Deriving forecast variances

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Mean

Assume ε_t has mean 0 and variance σ^2

$$y_{t} = \mu + \varepsilon_{t}$$

$$y_{T+h} = \mu + \varepsilon_{T+h}$$

$$E[y_{T+h|T}] = \mu$$

$$V[y_{T+h|T}] = V(\hat{\mu}) + V(\varepsilon_{t})$$

$$= \sigma^{2}/T + \sigma^{2}$$

$$= \sigma^{2}(1 + 1/T)$$

Naive

$$y_{t} = y_{t-1} + \varepsilon_{t}$$

$$y_{T+h} = y_{T+h-1} + \varepsilon_{T+h}$$

$$= y_{T+h-2} + \varepsilon_{T+h-1} + \varepsilon_{T+h}$$

$$\vdots$$

$$= y_{T} + \sum_{i=0}^{h-1} \varepsilon_{T+h-i}$$

$$E[y_{T+h|T}] = y_{T}$$

$$V[y_{T+h|T}] = h\sigma^{2}$$