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FORECASTING
PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.

3RD EDITION

oTexts

9. ARIMA models

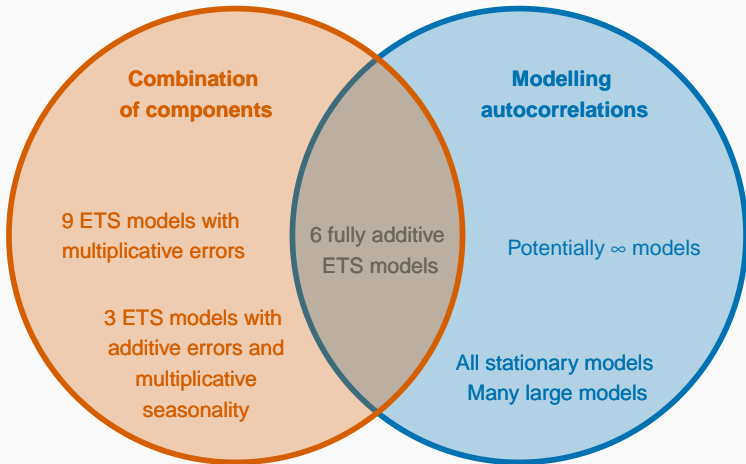9.10 ARIMA vs ETS

OTexts.org/fpp3/

ARIMA vs ETS

- Myth that ARIMA models are more general than exponential smoothing.
- Linear exponential smoothing models all special cases of ARIMA models.
- Non-linear exponential smoothing models have no equivalent ARIMA counterparts.
- Many ARIMA models have no exponential smoothing counterparts.
- ETS models all non-stationary. Models with seasonality or non-damped trend (or both) have two unit roots; all other models have one unit root.

ARIMA vs ETS

ETS models

ARIMA models

Combination of components

Modelling autocorrelations

9 ETS models with multiplicative errors

6 fully additive ETS models

Potentially ∞ models

3 ETS models with additive errors and multiplicative seasonality

All stationary models
Many large models

Equivalences

ETS model	ARIMA model	Parameters
ETS(A,N,N)	ARIMA(0,1,1)	$\theta_1 = \alpha - 1$
ETS(A,A,N)	ARIMA(0,2,2)	$\theta_1 = \alpha + \beta - 2$
		$\theta_2 = 1 - \alpha$
ETS(A,A$_d$,N)	ARIMA(1,1,2)	$\phi_1 = \phi$
		$\theta_1 = \alpha + \phi\beta - 1 - \phi$
		$\theta_2 = (1 - \alpha)\phi$
ETS(A,N,A)	ARIMA(0,0,m)(0,1,0)$_m$	
ETS(A,A,A)	ARIMA(0,1,$m + 1$)(0,1,0)$_m$	
ETS(A,A$_d$,A)	ARIMA(1,0,$m + 1$)(0,1,0)$_m$	

## Example: Australian population

```
aus_economy <- global_economy |>
  filter(Code == "AUS") |>
  mutate(Population = Population / 1e6)
aus_economy |>
  slice(-n()) |>
  stretch_tsibble(.init = 10) |>
  model(ets = ETS(Population), arima = ARIMA(Population)) |>
  forecast(h = 1) |>
  accuracy(aus_economy) |>
  select(.model, ME:RMSSE)
```

```
## # A tibble: 2 x 8
##    .model     ME    RMSE     MAE    MPE   MAPE   MASE  RMSSE
##    <chr>   <dbl>   <dbl>   <dbl>  <dbl>  <dbl>  <dbl>  <dbl>
## 1 arima   0.0420  0.194   0.0789  0.277  0.509  0.317  0.746
## 2 ets     0.0202  0.0774  0.0543  0.112  0.327  0.218  0.298
```

Example: Australian population

```
aus_economy |>
  model(ETS(Population)) |>
  forecast(h = "5 years") |>
  autoplot(aus_economy) +
  labs(title = "Australian population", y = "People (millions)")
```


Australian population
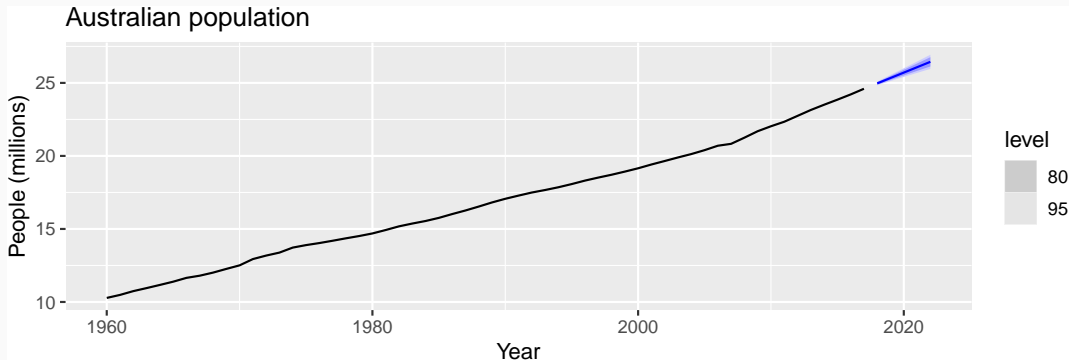
Example: Cement production

```r
cement <- aus_production |>
  select(Cement) |>
  filter_index("1988 Q1" ~ .)
train <- cement |> filter_index(. ~ "2007 Q4")
fit <- train |>
  model(
    arima = ARIMA(Cement),
    ets = ETS(Cement)
  )
```

Example: Cement production

```
fit |>
  select(arima) |>
  report()

## Series: Cement
## Model: ARIMA(1,0,1)(2,1,1)[4] w/ drift
##
## Coefficients:
##           ar1     ma1    sar1    sar2    sma1  constant
##        0.8886  -0.237   0.081  -0.234  -0.898      5.39
## s.e.   0.0842   0.133   0.157   0.139   0.178      1.48
##
## sigma^2 estimated as 11456:  log likelihood=-464
## AIC=941   AICc=943   BIC=957
```
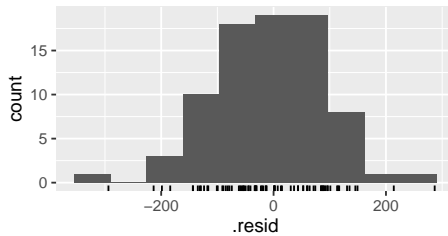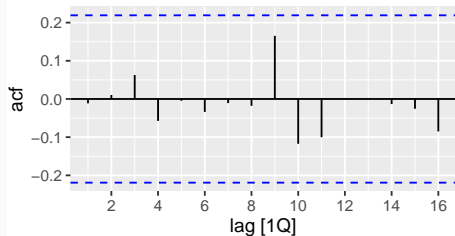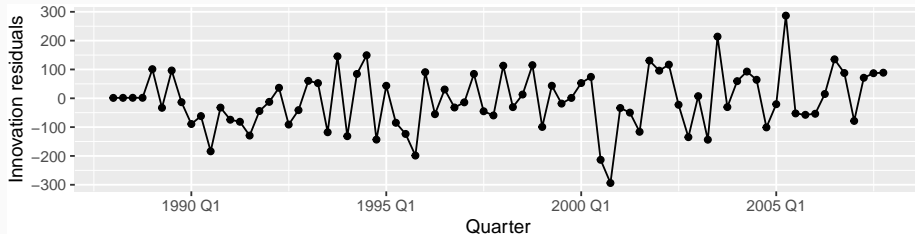
Example: Cement production

```
fit |>
  select(ets) |>
  report()

## Series: Cement
## Model: ETS(M,N,M)
##   Smoothing parameters:
##     alpha = 0.753
##     gamma = 1e-04
##
##   Initial states:
## l[0] s[0] s[-1] s[-2] s[-3]
## 1695 1.03  1.05  1.01 0.912
##
##   sigma^2:  0.0034
##
##  AIC AICc  BIC
## 1104 1106 1121
```
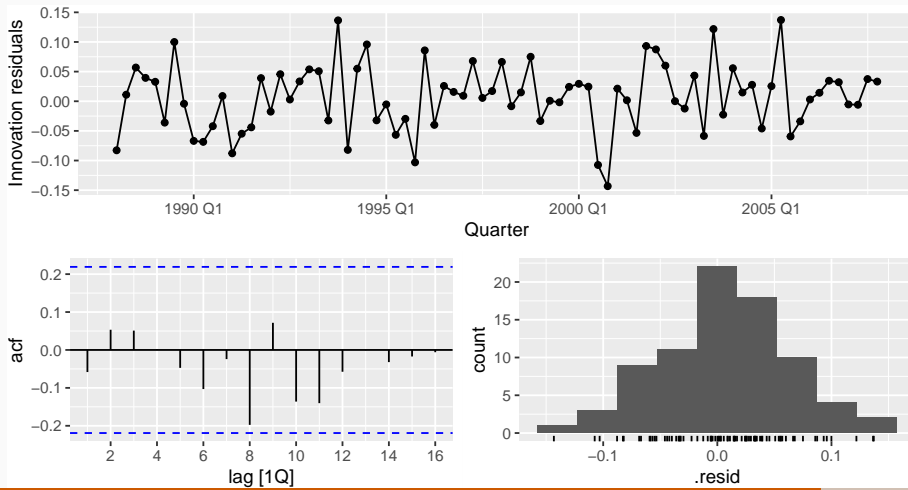
Example: Cement production

```
gg_tsresiduals(fit |> select(arima), lag_max = 16)
```

Example: Cement production

```
gg_tsresiduals(fit |> select(ets), lag_max = 16)
```

Example: Cement production

```
fit |>
  select(arima) |>
  augment() |>
  features(.innov, ljung_box, lag = 16, dof = 6)


## # A tibble: 1 x 3
##    .model lb_stat lb_pvalue
##    <chr>    <dbl>     <dbl>
## 1 arima     6.37     0.783
```

Example: Cement production

```
fit |>
  select(ets) |>
  augment() |>
  features(.innov, ljung_box, lag = 16)
```

```
## # A tibble: 1 x 3
##    .model lb_stat lb_pvalue
##    <chr>    <dbl>     <dbl>
## 1 ets       10.0     0.865
```

Example: Cement production

```
fit |>
  forecast(h = "2 years 6 months") |>
  accuracy(cement) |>
  select(-ME, -MPE, -ACF1)
```

```
## # A tibble: 2 x 7
##   .model .type  RMSE   MAE  MAPE  MASE RMSSE
##   <chr>  <chr> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1 arima  Test   216.  186.  8.68  1.27  1.26
## 2 ets    Test   222.  191.  8.85  1.30  1.29
```

Example: Cement production

```
fit |>
  select(arima) |>
  forecast(h = "3 years") |>
  autoplot(cement) +
  labs(title = "Cement production in Australia", y = "Tonnes ('000)")
```


Cement production in Australia

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