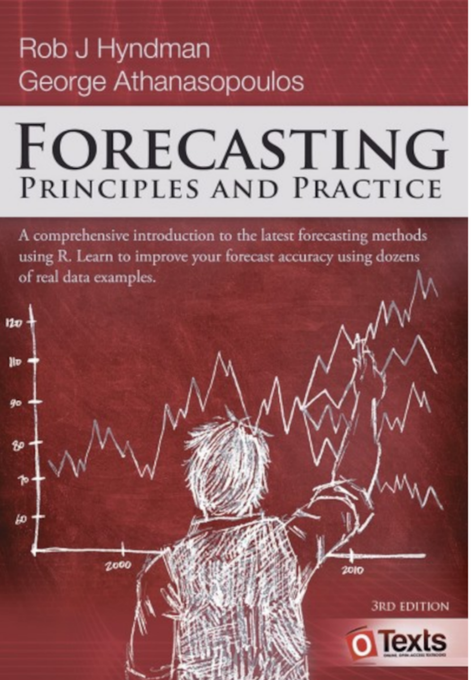

7. Time series regression models

7.5 Selecting predictors

OTexts.org/fpp3/

Comparing regression models

Computer output for regression will always give the R^2 value. This is a useful summary of the model.

- It is equal to the square of the correlation between y and $\hat{y}$.
- It is often called the "coefficient of determination' '.
- It can also be calculated as follows:

$$R^2 = \frac{\sum(\hat{y}_t - \bar{y})^2}{\sum(y_t - \bar{y})^2}$$

- It is the proportion of variance accounted for (explained) by the predictors.

Comparing regression models

However . . .

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where k = no. predictors and T = no. observations.

Maximizing $\bar{R}^2$ is equivalent to minimizing $\hat{\sigma}^2$.

$$\hat{\sigma}^2 = \frac{1}{T-k-1}\sum_{t=1}^{T}\varepsilon_t^2$$

Akaike's Information Criterion

$$\text{AIC} = -2\log(L) + 2(k+2)$$

where L is the likelihood and k is the number of predictors in the model.

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where L is the likelihood and k is the number of predictors in the model.

- AIC penalizes terms more heavily than $\bar{R}^2$.
- Minimizing the AIC is asymptotically equivalent to minimizing MSE via **leave-one-out cross-validation** (for any linear regression).

Corrected AIC

For small values of T, the AIC tends to select too many predictors, and so a bias-corrected version of the AIC has been developed.

$$\text{AIC}_{\text{C}} = \text{AIC} + \frac{2(k+2)(k+3)}{T-k-3}$$

As with the AIC, the AIC_{C} should be minimized.

Bayesian Information Criterion

$$\mathrm{BIC} = -2\log(L) + (k+2)\log(T)$$

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$$\text{BIC} = -2\log(L) + (k+2)\log(T)$$

where L is the likelihood and k is the number of predictors in the model.

- BIC penalizes terms more heavily than AIC
- Also called SBIC and SC.
- Minimizing BIC is asymptotically equivalent to leave-v-out cross-validation when $v = T[1 - 1/(log(T) - 1)]$.

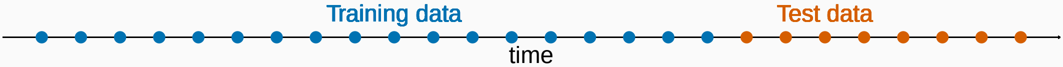
Leave-one-out cross-validation

For regression, leave-one-out cross-validation is faster and more efficient than time-series cross-validation.

- Select one observation for test set, and use *remaining* observations in training set. Compute error on test observation.
- Repeat using each possible observation as the test set.
- Compute accuracy measure over all errors.

Cross-validation

Traditional evaluation



Cross-validation

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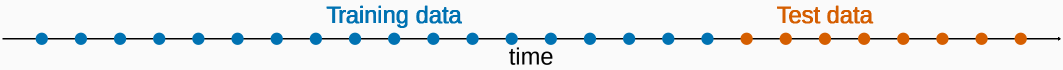
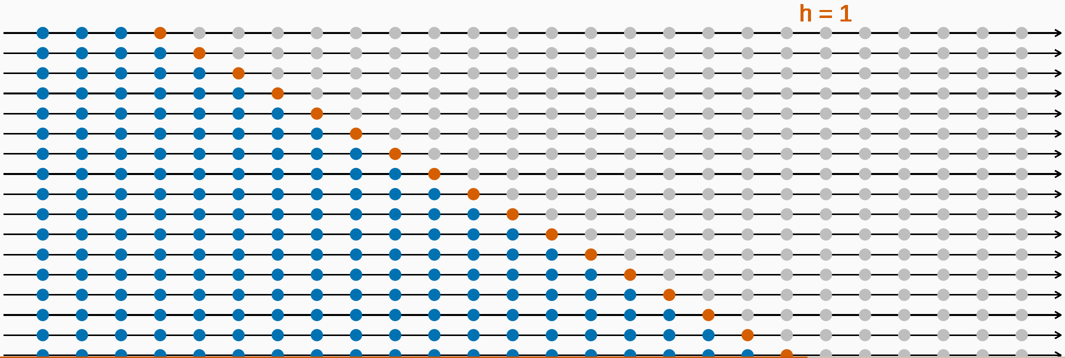


Time series cross-validation



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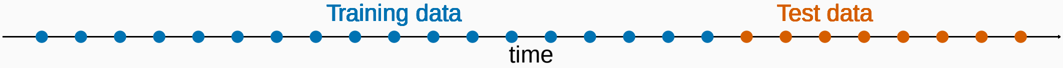
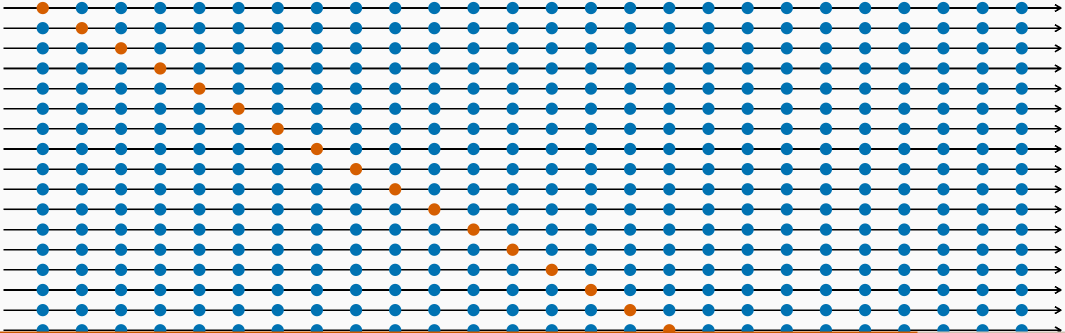


Leave-one-out cross-validation



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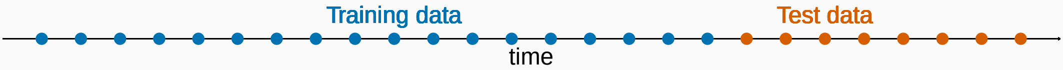

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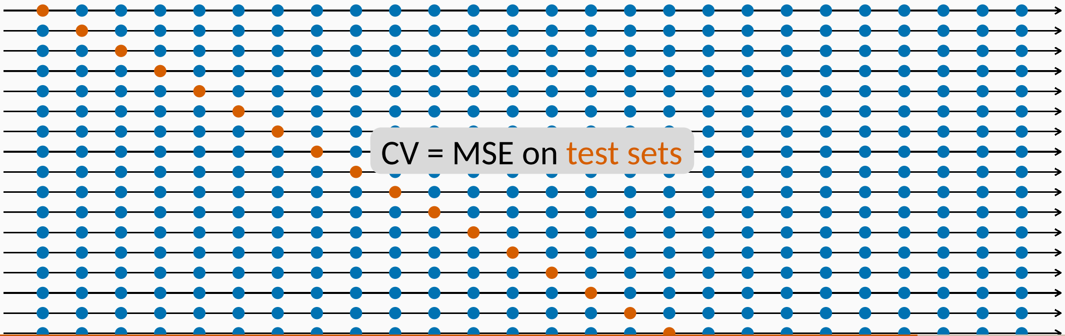

Choosing regression variables

Best subsets regression

- Fit all possible regression models using one or more of the predictors.
- Choose the best model based on one of the measures of predictive ability (CV, AIC, AICc).

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Warning!

- If there are a large number of predictors, this is not possible.
- For example, 44 predictors leads to 18 trillion possible models!

Choosing regression variables

Backwards stepwise regression

- Start with a model containing all variables.
- Try subtracting one variable at a time. Keep the model if it has lower CV or AICc.
- Iterate until no further improvement.

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Backwards stepwise regression

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Notes

- Stepwise regression is not guaranteed to lead to the best possible model.
- Inference on coefficients of final model will be wrong.