MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550 Applied forecasting

Ch10. Dynamic regression models

OTexts.org/fpp3/

Outline

1 Regression with ARIMA errors

2 Stochastic and deterministic trends

3 Dynamic harmonic regression

4 Lagged predictors

Regression with ARIMA errors

Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \ldots, x_{k,t}$.
- In regression, we assume that ε_t is WN.
- Now we want to allow ε_t to be autocorrelated.

Regression with ARIMA errors

Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \ldots, x_{k,t}$.
- In regression, we assume that ε_t is WN.
- Now we want to allow ε_t to be autocorrelated.

Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$

$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t$$

Residuals and errors

Example: η_t = ARIMA(1,1,1)

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

Residuals and errors

Example: η_t = ARIMA(1,1,1)

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

- Be careful in distinguishing η_t from ε_t.
- Only the errors ε_t are assumed to be white noise.
- In ordinary regression, η_t is assumed to be white noise and so $\eta_t = \varepsilon_t$.

Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are no longer optimal as some information ignored;
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
3. p-values for coefficients usually too small ("spurious regression'').
4. AIC of fitted models misleading.

Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are no longer optimal as some information ignored;
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
3. p-values for coefficients usually too small ("spurious regression'').
4. AIC of fitted models misleading.

- Minimizing $\sum \varepsilon_t^2$ avoids these problems.
- Maximizing likelihood similar to minimizing $\sum \varepsilon_t^2$

Regression with ARIMA errors

Model with ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

Regression with ARIMA errors

Model with ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

Equivalent to model with ARIMA(1,0,1) errors

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t,$$
$$(1 - \phi_1 B)\eta'_t = (1 + \theta_1 B)\varepsilon_t,$$

where $y'_t = y_t - y_{t-1}$, $x'_{t,i} = x_{t,i} - x_{t-1,i}$ and $\eta'_t = \eta_t - \eta_{t-1}$.

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

Original data

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t$$

$$\text{where} \quad \phi(B)(1-B)^d \eta_t = \theta(B)\varepsilon_t$$

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

Original data

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t$$

$$\text{where} \quad \phi(B)(1-B)^d \eta_t = \theta(B)\varepsilon_t$$

After differencing all variables

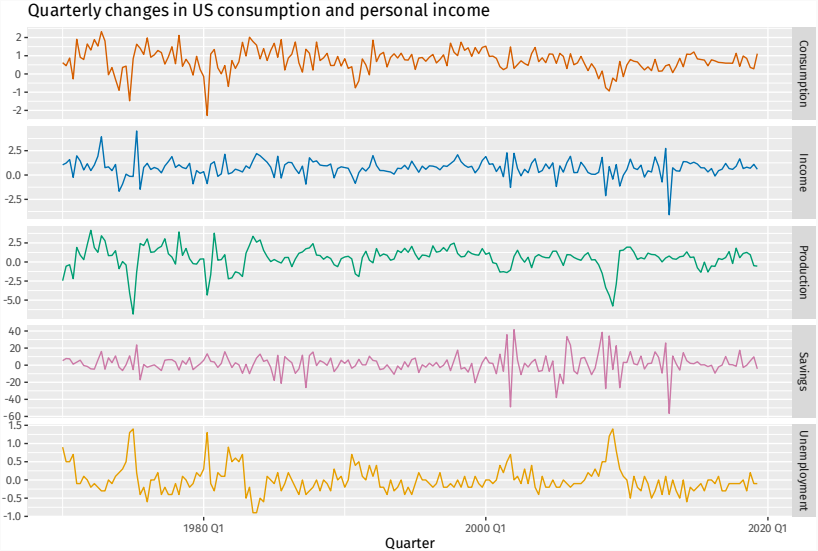
$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t.$$

where $\phi(B)\eta'_t = \theta(B)\varepsilon_t$,

Regression with ARIMA errors

- In R, we can specify an ARIMA(p, d, q) for the errors, and d levels of differencing will be applied to all variables $(y, x_{1,t}, \ldots, x_{k,t})$ during estimation.
- Check that ε_t series looks like white noise.
- AICc can be calculated for final model.
- Repeat procedure for all subsets of predictors to be considered, and select model with lowest AICc value.

US personal consumption and income



US personal consumption and income

```r
fit <- us_change |> model(ARIMA(Consumption ~ Income))
report(fit)
```

```
Series: Consumption
Model: LM w/ ARIMA(1,0,2) errors

Coefficients:
        ar1     ma1     ma2  Income  intercept
      0.707  -0.617  0.2066  0.1976      0.595
s.e.  0.107   0.122  0.0741  0.0462      0.085

sigma^2 estimated as 0.3113:  log likelihood=-163
AIC=338   AICc=339   BIC=358
```

US personal consumption and income

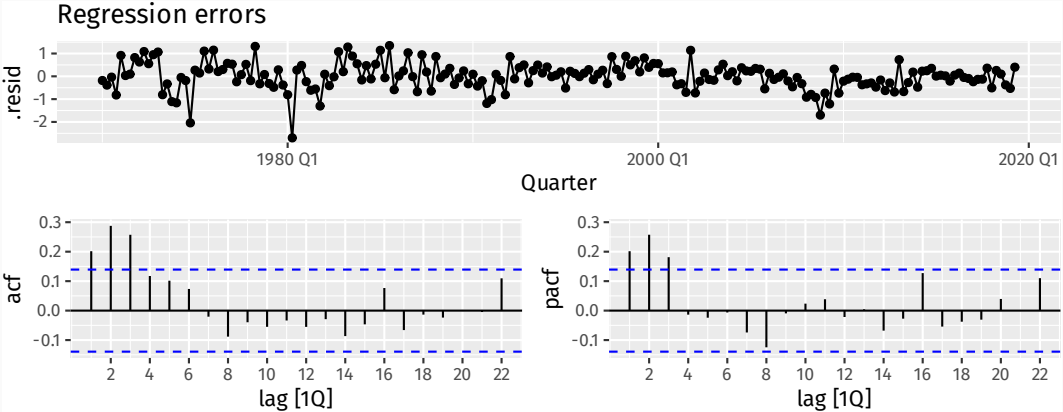
```r
fit <- us_change |> model(ARIMA(Consumption ~ Income))
report(fit)
```

```
Series: Consumption
Model: LM w/ ARIMA(1,0,2) errors

Coefficients:
        ar1     ma1     ma2  Income  intercept
      0.707  -0.617  0.2066  0.1976      0.595
s.e.  0.107   0.122  0.0741  0.0462      0.085

sigma^2 estimated as 0.3113:  log likelihood=-163
AIC=338   AICc=339   BIC=358
```

Write down the equations for the fitted model.

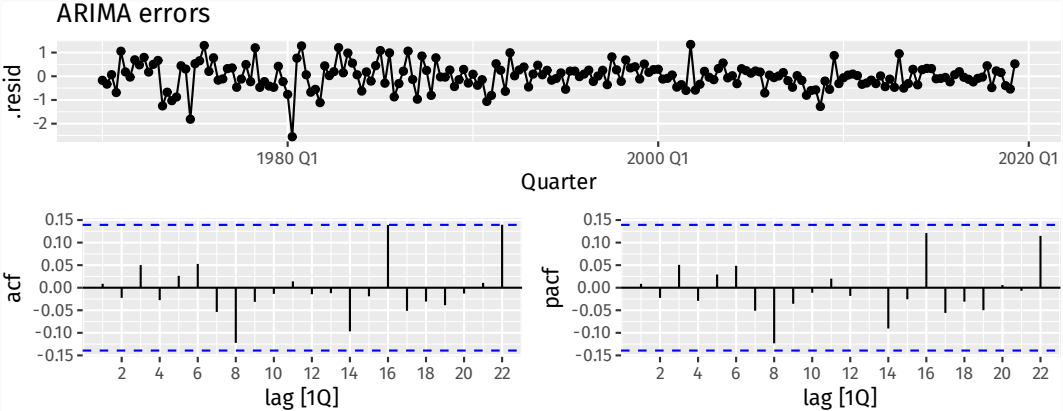
US personal consumption and income

```
residuals(fit, type = "regression") |>
  gg_tsdisplay(.resid, plot_type = "partial") +
  labs(title = "Regression errors")
```



US personal consumption and income

```
residuals(fit, type = "innovation") |>
  gg_tsdisplay(.resid, plot_type = "partial") +
  labs(title = "ARIMA errors")
```



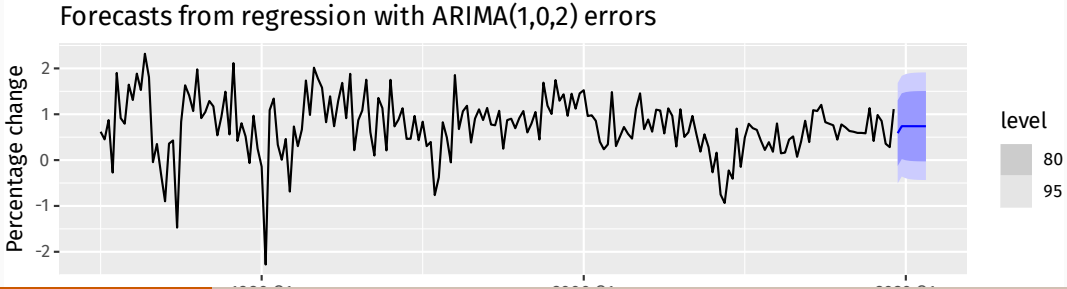
US personal consumption and income

```r
augment(fit) |>
  features(.innov, ljung_box, dof = 5, lag = 12)
```

```
# A tibble: 1 x 3
  .model                       lb_stat lb_pvalue
  <chr>                          <dbl>     <dbl>
1 ARIMA(Consumption ~ Income)     5.54     0.595
```

US personal consumption and income

```
us_change_future <- new_data(us_change, 8) |>
  mutate(Income = mean(us_change$Income))
forecast(fit, new_data = us_change_future) |>
  autoplot(us_change) +
  labs(
    x = "Year", y = "Percentage change",
    title = "Forecasts from regression with ARIMA(1,0,2) errors"
  )
```

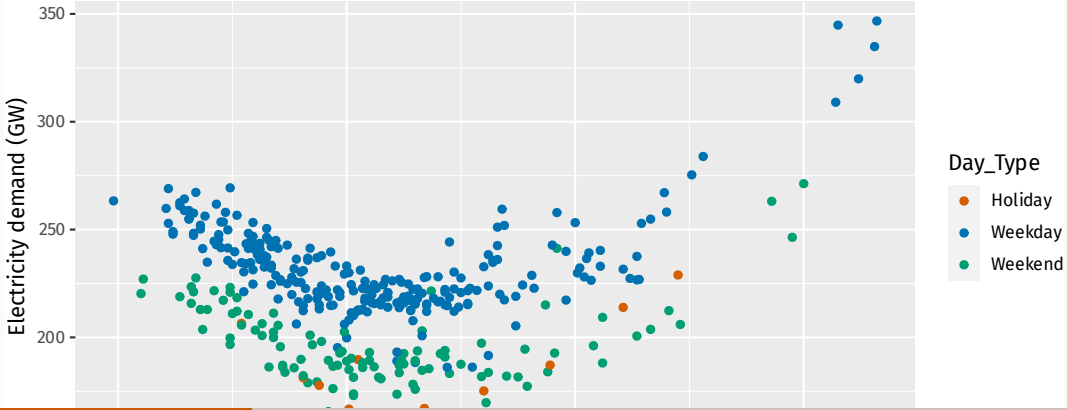


Forecasting

- To forecast a regression model with ARIMA errors, we need to forecast the regression part of the model and the ARIMA part of the model and combine the results.
- Some predictors are known into the future (e.g., time, dummies).
- Separate forecasting models may be needed for other predictors.
- Forecast intervals ignore the uncertainty in forecasting the predictors.

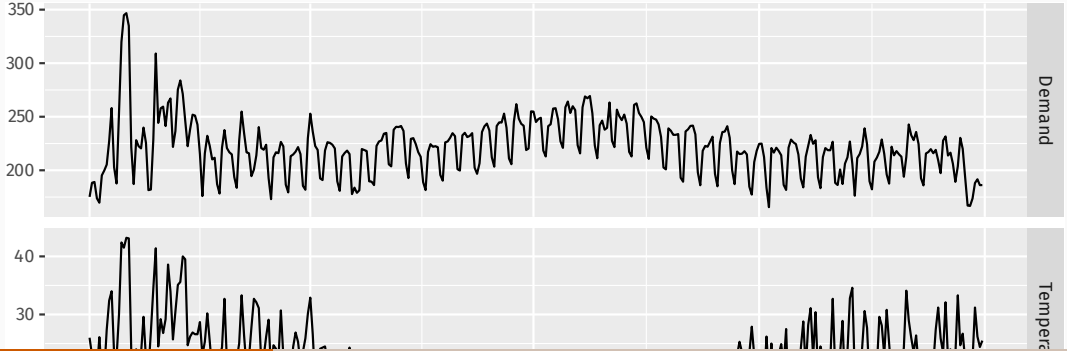
Daily electricity demand

```
vic_elec_daily |>
  ggplot(aes(x = Temperature, y = Demand, colour = Day_Type)) +
  geom_point() +
  labs(x = "Maximum temperature", y = "Electricity demand (GW)")
```



Daily electricity demand

```
vic_elec_daily |>
  pivot_longer(c(Demand, Temperature)) |>
  ggplot(aes(x = Date, y = value)) +
  geom_line() +
  facet_grid(name ~ ., scales = "free_y") +
  ylab("")
```



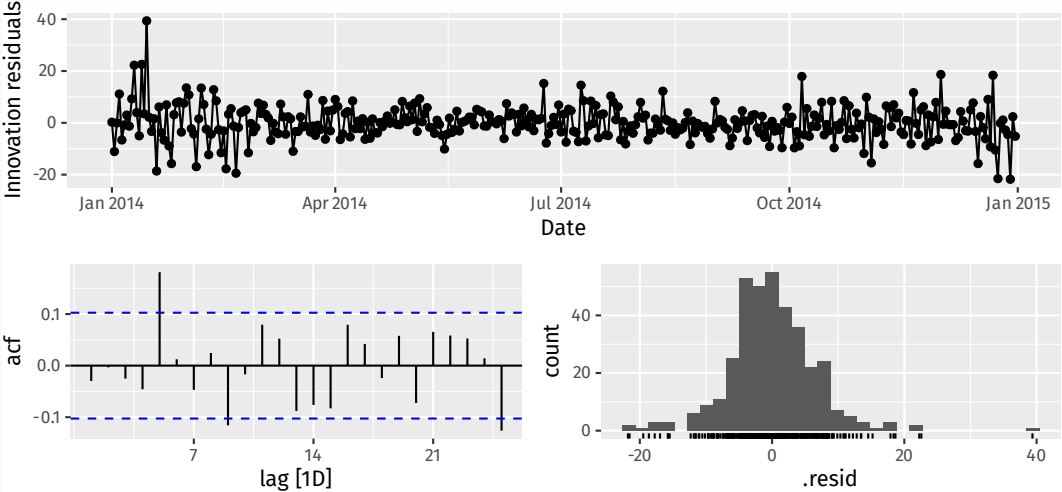
Daily electricity demand

```r
fit <- vic_elec_daily |>
  model(ARIMA(Demand ~ Temperature + I(Temperature^2) +
    (Day_Type == "Weekday")))
report(fit)
```

```
Series: Demand
Model: LM w/ ARIMA(2,1,2)(2,0,0)[7] errors

Coefficients:
          ar1     ar2      ma1      ma2    sar1   sar2  Temperature
      -0.1093  0.7226  -0.0182  -0.9381  0.1958  0.417       -7.614
s.e.   0.0779  0.0739   0.0494   0.0493  0.0525  0.057        0.448
      I(Temperature^2)  Day_Type == "Weekday"TRUE
                0.1810                      30.40
s.e.            0.0085                       1.33

sigma^2 estimated as 44.91:  log likelihood=-1206
AIC=2432   AICc=2433   BIC=2471
```

Daily electricity demand

```
gg_tsresiduals(fit)
```



Daily electricity demand

```r
augment(fit) |>
  features(.resid, ljung_box, dof = 9, lag = 14)
```

```
# A tibble: 1 x 3
  .model                                                   lb_stat lb_pvalue
  <chr>                                                      <dbl>     <dbl>
1 "ARIMA(Demand ~ Temperature + I(Temperature^2) + (Day_~     28.4 0.0000304
```

Daily electricity demand

```r
# Forecast one day ahead
vic_next_day <- new_data(vic_elec_daily, 1) |>
  mutate(Temperature = 26, Day_Type = "Holiday")
forecast(fit, vic_next_day)
```

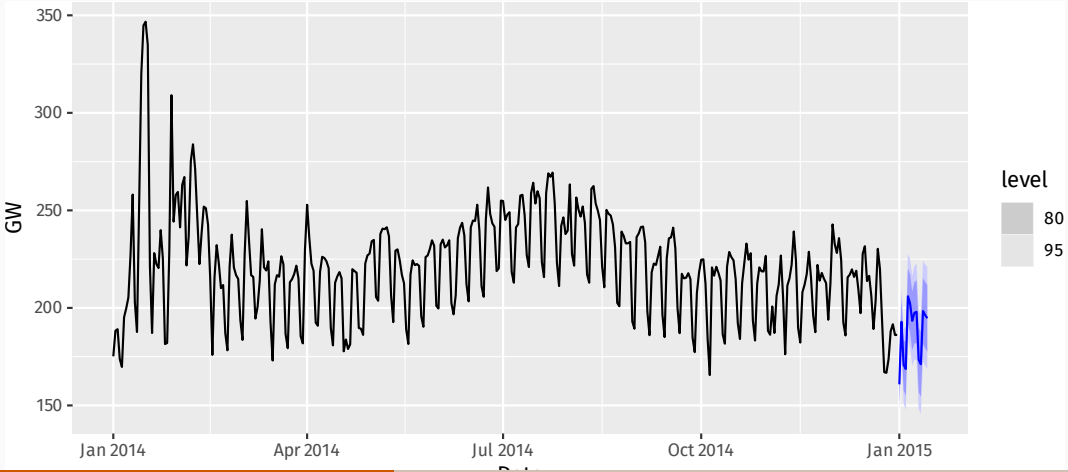
```
# A fable: 1 x 6 [1D]
# Key:     .model [1]
  .model                    Date           Demand .mean Temperature Day_Type
  <chr>                     <date>         <dist> <dbl>       <dbl> <chr>
1 "ARIMA(Demand ~ Tempera~ 2015-01-01 N(161, 45)  161.          26 Holiday
```

Daily electricity demand

```r
vic_elec_future <- new_data(vic_elec_daily, 14) |>
  mutate(
    Temperature = 26,
    Holiday = c(TRUE, rep(FALSE, 13)),
    Day_Type = case_when(
      Holiday ~ "Holiday",
      wday(Date) %in% 2:6 ~ "Weekday",
      TRUE ~ "Weekend"
    )
  )
```

Daily electricity demand

```
forecast(fit, new_data = vic_elec_future) |>
  autoplot(vic_elec_daily) + labs(y = "GW")
```



Outline

1 Regression with ARIMA errors

2 Stochastic and deterministic trends

3 Dynamic harmonic regression

4 Lagged predictors

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

Stochastic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARIMA process with $d \geq 1$.

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

Stochastic trend

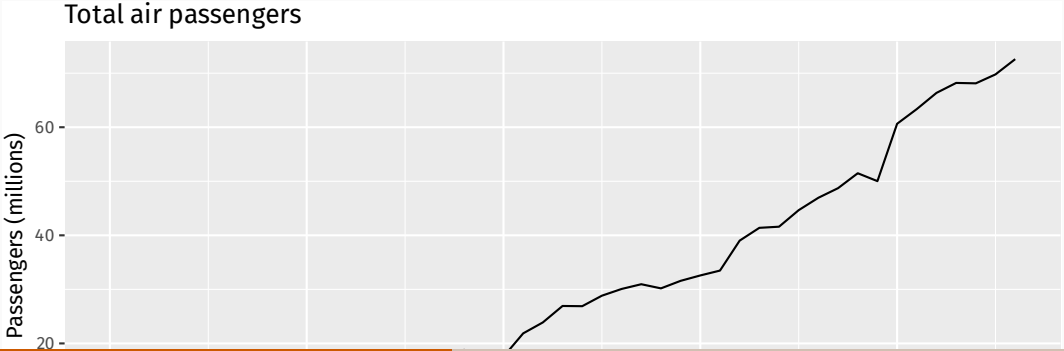
$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARIMA process with $d \geq 1$.
Difference both sides until η_t is stationary:

$$y'_t = \beta_1 + \eta'_t$$

Air transport passengers Australia

```
aus_airpassengers |>
  autoplot(Passengers) +
  labs(
    y = "Passengers (millions)",
    title = "Total air passengers"
  )
```



Air transport passengers Australia

Deterministic trend

```r
fit_deterministic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
Series: Passengers
Model: LM w/ ARIMA(1,0,0) errors

Coefficients:
         ar1  trend()  intercept
      0.9564    1.415      0.901
s.e.  0.0362    0.197      7.075

sigma^2 estimated as 4.343:  log likelihood=-101
AIC=210   AICc=211   BIC=217
```

Air transport passengers Australia

Deterministic trend

```
fit_deterministic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ 1 + trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
Series: Passengers
Model: LM w/ ARIMA(1,0,0) errors

Coefficients:
         ar1  trend()  intercept
      0.9564    1.415      0.901
s.e.  0.0362    0.197      7.075

sigma^2 estimated as 4.343:  log likelihood=-101
AIC=210   AICc=211   BIC=217
```

$$y_t = 0.901 + 1.415t + \eta_t$$
$$\eta_t = 0.956\eta_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 4.343).$$

Air transport passengers Australia

Stochastic trend

```r
fit_stochastic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ pdq(d = 1)))
report(fit_stochastic)
```

```
Series: Passengers
Model: ARIMA(0,1,0) w/ drift

Coefficients:
      constant
         1.419
s.e.     0.301

sigma^2 estimated as 4.271:  log likelihood=-98.2
AIC=200   AICc=201   BIC=204
```

Air transport passengers Australia

Stochastic trend

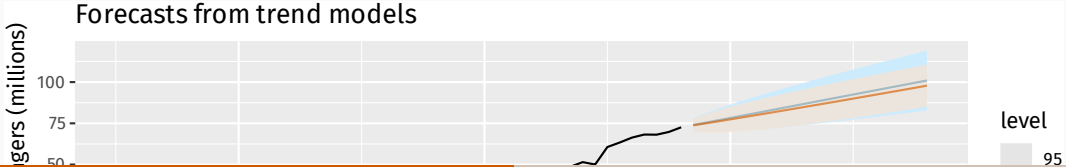
```r
fit_stochastic <- aus_airpassengers |>
  model(ARIMA(Passengers ~ pdq(d = 1)))
report(fit_stochastic)
```

```
Series: Passengers
Model: ARIMA(0,1,0) w/ drift

Coefficients:
      constant
         1.419
s.e.     0.301

sigma^2 estimated as 4.271:  log likelihood=
AIC=200   AICc=201   BIC=204
```

$$
\begin{aligned}
y_t - y_{t-1} &= 1.419 + \varepsilon_t, \\
y_t &= y_0 + 1.419t + \eta_t \\
\eta_t &= \eta_{t-1} + \varepsilon_t \\
\varepsilon_t &\sim \text{NID}(0, 4.271).
\end{aligned}
$$

Air transport passengers Australia

```
aus_airpassengers |>
  autoplot(Passengers) +
  autolayer(fit_stochastic |> forecast(h = 20),
    colour = "#0072B2", level = 95
  ) +
  autolayer(fit_deterministic |> forecast(h = 20),
    colour = "#D55E00", alpha = 0.65, level = 95
  ) +
  labs(
    y = "Air passengers (millions)",
    title = "Forecasts from trend models"
  )
```



Forecasting with trend

- Point forecasts are almost identical, but prediction intervals differ.
- Stochastic trends have much wider prediction intervals because the errors are non-stationary.
- Be careful of forecasting with deterministic trends too far ahead.

Outline

1 Regression with ARIMA errors

2 Stochastic and deterministic trends

3 Dynamic harmonic regression

4 Lagged predictors

Dynamic harmonic regression

Combine Fourier terms with ARIMA errors

Advantages

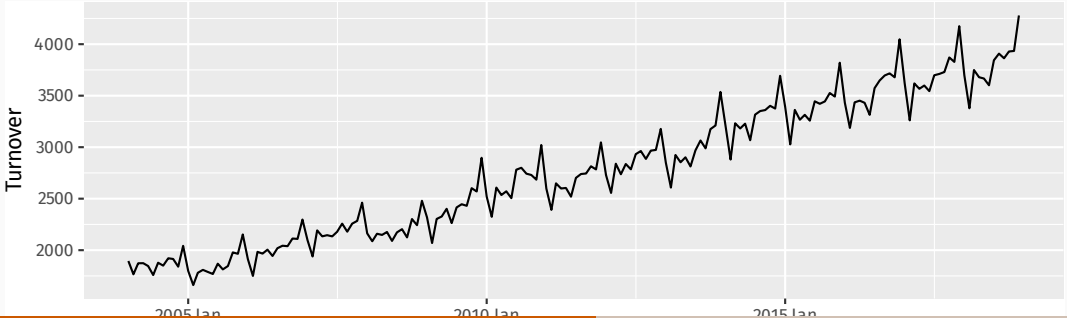
- it allows any length seasonality;
- for data with more than one seasonal period, you can include Fourier terms of different frequencies;
- the seasonal pattern is smooth for small values of K (but more wiggly seasonality can be handled by increasing K);
- the short-term dynamics are easily handled with a simple ARMA error.

Disadvantages

- seasonality is assumed to be fixed

Eating-out expenditure

```r
aus_cafe <- aus_retail |>
  filter(
    Industry == "Cafes, restaurants and takeaway food services",
    year(Month) %in% 2004:2018
  ) |>
  summarise(Turnover = sum(Turnover))
aus_cafe |> autoplot(Turnover)
```

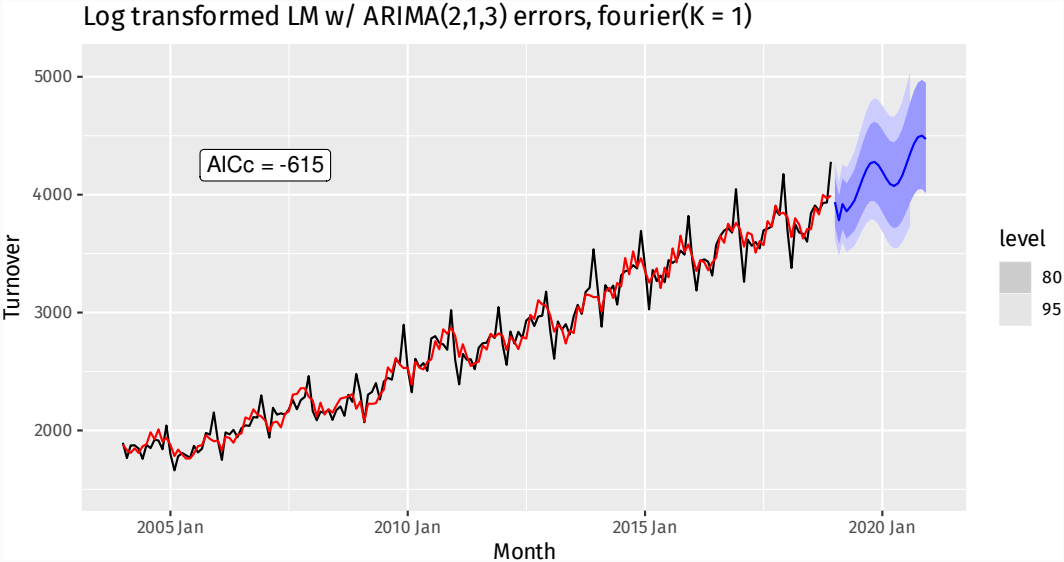


Eating-out expenditure

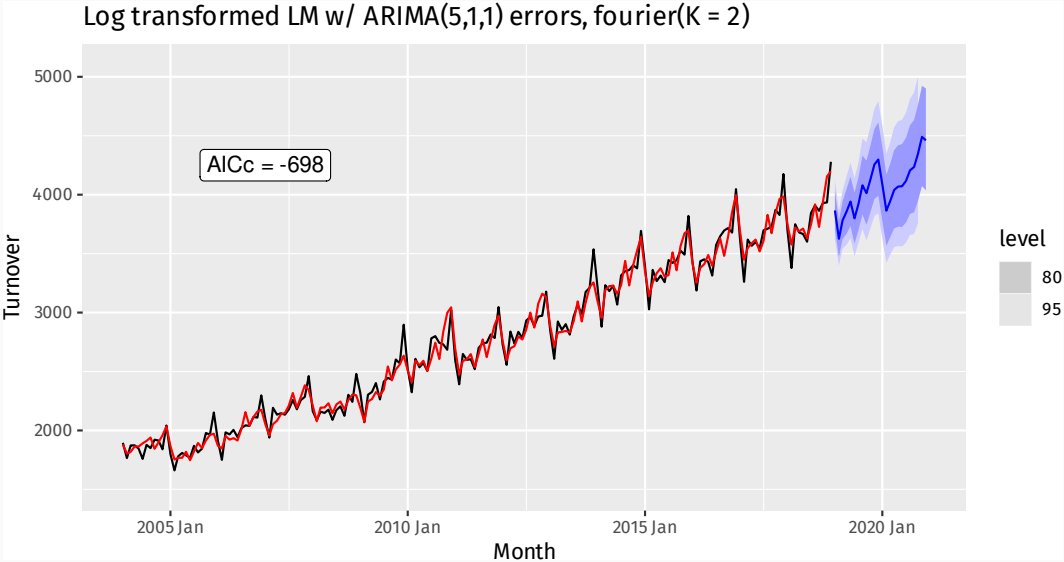
```r
fit <- aus_cafe |> model(
  `K = 1` = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0, 0, 0)),
  `K = 2` = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0, 0, 0)),
  `K = 3` = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0, 0, 0)),
  `K = 4` = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0, 0, 0)),
  `K = 5` = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0, 0, 0)),
  `K = 6` = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0, 0, 0))
)
glance(fit)
```

.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.002	317	-616	-615	-588
K = 2	0.001	362	-700	-698	-661
K = 3	0.001	394	-763	-761	-725
K = 4	0.001	427	-822	-818	-771
K = 5	0.000	474	-919	-917	-875
K = 6	0.000	474	-920	-918	-875

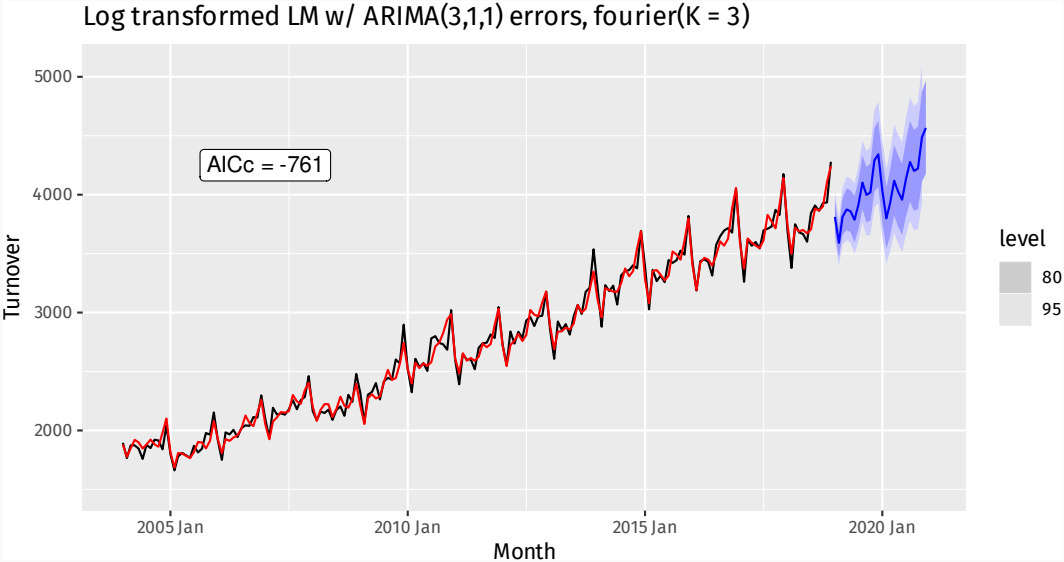
Eating-out expenditure



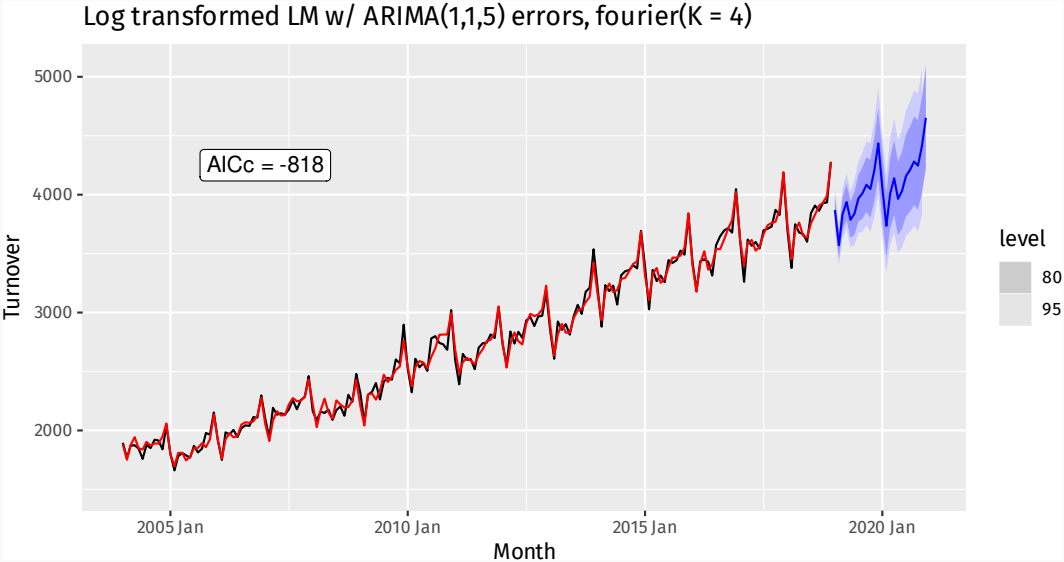
Eating-out expenditure



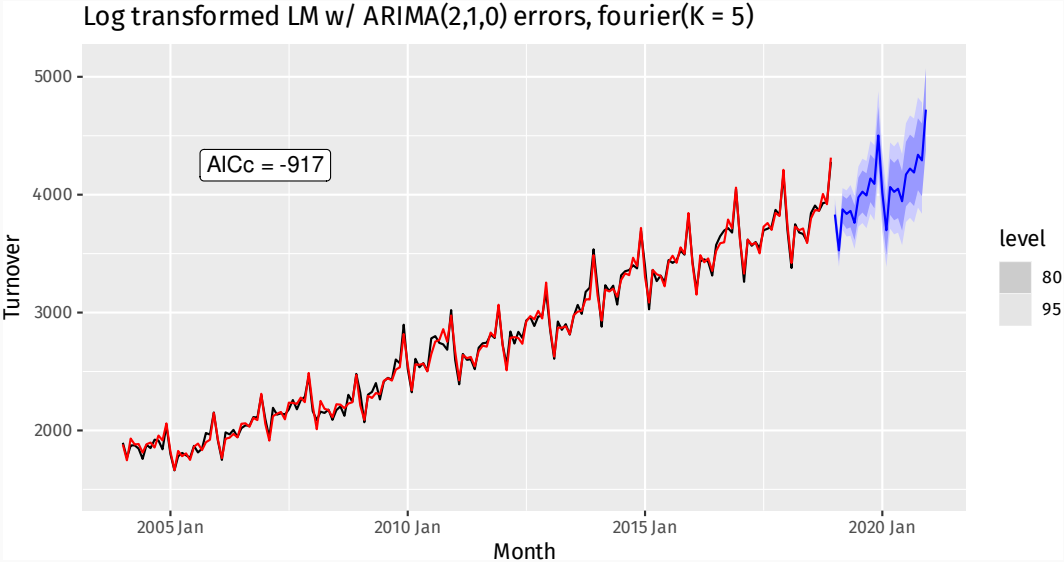
Eating-out expenditure



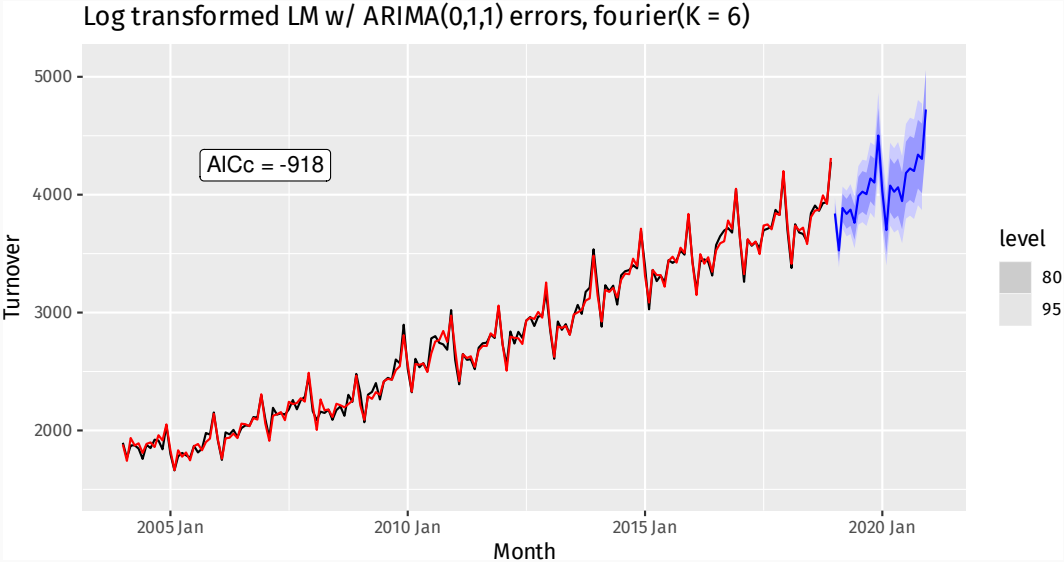
Eating-out expenditure



Eating-out expenditure



Eating-out expenditure



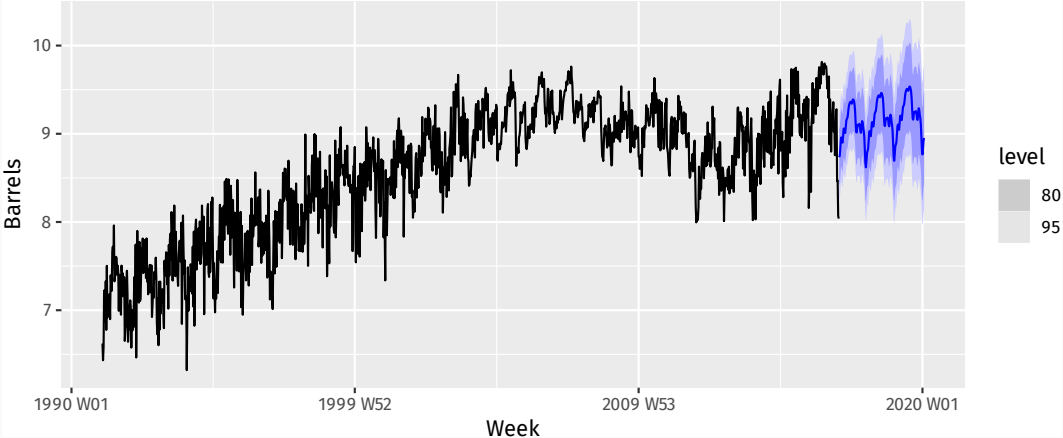
Example: weekly gasoline products

```r
fit <- us_gasoline |>
  model(ARIMA(Barrels ~ fourier(K = 13) + PDQ(0, 0, 0)))
report(fit)
```

```
Series: Barrels
Model: LM w/ ARIMA(0,1,1) errors

Coefficients:
          ma1  fourier(K = 13)C1_52  fourier(K = 13)S1_52
      -0.8934               -0.1121               -0.2300
s.e.   0.0132                0.0123                0.0122
      fourier(K = 13)C2_52  fourier(K = 13)S2_52
                    0.0420                0.0317
s.e.                0.0099                0.0099
      fourier(K = 13)C3_52  fourier(K = 13)S3_52
                    0.0832                0.0346
s.e.                0.0094                0.0094
      fourier(K = 13)C4_52  fourier(K = 13)S4_52
                    0.0185                0.0398
s.e.                0.0092                0.0092
      fourier(K = 13)C5_52  fourier(K = 13)S5_52
                   -0.0315                0.0009
```

Example: weekly gasoline products

```
forecast(fit, h = "3 years") |>
  autoplot(us_gasoline)
```



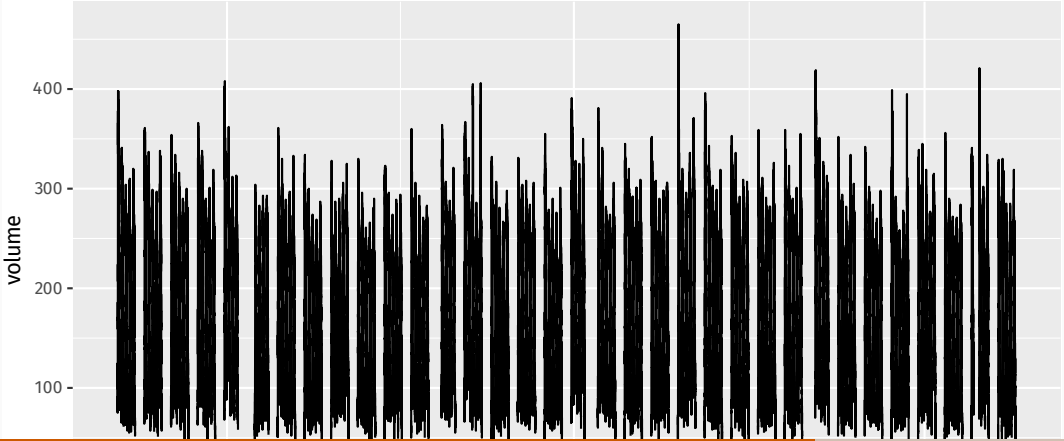
5-minute call centre volume

```r
(calls <- readr::read_tsv("http://robjhyndman.com/data/callcenter.txt") |>
  rename(time = `...1`) |>
  pivot_longer(-time, names_to = "date", values_to = "volume") |>
  mutate(
    date = as.Date(date, format = "%d/%m/%Y"),
    datetime = as_datetime(date) + time
  ) |>
  as_tsibble(index = datetime))
```

```
# A tsibble: 27,716 x 4 [5m] <UTC>
   time   date       volume datetime
   <time> <date>      <dbl> <dttm>
 1 07:00  2003-03-03    111 2003-03-03 07:00:00
 2 07:05  2003-03-03    113 2003-03-03 07:05:00
 3 07:10  2003-03-03     76 2003-03-03 07:10:00
 4 07:15  2003-03-03     82 2003-03-03 07:15:00
 5 07:20  2003-03-03     91 2003-03-03 07:20:00
 6 07:25  2003-03-03     87 2003-03-03 07:25:00
```

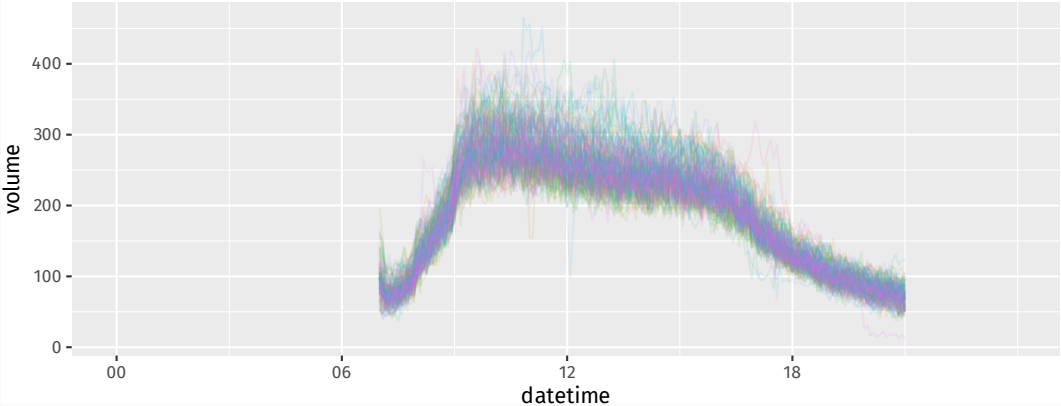
5-minute call centre volume

```
calls |>
  fill_gaps() |>
  autoplot(volume)
```



5-minute call centre volume

```
calls |>
  fill_gaps() |>
  gg_season(volume, period = "day", alpha = 0.1) +
  guides(colour = FALSE)
```



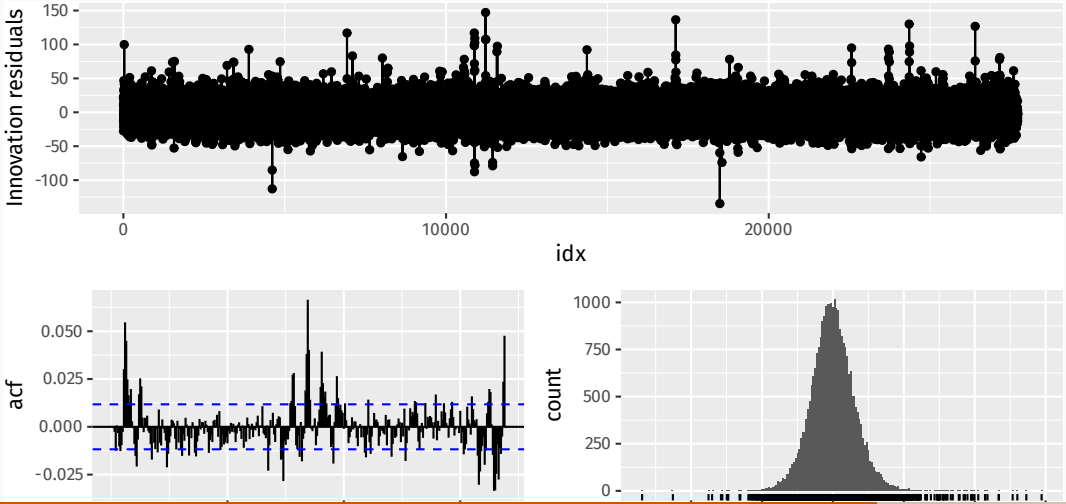
5-minute call centre volume

```r
calls_mdl <- calls |>
  mutate(idx = row_number()) |>
  update_tsibble(index = idx)
fit <- calls_mdl |>
  model(ARIMA(volume ~ fourier(169, K = 10) + pdq(d = 0) + PDQ(0, 0, 0)))
report(fit)
```

```
Series: volume
Model: LM w/ ARIMA(1,0,3) errors

Coefficients:
        ar1      ma1      ma2      ma3  fourier(169, K = 10)C1_169
      0.989  -0.7383  -0.0333  -0.0282                       -79.1
s.e.  0.001   0.0061   0.0075   0.0060                         0.7
      fourier(169, K = 10)S1_169  fourier(169, K = 10)C2_169
                          55.298                     -32.361
s.e.                       0.701                       0.378
      fourier(169, K = 10)S2_169  fourier(169, K = 10)C3_169
```

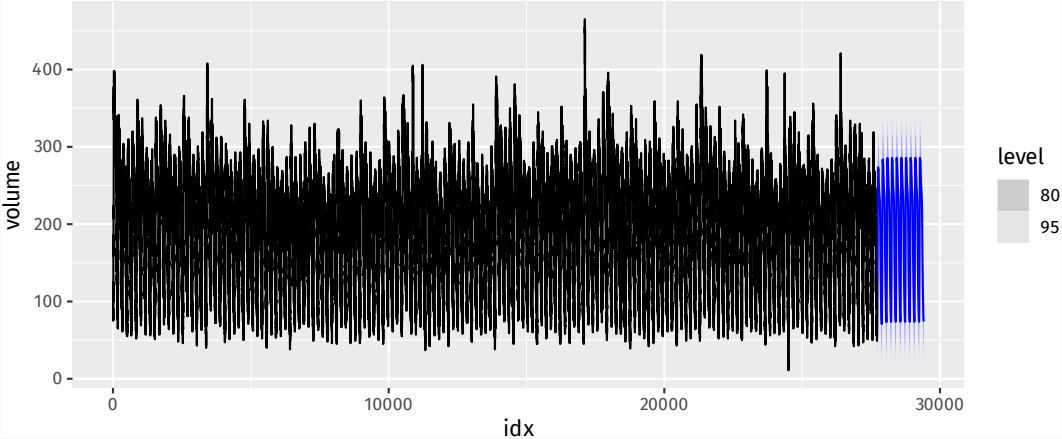
5-minute call centre volume

```
gg_tsresiduals(fit, lag = 338)
```



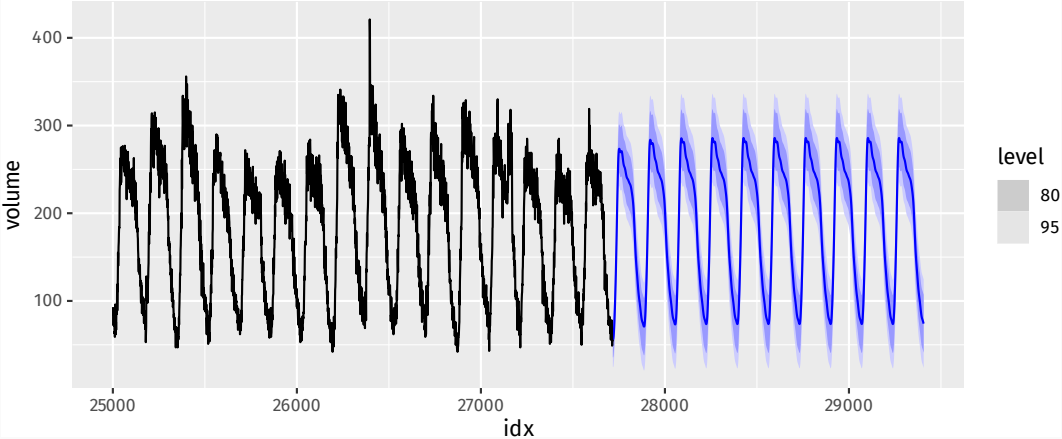
5-minute call centre volume

```
fit |>
  forecast(h = 1690) |>
  autoplot(calls_mdl)
```



5-minute call centre volume

```
fit |>
  forecast(h = 1690) |>
  autoplot(filter(calls_mdl, idx > 25000))
```



Outline

1 Regression with ARIMA errors

2 Stochastic and deterministic trends

3 Dynamic harmonic regression

4 Lagged predictors

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

- y_t = sales, x_t = advertising.
- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

- y_t = sales, x_t = advertising.
- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

- These are dynamic systems with input (x_t) and output (y_t).
- x_t is often a leading indicator.
- There can be multiple predictors.

Lagged predictors

The model include present and past values of predictor:

$$y_t = a + \gamma_0 x_t + \gamma_1 x_{t-1} + \cdots + \gamma_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

Lagged predictors

The model include present and past values of predictor:

$$y_t = a + \gamma_0 x_t + \gamma_1 x_{t-1} + \cdots + \gamma_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

Rewrite model as

$$\begin{aligned} y_t &= a + (\gamma_0 + \gamma_1 B + \gamma_2 B^2 + \cdots + \gamma_k B^k) x_t + \eta_t \\ &= a + \gamma(B) x_t + \eta_t. \end{aligned}$$

Lagged predictors

The model include present and past values of predictor:

$$y_t = a + \gamma_0 x_t + \gamma_1 x_{t-1} + \cdots + \gamma_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

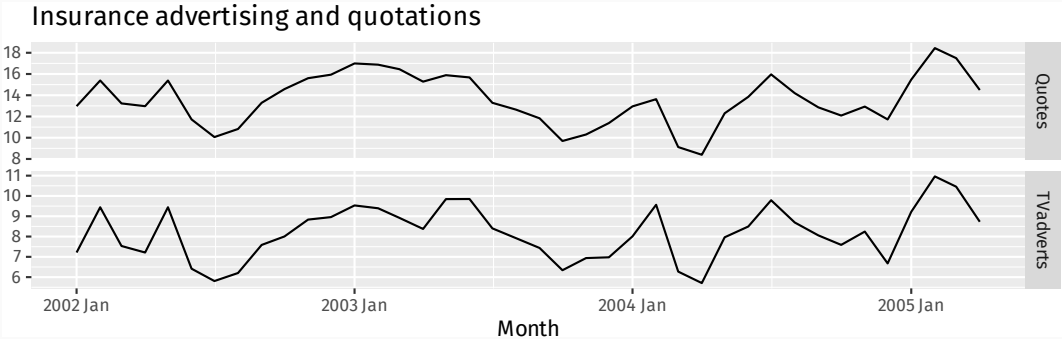
Rewrite model as

$$\begin{aligned} y_t &= a + (\gamma_0 + \gamma_1 B + \gamma_2 B^2 + \cdots + \gamma_k B^k) x_t + \eta_t \\ &= a + \gamma(B) x_t + \eta_t. \end{aligned}$$

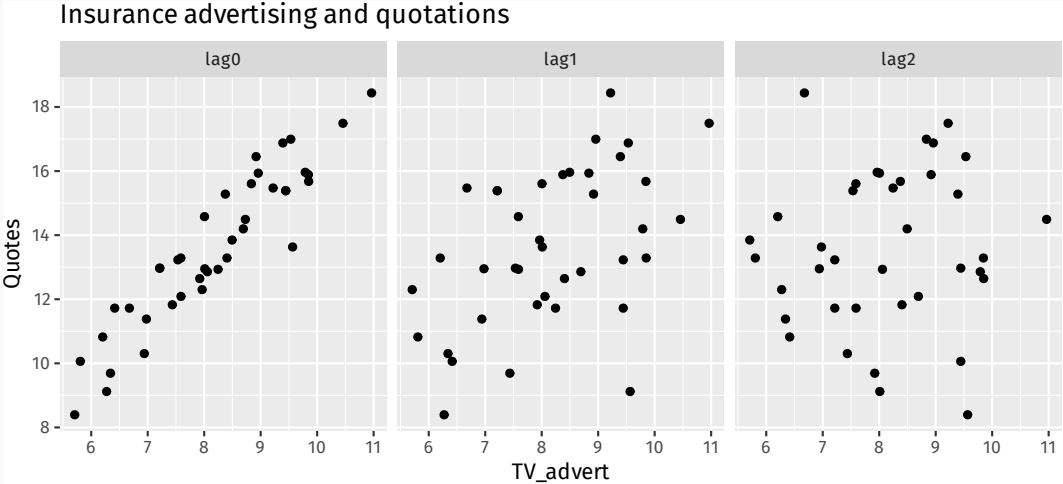
- $\gamma(B)$ is called a *transfer function* since it describes how

Example: Insurance quotes and TV adverts

```
insurance |>
  pivot_longer(Quotes:TVadverts) |>
  ggplot(aes(x = Month, y = value)) +
  geom_line() +
  facet_grid(vars(name), scales = "free_y") +
  labs(y = NULL, title = "Insurance advertising and quotations")
```



Example: Insurance quotes and TV adverts



Example: Insurance quotes and TV adverts

```r
fit <- insurance |>
  # Restrict data so models use same fitting period
  mutate(Quotes = c(NA, NA, NA, Quotes[4:40])) |>
  # Estimate models
  model(
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts) +
      lag(TVadverts, 2)),
    ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts) +
      lag(TVadverts, 2) + lag(TVadverts, 3))
  )
```

Example: Insurance quotes and TV adverts

```
glance(fit)
```

Lag order	sigma2	log_lik	AIC	AICc	BIC
0	0.265	-28.3	66.6	68.3	75.0
1	0.209	-24.0	58.1	59.9	66.5
2	0.215	-24.0	60.0	62.6	70.2
3	0.206	-22.2	60.3	65.0	73.8

Example: Insurance quotes and TV adverts

```r
fit_best <- insurance |>
  model(ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts)))
report(fit_best)
```

```
Series: Quotes
Model: LM w/ ARIMA(1,0,2) errors

Coefficients:
        ar1    ma1    ma2  TVadverts  lag(TVadverts)  intercept
      0.512  0.917  0.459     1.2527          0.1464       2.16
s.e.  0.185  0.205  0.190     0.0588          0.0531       0.86

sigma^2 estimated as 0.2166:  log likelihood=-23.9
AIC=61.9   AICc=65.4   BIC=73.7
```

Example: Insurance quotes and TV adverts

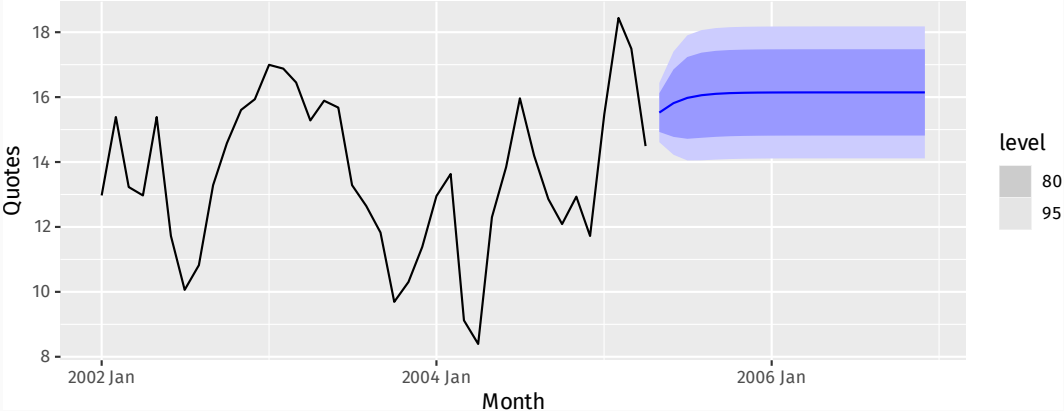
```r
fit_best <- insurance |>
  model(ARIMA(Quotes ~ pdq(d = 0) + TVadverts + lag(TVadverts)))
report(fit_best)
```

```
Series: Quotes
Model: LM w/ ARIMA(1,0,2) errors

Coefficients:
        ar1    ma1    ma2  TVadverts  lag(TVadverts)  intercept
      0.512  0.917  0.459     1.2527          0.1464       2.16
s.e.  0.185  0.205  0.190     0.0588          0.0531       0.86

sigma^2 estimated as 0.2166:  log likelihood=-23.9
AIC=61.9   AICc=65.4   BIC=73.7
```

$$y_t = 2.155 + 1.253x_t + 0.146x_{t-1} + \eta_t,$$
$$\eta_t = 0.512\eta_{t-1} + \varepsilon_t + 0.917\varepsilon_{t-1} + 0.459\varepsilon_{t-2},$$

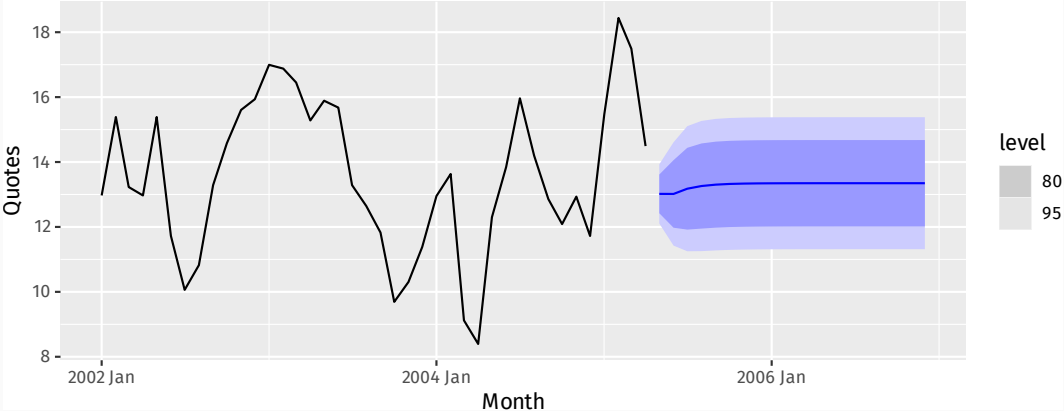
Example: Insurance quotes and TV adverts

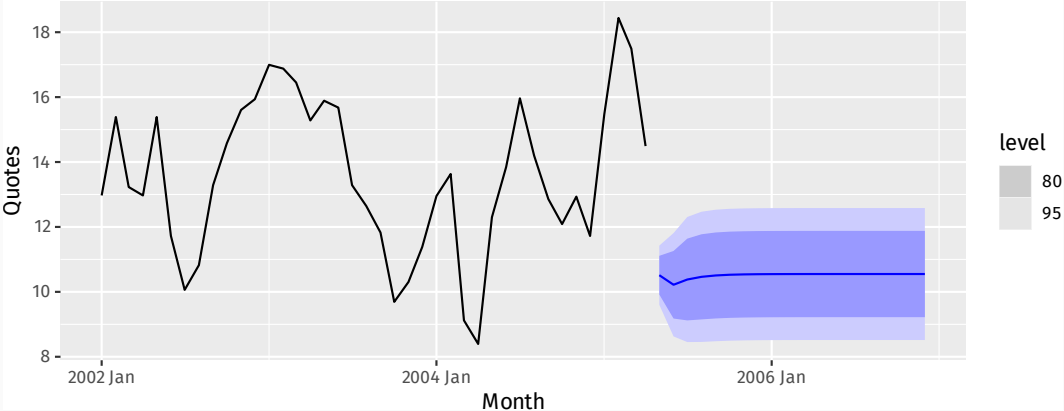
```
advert_a <- new_data(insurance, 20) |>
  mutate(TVadverts = 10)
forecast(fit_best, advert_a) |> autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```r
advert_b <- new_data(insurance, 20) |>
  mutate(TVadverts = 8)
forecast(fit_best, advert_b) |> autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_c <- new_data(insurance, 20) |>
  mutate(TVadverts = 6)
forecast(fit_best, advert_c) |> autoplot(insurance)
```