

MONASH BUSINESS SCHOOL

# ETC3550/ETC5550 Applied forecasting

Ch9. ARIMA models
OTexts.org/fpp3/



#### **Point forecasts**

- Rearrange ARIMA equation so  $y_t$  is on LHS.
- Rewrite equation by replacing t by T + h.
- On RHS, replace future observations by their forecasts, future errors by zero, and past errors by corresponding residuals.

Start with h = 1. Repeat for h = 2, 3, ...

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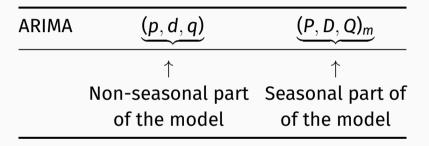
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- $\mathbf{v}_{T+1|T} = \hat{\sigma}^2$  for all ARIMA models regardless of parameters and orders.
- Multi-step prediction intervals for ARIMA(0,0,q):

$$y_{t} = \varepsilon_{t} + \sum_{i=1}^{q} \theta_{i} \varepsilon_{t-i}.$$

$$v_{T|T+h} = \hat{\sigma}^{2} \left[ 1 + \sum_{i=1}^{h-1} \theta_{i}^{2} \right], \quad \text{for } h = 2, 3, \dots.$$

- Prediction intervals increase in size with forecast horizon.
- Prediction intervals can be difficult to calculate by hand
- Calculations assume residuals are uncorrelated and normally distributed.
- Prediction intervals tend to be too narrow.
  - the uncertainty in the parameter estimates has not been accounted for.
  - the ARIMA model assumes historical patterns will not change during the forecast period.
  - the ARIMA model assumes uncorrelated future errors



where m = number of observations per year.

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All the factors can be multiplied out and the general model written as follows:

$$y_{t} = (1 + \phi_{1})y_{t-1} - \phi_{1}y_{t-2} + (1 + \Phi_{1})y_{t-4}$$

$$- (1 + \phi_{1} + \Phi_{1} + \phi_{1}\Phi_{1})y_{t-5} + (\phi_{1} + \phi_{1}\Phi_{1})y_{t-6}$$

$$- \Phi_{1}y_{t-8} + (\Phi_{1} + \phi_{1}\Phi_{1})y_{t-9} - \phi_{1}\Phi_{1}y_{t-10}$$

$$+ \varepsilon_{t} + \theta_{1}\varepsilon_{t-1} + \Theta_{1}\varepsilon_{t-4} + \theta_{1}\Theta_{1}\varepsilon_{t-5}.$$

The seasonal part of an AR or MA model will be seen in the seasonal lags of the PACF and ACF.

#### $ARIMA(0,0,0)(0,0,1)_{12}$ will show:

- a spike at lag 12 in the ACF but no other significant spikes.
- The PACF will show exponential decay in the seasonal lags; that is, at lags 12, 24, 36, ....

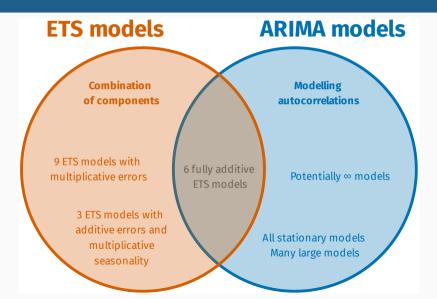
### **ARIMA(0,0,0)(1,0,0)**<sub>12</sub> will show:

- exponential decay in the seasonal lags of the ACF
- a single significant spike at lag 12 in the PACF.

#### **ARIMA vs ETS**

- Myth that ARIMA models are more general than exponential smoothing.
- Linear exponential smoothing models all special cases of ARIMA models.
- Non-linear exponential smoothing models have no equivalent ARIMA counterparts.
- Many ARIMA models have no exponential smoothing counterparts.
- ETS models all non-stationary. Models with seasonality or non-damped trend (or both) have two unit roots; all other models have one unit root.

#### **ARIMA vs ETS**



# **Equivalences**

ETS model	ARIMA model	Parameters
ETS(A,N,N)	ARIMA(0,1,1)	$\theta_1 = \alpha - 1$
ETS(A,A,N)	ARIMA(0,2,2)	$\theta_1$ = $\alpha$ + $\beta$ $-$ 2
		$\theta_{2}$ = 1 $-\alpha$
$ETS(A,A_d,N)$	ARIMA(1,1,2)	$\phi_1 = \phi$
		$\theta_1$ = $\alpha$ + $\phi\beta$ $-$ 1 $ \phi$
		$\theta_2$ = $(1 - \alpha)\phi$
ETS(A,N,A)	$ARIMA(0,0,m)(0,1,0)_m$	
ETS(A,A,A)	$ARIMA(0,1,m+1)(0,1,0)_m$	
ETS(A,A <sub>d</sub> ,A)	ARIMA(1,0, $m$ + 1)(0,1,0) $_m$	