Rob J Hyndman George Athanasopoulos

FORECASTING PRINCIPLES AND PRACTICE



3. Time series decomposition

3.2 Time series components

OTexts.org/fpp3/

Time series patterns

Recall

- **Trend** pattern exists when there is a long-term increase or decrease in the data.
- Cyclic pattern exists when data exhibit rises and falls that are not of fixed period (duration usually of at least 2 years).
- Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where $y_t = \text{data at period } t$

 T_t = trend-cycle component at period t

 S_t = seasonal component at period t

 R_t = remainder component at period t

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where $y_t = \text{data at period } t$

 T_t = trend-cycle component at period t

 S_t = seasonal component at period t

 R_t = remainder component at period t

Additive decomposition: $y_t = S_t + T_t + R_t$.

Multiplicative decomposition: $y_t = S_t \times T_t \times R_t$.

Time series decomposition

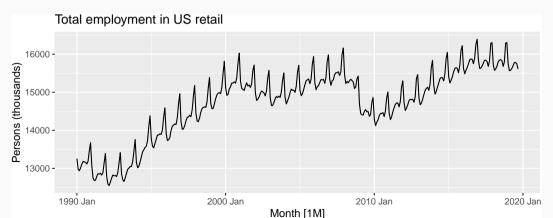
- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$y_t = S_t \times T_t \times R_t \implies \log y_t = \log S_t + \log T_t + \log R_t.$$

```
us_retail_employment <- us_employment |>
  filter(year(Month) >= 1990, Title == "Retail Trade") |>
  select(-Series_ID)
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]
         Month Title
                            Employed
##
         <mth> <chr>
                               <dbl>
##
##
   1 1990 Jan Retail Trade
                              13256.
##
   2 1990 Feb Retail Trade
                              12966.
   3 1990 Mar Retail Trade
##
                              12938.
##
   4 1990 Apr Retail Trade
                              13012.
##
   5 1990 May Retail Trade
                              13108.
   6 1990 Jun Retail Trade
                              13183.
##
##
   7 1990 Jul Retail Trade
                              13170.
##
   8 1990 Aug Retail Trade
                              13160.
"" O 1000 Can Datail Tarada
```

```
us_retail_employment |>
  autoplot(Employed) +
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



```
us_retail_employment |>
  model(stl = STL(Employed))

## # A mable: 1 x 1

## stl

## <model>
## 1 <STL>
```

7 stl

##

1990 Jul

```
dcmp <- us retail employment |>
 model(stl = STL(Employed))
components(dcmp)
## # A dable: 357 x 7 [1M]
## # Key: .model [1]
## # :
            Employed = trend + season_year + remainder
              Month Employed trend season year remainder season adjust
##
     .model
##
  <chr>
              <mth> <dbl> <dbl>
                                      <dbl>
                                               <dbl>
                                                            <dbl>
  1 stl
           1990 Jan 13256. 13288. -33.0 0.836
                                                           13289.
##
##
   2 stl
           1990 Feb 12966. 13269. -258. -44.6
                                                           13224.
##
   3 stl
           1990 Mar 12938. 13250. -290.
                                             -22.1
                                                           13228.
##
   4 stl
           1990 Apr
                    13012. 13231. -220.
                                             1.05
                                                           13232.
           1990 May
##
   5 stl
                    13108. 13211. -114.
                                              11.3
                                                           13223.
##
   6 stl
           1990 Jun
                    13183. 13192. -24.3
                                              15.5
                                                           13207.
```

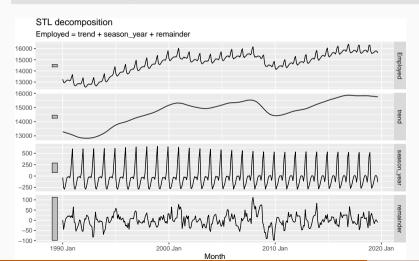
-23.2

21.6

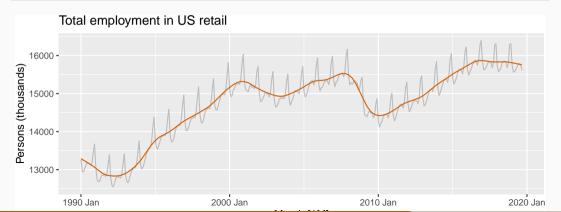
13170, 13172,

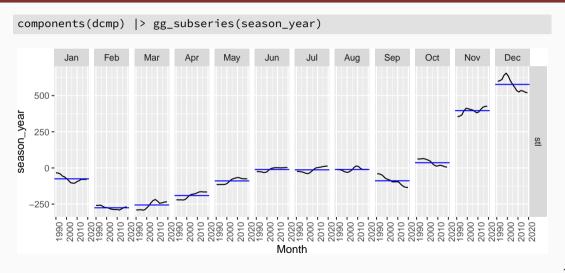
13193.⁸

components(dcmp) |> autoplot()



```
us_retail_employment |>
autoplot(Employed, color = "gray") +
autolayer(components(dcmp), trend, color = "#D55E00") +
labs(y = "Persons (thousands)", title = "Total employment in US retail")
```





Seasonal adjustment

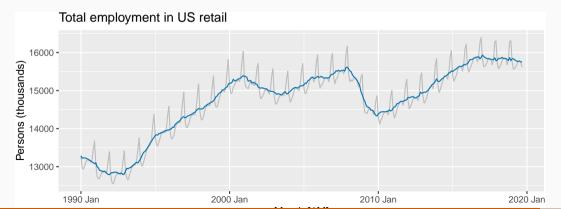
- Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.
- Additive decomposition: seasonally adjusted data given by

$$y_t - S_t = T_t + R_t$$

Multiplicative decomposition: seasonally adjusted data given by

$$y_t/S_t = T_t \times R_t$$

```
us_retail_employment |>
  autoplot(Employed, color = "gray") +
  autolayer(components(dcmp), season_adjust, color = "#0072B2") +
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



Seasonal adjustment

- We use estimates of S based on past values to seasonally adjust a current value.
- Seasonally adjusted series reflect remainders as well as trend. Therefore they are not "smooth" and "downturns" or "upturns" can be misleading.
- It is better to use the trend-cycle component to look for turning points.