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FORECASTING

PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**
OPEN TEXTS FOR PRACTICE

9. ARIMA models

9.3 Autoregressive models

OTexts.org/fpp3/

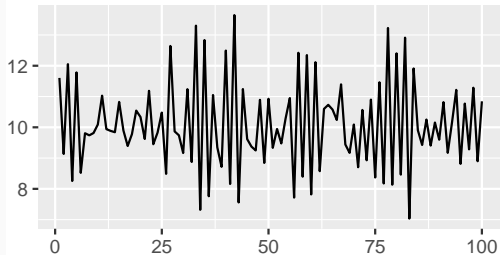
Autoregressive models

Autoregressive (AR) models:

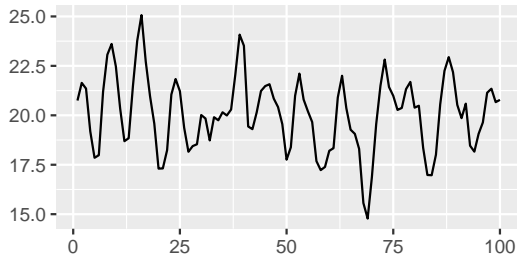
$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

where ε_t is white noise. This is a multiple regression with **lagged values** of y_t as predictors.

AR(1)



AR(2)

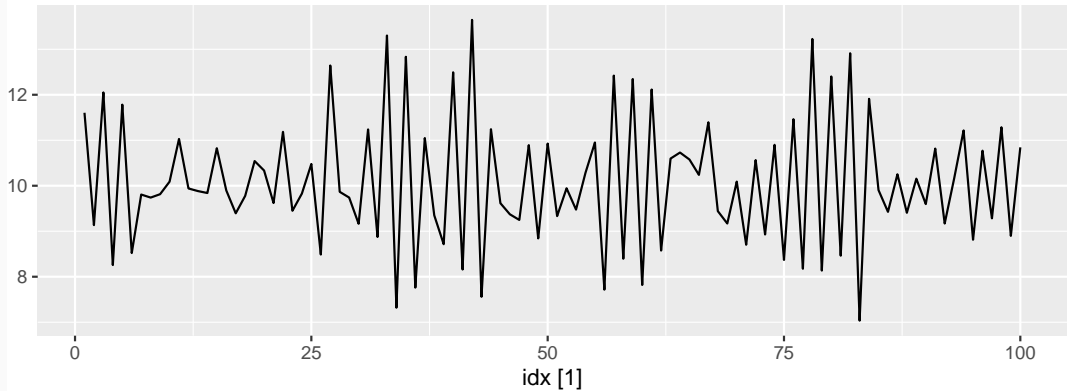


AR(1) model

$$y_t = 18 - 0.8y_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$

AR(1)



AR(1) model

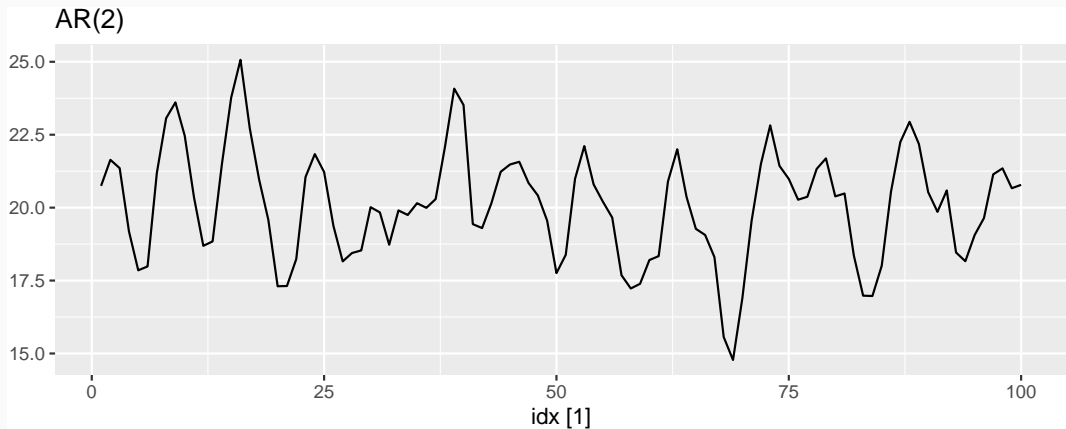
$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t$$

- When $\phi_1 = 0$, y_t is **equivalent to WN**
- When $\phi_1 = 1$ and $c = 0$, y_t is **equivalent to a RW**
- When $\phi_1 = 1$ and $c \neq 0$, y_t is **equivalent to a RW with drift**
- When $\phi_1 < 0$, y_t tends to **oscillate between positive and negative values.**

AR(2) model

$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, 1), \quad T = 100.$$



Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

General condition for stationarity

Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \dots - \phi_p z^p$ lie outside the unit circle on the complex plane.

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- For $p = 1$: $-1 < \phi_1 < 1$.
- For $p = 2$: $-1 < \phi_2 < 1$ $\phi_2 + \phi_1 < 1$ $\phi_2 - \phi_1 < 1$.
- More complicated conditions hold for $p \geq 3$.
- Estimation software takes care of this.