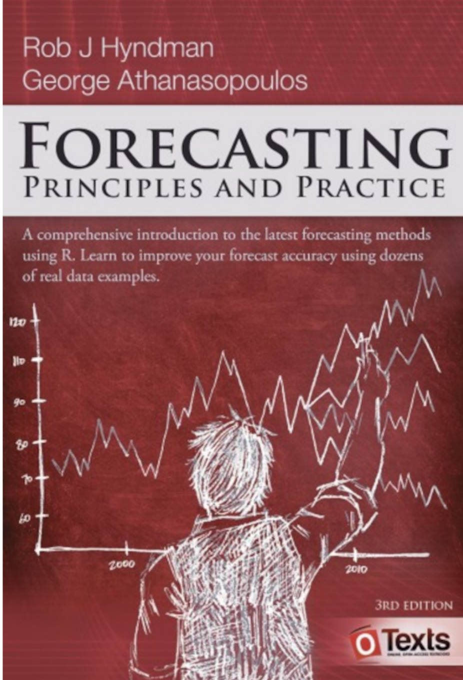
3. Time series decomposition

3.2 Time series components

OTexts.org/fpp3/



Time series patterns

Recall

Trend pattern exists when there is a long-term increase or decrease in the data.

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t
T_t = trend-cycle component at period t
S_t = seasonal component at period t
R_t = remainder component at period t

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where $y_t =$ data at period t

$T_t =$ trend-cycle component at period t

$S_t =$ seasonal component at period t

$R_t =$ remainder component at period t

Additive decomposition: $y_t = S_t + T_t + R_t$.

Multiplicative decomposition: $y_t = S_t \times T_t \times R_t$.

Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$y_t = S_t \times T_t \times R_t \quad \Rightarrow \quad \log y_t = \log S_t + \log T_t + \log R_t.$$

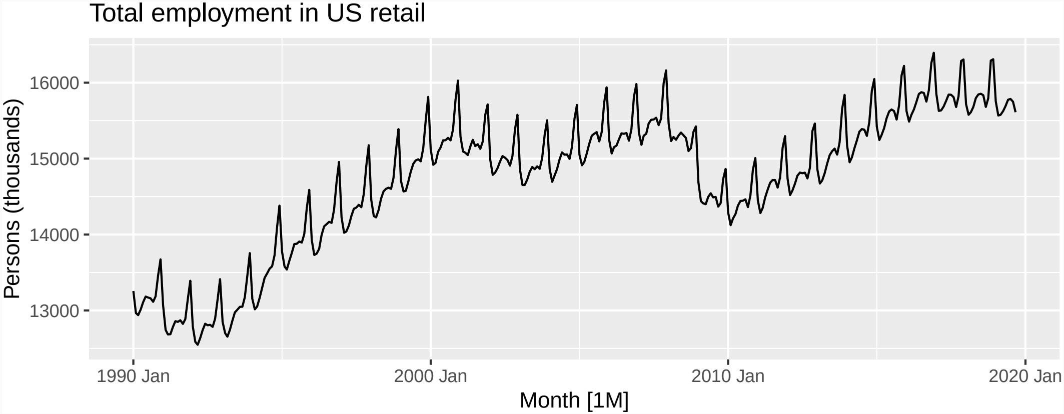
US Retail Employment

```
us_retail_employment <- us_employment |>
  filter(year(Month) >= 1990, Title == "Retail Trade") |>
  select(-Series_ID)
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]
##       Month Title        Employed
##       <mth> <chr>           <dbl>
##  1 1990 Jan Retail Trade   13256.
##  2 1990 Feb Retail Trade   12966.
##  3 1990 Mar Retail Trade   12938.
##  4 1990 Apr Retail Trade   13012.
##  5 1990 May Retail Trade   13108.
##  6 1990 Jun Retail Trade   13183.
##  7 1990 Jul Retail Trade   13170.
##  8 1990 Aug Retail Trade   13160.
##  9 1990 Sep Retail Trade   13113
```

US Retail Employment

```
us_retail_employment |>
  autoplot(Employed) +
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



US Retail Employment

```
us_retail_employment |>
  model(stl = STL(Employed))
```

```
## # A mable: 1 x 1
##          stl
##      <model>
## 1      <STL>
```

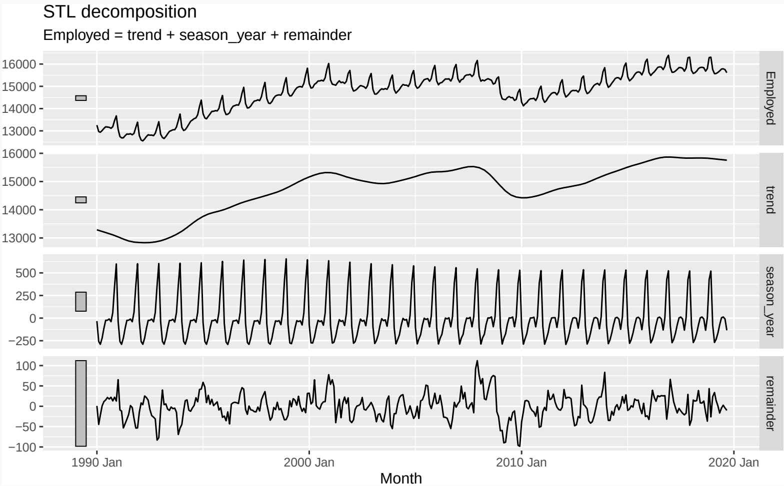
US Retail Employment

```
dcmp <- us_retail_employment |>
  model(stl = STL(Employed))
components(dcmp)
```

```
## # A dable: 357 x 7 [1M]
## # Key:      .model [1]
## # :         Employed = trend + season_year + remainder
##    .model    Month Employed  trend season_year remainder season_adjust
##    <chr>     <mth>    <dbl>  <dbl>       <dbl>     <dbl>         <dbl>
##  1 stl    1990 Jan   13256. 13288.       -33.0     0.836        13289.
##  2 stl    1990 Feb   12966. 13269.      -258.    -44.6          13224.
##  3 stl    1990 Mar   12938. 13250.      -290.    -22.1          13228.
##  4 stl    1990 Apr   13012. 13231.      -220.      1.05         13232.
##  5 stl    1990 May   13108. 13211.      -114.     11.3          13223.
##  6 stl    1990 Jun   13183. 13192.       -24.3    15.5          13207.
##  7 stl    1990 Jul   13170. 13172.       -23.2    21.6          13193.
```

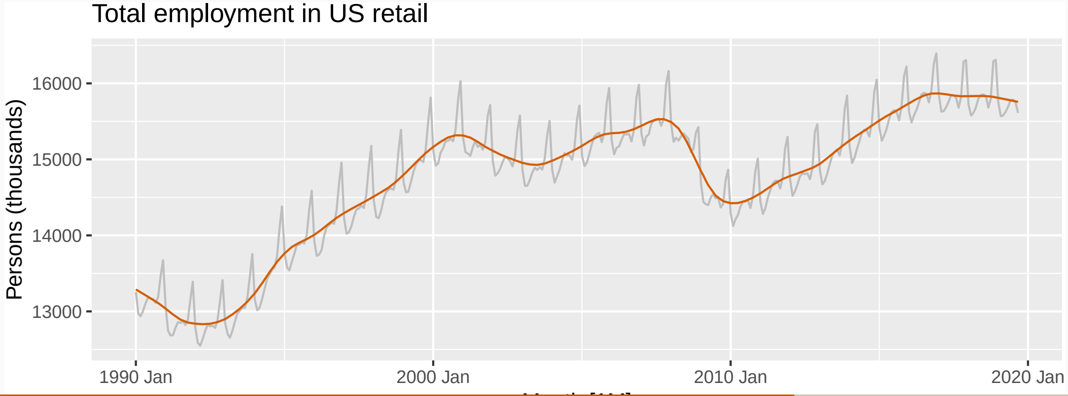
US Retail Employment

```
components(dcmp) |> autoplot()
```



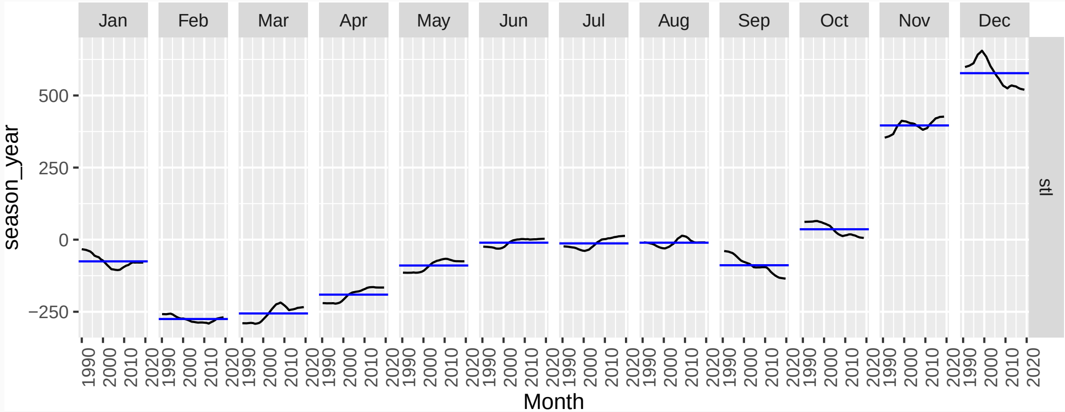
US Retail Employment

```
us_retail_employment |>
  autoplot(Employed, color = "gray") +
  autolayer(components(dcmp), trend, color = "#D55E00") +
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



US Retail Employment

```
components(dcmp) |> gg_subseries(season_year)
```



Seasonal adjustment

- Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.
- Additive decomposition: seasonally adjusted data given by

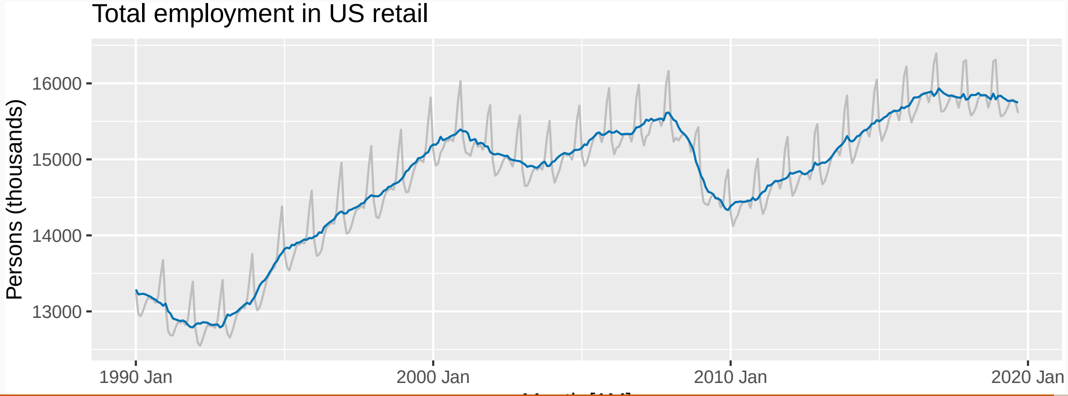
$$y_t - S_t = T_t + R_t$$

- Multiplicative decomposition: seasonally adjusted data given by

$$y_t / S_t = T_t \times R_t$$

US Retail Employment

```
us_retail_employment |>
  autoplot(Employed, color = "gray") +
  autolayer(components(dcmp), season_adjust, color = "#0072B2") +
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



Seasonal adjustment

- We use estimates of S based on past values to seasonally adjust a current value.
- Seasonally adjusted series reflect **remainders** as well as **trend**. Therefore they are not “smooth” and “downturns” or “upturns” can be misleading.
- It is better to use the trend-cycle component to look for turning points.