	1	0	0
3	Wednesday	0	0	1	0
4	Thursday	0	0	0	1
5	Friday	0	0	0	0
6	Monday	1	0	0	0
7	Tuesday	0	1	0	0
8	Wednesday	0	0	1	0
9	Thursday	0	0	0	1
10	Friday	0	0	0	0
11	Monday	1	0	0	0
12	Tuesday	0	1	0	0
13	Wednesday	0	0	1	0
14	Thursday	0	0	0	1
15	Friday	0	0	0	0

Beware of the dummy variable trap!

- Using one dummy for each category gives too many dummy variables!
- The regression will then be singular and inestimable.
- Either omit the constant, or omit the dummy for one category.
- The coefficients of the dummies are relative to the omitted category.

Uses of dummy variables

Seasonal dummies

- For quarterly data: use 3 dummies
- For monthly data: use 11 dummies
- For daily data: use 6 dummies
- What to do with weekly data?

Uses of dummy variables

Seasonal dummies

- For quarterly data: use 3 dummies
- For monthly data: use 11 dummies
- For daily data: use 6 dummies
- What to do with weekly data?

Outliers

- If there is an outlier, you can use a dummy variable to remove its effect.

Uses of dummy variables

Seasonal dummies

- For quarterly data: use 3 dummies
- For monthly data: use 11 dummies
- For daily data: use 6 dummies
- What to do with weekly data?

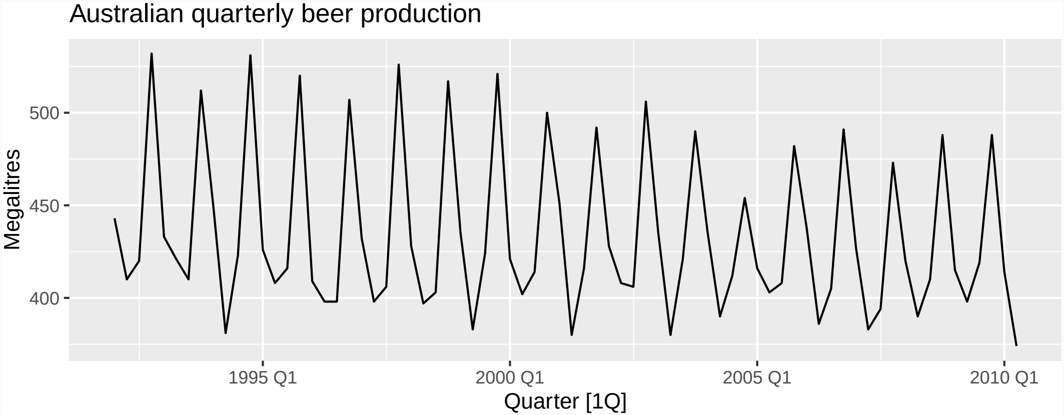
Outliers

- If there is an outlier, you can use a dummy variable to remove its effect.

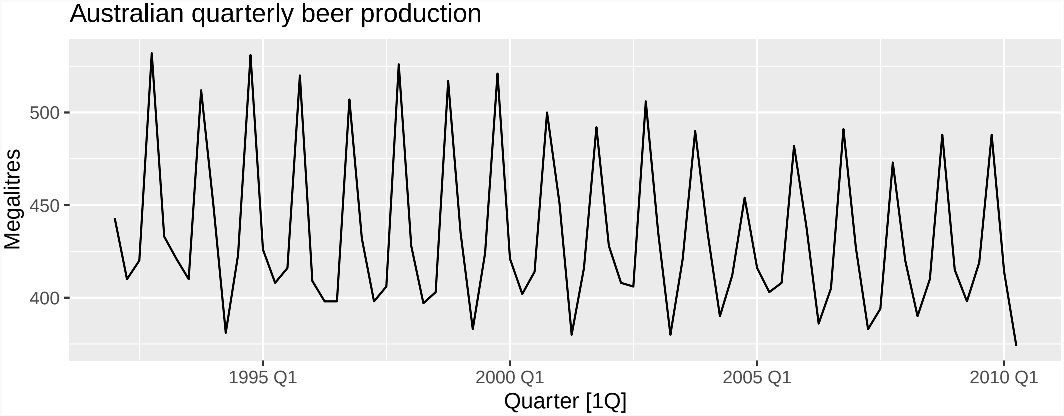
Public holidays

- For daily data: if it is a public holiday, dummy=1, otherwise dummy=0.

Beer production revisited



Beer production revisited



Regression model

$$y_t = \beta_0 + \beta_1 t + \beta_2 d_{2,t} + \beta_3 d_{3,t} + \beta_4 d_{4,t} + \varepsilon_t$$

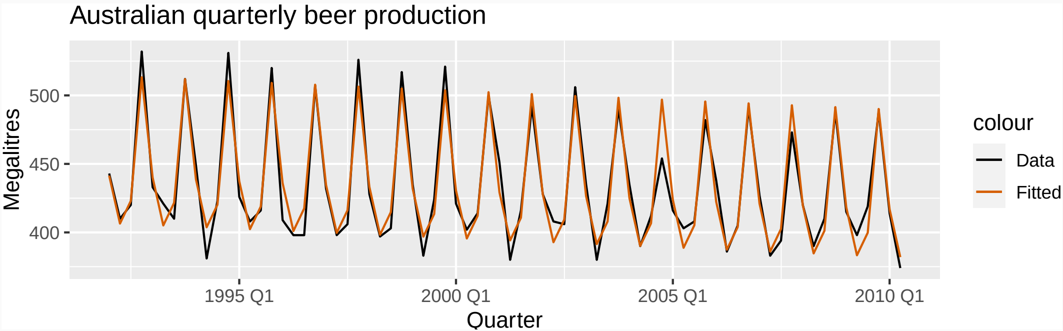
Beer production revisited

```
fit_beer <- recent_production |> model(TSLM(Beer ~ trend() + season()))
report(fit_beer)
```

```
## Series: Beer
## Model: TSLM
##
## Residuals:
##     Min      1Q  Median      3Q     Max
##   -42.9    -7.6    -0.5     8.0    21.8
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    441.8004     3.7335  118.33  < 2e-16 ***
## trend()         -0.3403     0.0666   -5.11  2.7e-06 ***
## season()year2  -34.6597     3.9683   -8.73  9.1e-13 ***
## season()year3  -17.8216     4.0225   -4.43  3.4e-05 ***
## season()year4   72.7964     4.0230   18.09  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.2 on 69 degrees of freedom
```

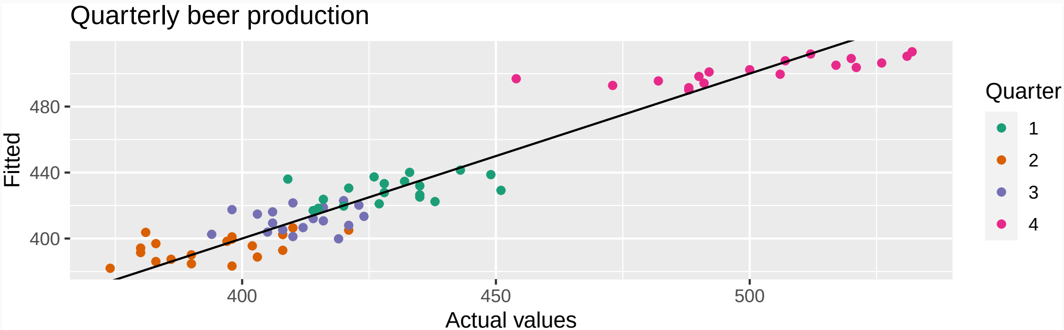
Beer production revisited

```
augment(fit_beer) |>
  ggplot(aes(x = Quarter)) +
  geom_line(aes(y = Beer, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted")) +
  labs(y = "Megalitres", title = "Australian quarterly beer production") +
  scale_colour_manual(values = c(Data = "black", Fitted = "#D55E00"))
```



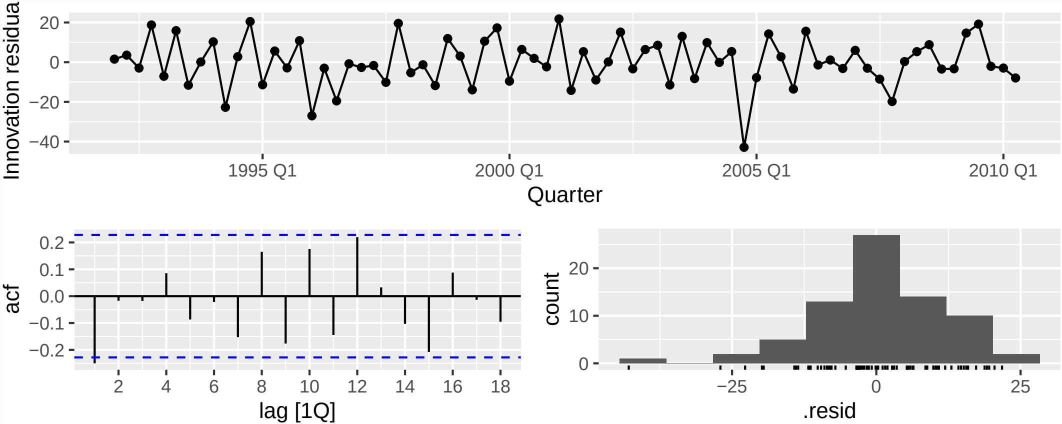
Beer production revisited

```
augment(fit_beer) |>
  ggplot(aes(x = Beer, y = .fitted, colour = factor(quarter(Quarter)))) +
  geom_point() +
  labs(y = "Fitted", x = "Actual values", title = "Quarterly beer production") +
  scale_colour_brewer(palette = "Dark2", name = "Quarter") +
  geom_abline(intercept = 0, slope = 1)
```



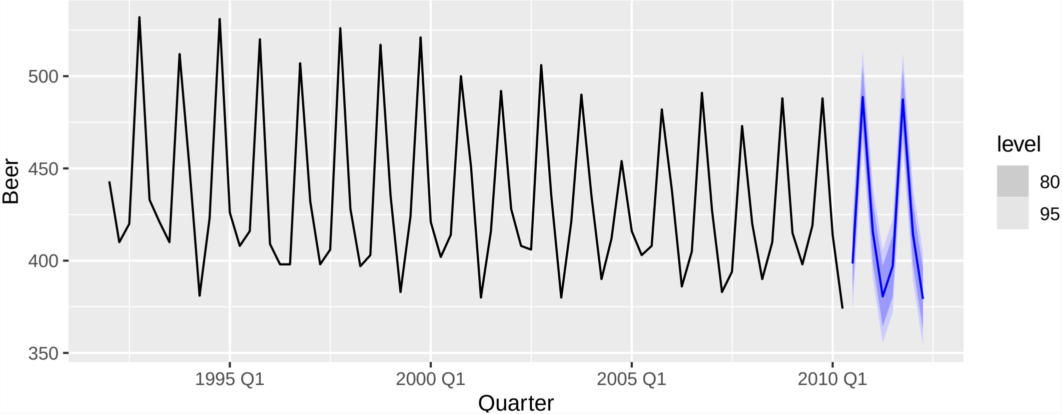
Beer production revisited

```
fit_beer |> gg_tsresiduals()
```



Beer production revisited

```
fit_beer |>
  forecast() |>
  autoplot(recent_production)
```



Fourier series

Periodic seasonality can be handled using pairs of Fourier terms:

$$s_k(t) = \sin\left(\frac{2\pi kt}{m}\right) \qquad c_k(t) = \cos\left(\frac{2\pi kt}{m}\right)$$

$$y_t = a + bt + \sum_{k=1}^{K}\left[\alpha_k s_k(t) + \beta_k c_k(t)\right] + \varepsilon_t$$

- Every periodic function can be approximated by sums of sin and cos terms for large enough K.
- Choose K by minimizing AICc.
- Called "harmonic regression"

```
TSLM(y ~ trend() + fourier(K))
```

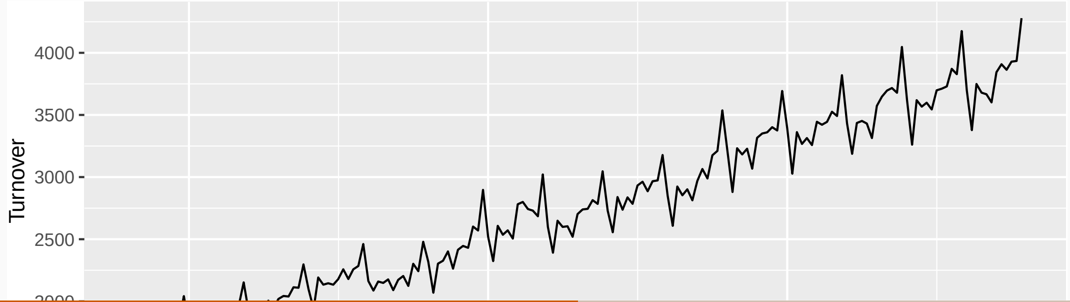
Harmonic regression: beer production

```
fourier_beer <- recent_production |> model(TSLM(Beer ~ trend() + fourier(K = 2)))
report(fourier_beer)
```

```
## Series: Beer
## Model: TSLM
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -42.9   -7.6   -0.5    8.0   21.8
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)         446.8792     2.8732  155.53  < 2e-16 ***
## trend()              -0.3403     0.0666   -5.11  2.7e-06 ***
## fourier(K = 2)C1_4    8.9108     2.0112    4.43  3.4e-05 ***
## fourier(K = 2)S1_4 -53.7281     2.0112  -26.71  < 2e-16 ***
## fourier(K = 2)C2_4 -13.9896     1.4226   -9.83  9.3e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.2 on 69 degrees of freedom
```

Harmonic regression: eating-out expenditure

```
aus_cafe <- aus_retail |>
  filter(
    Industry == "Cafes, restaurants and takeaway food services",
    year(Month) %in% 2004:2018
  ) |>
  summarise(Turnover = sum(Turnover))
aus_cafe |> autoplot(Turnover)
```

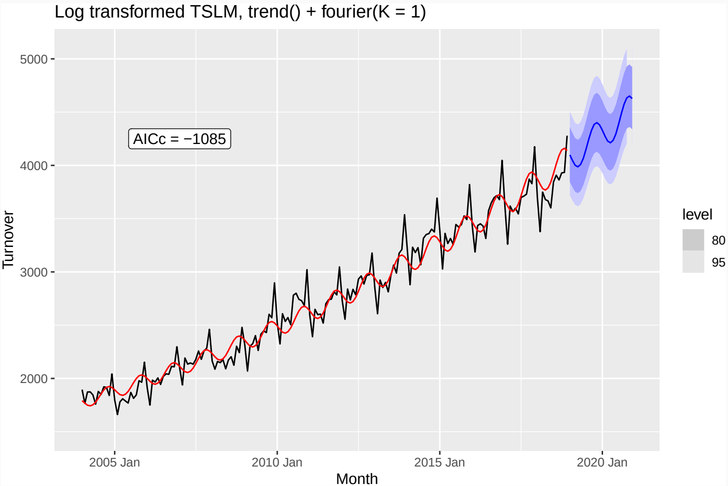


Harmonic regression: eating-out expenditure

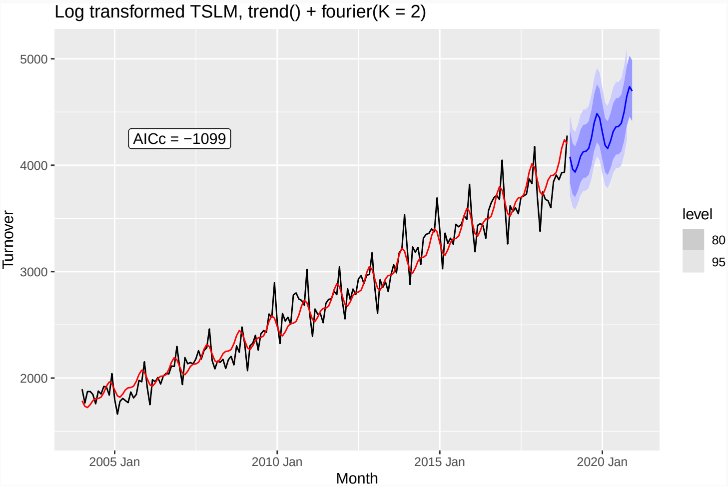
```
fit <- aus_cafe |>
  model(
    K1 = TSLM(log(Turnover) ~ trend() + fourier(K = 1)),
    K2 = TSLM(log(Turnover) ~ trend() + fourier(K = 2)),
    K3 = TSLM(log(Turnover) ~ trend() + fourier(K = 3)),
    K4 = TSLM(log(Turnover) ~ trend() + fourier(K = 4)),
    K5 = TSLM(log(Turnover) ~ trend() + fourier(K = 5)),
    K6 = TSLM(log(Turnover) ~ trend() + fourier(K = 6))
  )
glance(fit) |> select(.model, r_squared, adj_r_squared, AICc)
```

```
## # A tibble: 6 x 4
##   .model r_squared adj_r_squared   AICc
##   <chr>      <dbl>         <dbl>  <dbl>
## 1 K1         0.962         0.962 -1085.
## 2 K2         0.966         0.965 -1099.
## 3 K3         0.976         0.975 -1160.
## 4 K4         0.980         0.979 -1183.
```

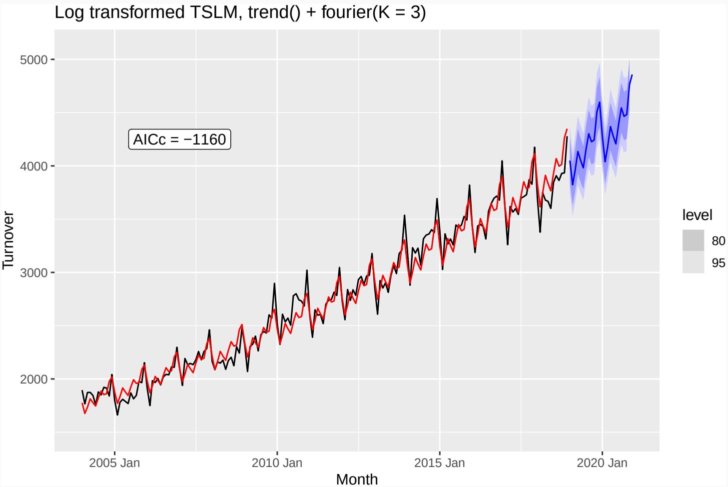
Harmonic regression: eating-out expenditure



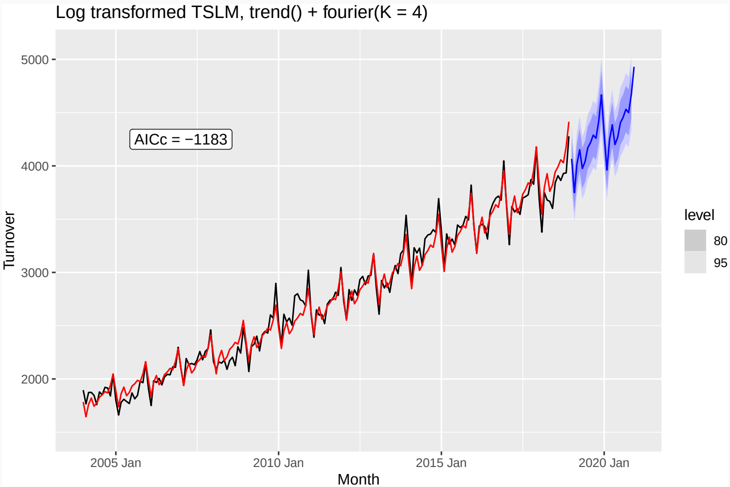
Harmonic regression: eating-out expenditure



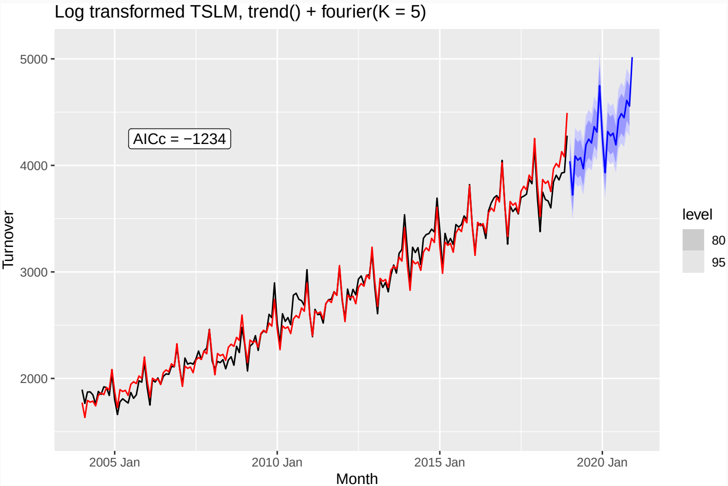
Harmonic regression: eating-out expenditure



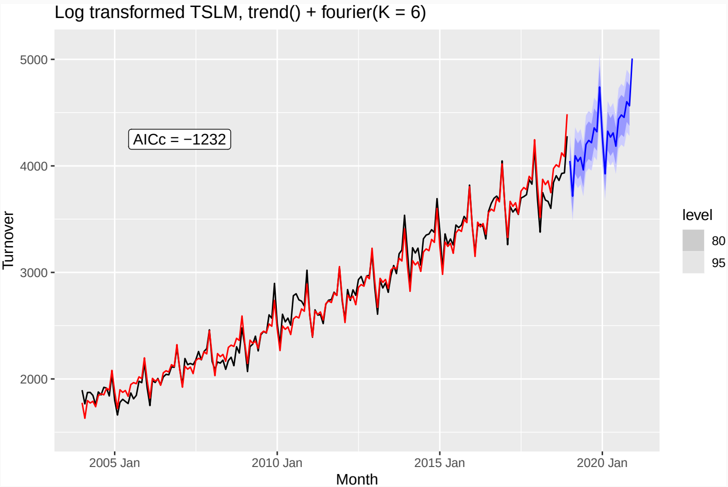
Harmonic regression: eating-out expenditure



Harmonic regression: eating-out expenditure



Harmonic regression: eating-out expenditure



Intervention variables

Spikes

- Equivalent to a dummy variable for handling an outlier.

Intervention variables

Spikes

- Equivalent to a dummy variable for handling an outlier.

Steps

- Variable takes value 0 before the intervention and 1 afterwards.

Intervention variables

Spikes

- Equivalent to a dummy variable for handling an outlier.

Steps

- Variable takes value 0 before the intervention and 1 afterwards.

Change of slope

- Variables take values 0 before the intervention and values $\{1, 2, 3, \ldots\}$ afterwards.

Holidays

For monthly data

- Christmas: always in December so part of monthly seasonal effect
- Easter: use a dummy variable $v_t = 1$ if any part of Easter is in that month, $v_t = 0$ otherwise.
- Ramadan and Chinese new year similar.

Distributed lags

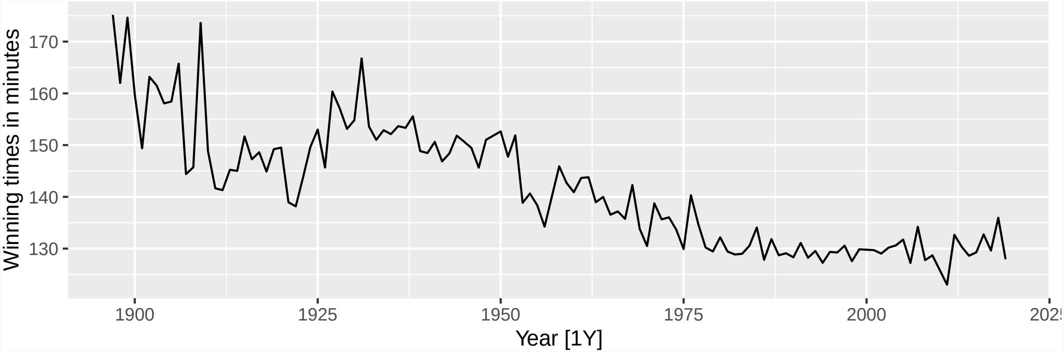
Lagged values of a predictor.

Example: x is advertising which has a delayed effect

$$
\begin{aligned}
x_1 &= \text{advertising for previous month;} \\
x_2 &= \text{advertising for two months previously;} \\
&\vdots \\
x_m &= \text{advertising for } m \text{ months previously.}
\end{aligned}
$$

Example: Boston marathon winning times

```
marathon <- boston_marathon |>
  filter(Event == "Men's open division") |>
  select(-Event) |>
  mutate(Minutes = as.numeric(Time) / 60)
marathon |> autoplot(Minutes) + labs(y = "Winning times in minutes")
```



Example: Boston marathon winning times

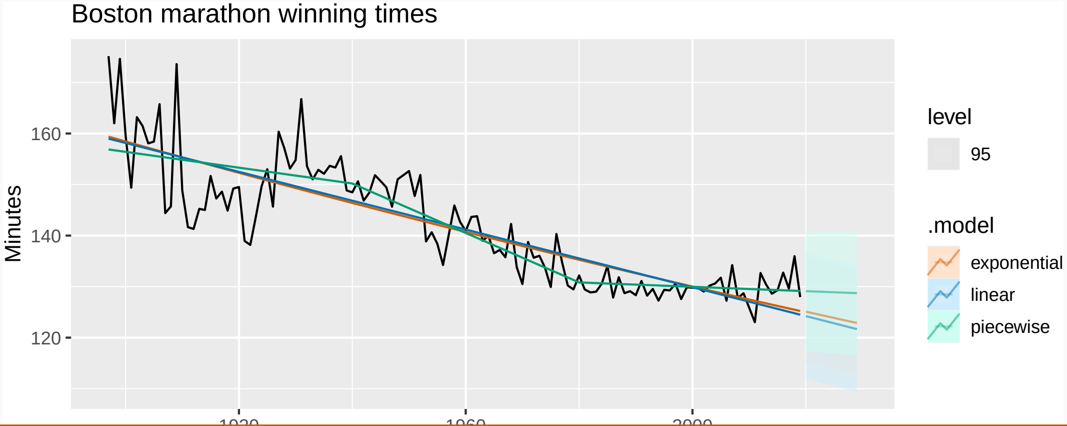
```
fit_trends <- marathon |>
  model(
    # Linear trend
    linear = TSLM(Minutes ~ trend()),
    # Exponential trend
    exponential = TSLM(log(Minutes) ~ trend()),
    # Piecewise linear trend
    piecewise = TSLM(Minutes ~ trend(knots = c(1940, 1980)))
  )
```

```
fit_trends
```

```
## # A mable: 1 x 3
##    linear exponential piecewise
##   <model>     <model>   <model>
## 1  <TSLM>      <TSLM>    <TSLM>
```

Example: Boston marathon winning times

```r
fit_trends |>
  forecast(h = 10) |>
  autoplot(marathon)
```



Example: Boston marathon winning times

```
fit_trends |>
  select(piecewise) |>
  gg_tsresiduals()
```