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FORECASTING

PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**
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8. Exponential smoothing

8.4 A taxonomy of methods

OTexts.org/fpp3/

Exponential smoothing methods

		Seasonal Component		
		N	A	M
Trend Component		(None)	(Additive)	(Multiplicative)
N	(None)	(N,N)	(N,A)	(N,M)
A	(Additive)	(A,N)	(A,A)	(A,M)
A _d	(Additive damped)	(A _d ,N)	(A _d ,A)	(A _d ,M)

(N,N): Simple exponential smoothing

(A,N): Holt's linear method

(A_d,N): Additive damped trend method

(A,A): Additive Holt-Winters' method

(A,M): Multiplicative Holt-Winters' method

(A_d,M): Damped multiplicative Holt-Winters' method

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There are also multiplicative trend methods (not recommended).

Exponential smoothing methods

Trend	Seasonal		
	N	A	M
N	$\hat{y}_{t+h t} = \ell_t$	$\hat{y}_{t+h t} = \ell_t + s_{t+h-m(k+1)}$	$\hat{y}_{t+h t} = \ell_t s_{t+h-m(k+1)}$
	$\ell_t = \alpha y_t + (1 - \alpha)\ell_{t-1}$	$\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)\ell_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1}) + (1 - \gamma)s_{t-m}$	$\ell_t = \alpha(y_t/s_{t-m}) + (1 - \alpha)\ell_{t-1}$ $s_t = \gamma(y_t/\ell_{t-1}) + (1 - \gamma)s_{t-m}$
A	$\hat{y}_{t+h t} = \ell_t + hb_t$	$\hat{y}_{t+h t} = \ell_t + hb_t + s_{t+h-m(k+1)}$	$\hat{y}_{t+h t} = (\ell_t + hb_t)s_{t+h-m(k+1)}$
	$\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$	$\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1 - \gamma)s_{t-m}$	$\ell_t = \alpha(y_t/s_{t-m}) + (1 - \alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$ $s_t = \gamma(y_t/(\ell_{t-1} + b_{t-1})) + (1 - \gamma)s_{t-m}$
A _d	$\hat{y}_{t+h t} = \ell_t + \phi_h b_t$	$\hat{y}_{t+h t} = \ell_t + \phi_h b_t + s_{t+h-m(k+1)}$	$\hat{y}_{t+h t} = (\ell_t + \phi_h b_t)s_{t+h-m(k+1)}$
	$\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}$	$\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - \phi b_{t-1}) + (1 - \gamma)s_{t-m}$	$\ell_t = \alpha(y_t/s_{t-m}) + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}$ $s_t = \gamma(y_t/(\ell_{t-1} + \phi b_{t-1})) + (1 - \gamma)s_{t-m}$

$$\phi_h = \phi + \phi^2 + \dots + \phi^h$$

k is the integer part of $(h - 1)/m$