

Deriving models for ETS trend models

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ETS(A,A,N)

Component form:

$$\begin{aligned}\hat{y}_{t+h|t} &= \ell_t + hb_t \\ \hat{y}_{t|t-1} &= \ell_{t-1} + b_{t-1} \\ \ell_t &= \alpha y_t + (1 - \alpha)(\ell_{t-1} + b_{t-1}) \\ b_t &= \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}.\end{aligned}$$

Model form:

$$\begin{aligned}y_t &= \hat{y}_{t|t-1} + \varepsilon_t \\ &= \ell_{t-1} + b_{t-1} + \varepsilon_t \\ \ell_t &= \alpha y_t + (1 - \alpha)(\ell_{t-1} + b_{t-1}) \\ &= \alpha(\ell_{t-1} + b_{t-1} + \varepsilon_t) + (1 - \alpha)(\ell_{t-1} + b_{t-1}) \\ &= \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t \\ b_t &= \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1} \\ &= \beta^*(b_{t-1} + \alpha\varepsilon_t) + (1 - \beta^*)b_{t-1} \\ &= b_{t-1} + \alpha\beta^*\varepsilon_t \\ &= b_{t-1} + \beta\varepsilon_t \quad \text{where } \beta = \alpha\beta^*.\end{aligned}$$

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Component form:

$$\begin{aligned}\hat{y}_{t+h|t} &= \ell_t + (\phi + \phi^2 + \dots + \phi^h)b_t \\ \hat{y}_{t|t-1} &= \ell_{t-1} + \phi b_{t-1} \\ \ell_t &= \alpha y_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1}) \\ b_t &= \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}.\end{aligned}$$

Model form:

$$\begin{aligned}y_t &= \hat{y}_{t|t-1} + \varepsilon_t \\ &= \ell_{t-1} + \phi b_{t-1} + \varepsilon_t \\ \ell_t &= \alpha y_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1}) \\ &= \alpha(\ell_{t-1} + \phi b_{t-1} + \varepsilon_t) + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1}) \\ &= \ell_{t-1} + \phi b_{t-1} + \alpha\varepsilon_t \\ b_t &= \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1} \\ &= \beta^*(\phi b_{t-1} + \alpha\varepsilon_t) + (1 - \beta^*)\phi b_{t-1} \\ &= \phi b_{t-1} + \alpha\beta^*\varepsilon_t \\ &= \phi b_{t-1} + \beta\varepsilon_t \quad \text{where } \beta = \alpha\beta^*.\end{aligned}$$