

Code/Proofs (Week 9 Wednesday)

```
import Data.Void
```

$$\frac{\overline{A \vdash A}^x}{A \rightarrow A} \rightarrow\text{-I}_x$$

```
prop_id :: a -> a
prop_id = \x -> x
```

$$\frac{\frac{\overline{A \vdash A}^x}{A \vdash A \vee \perp} \vee\text{-I}_L}{A \rightarrow (A \vee \perp)} \rightarrow\text{-I}_x$$

```
prop_or_false :: a -> Either a Void
prop_or_false = \x -> Left x
```

$$\frac{\frac{\overline{A \vdash A}^x}{A \vdash A \vee \top} \vee\text{-I}_L}{A \rightarrow (A \vee \top)} \rightarrow\text{-I}_x$$

```
prop_or_true_a :: a -> Either a ()
prop_or_true_a = \x -> Left x
```

$$\frac{\frac{\overline{A \vdash \top}^{\top\text{-I}}}{A \vdash A \vee \top} \vee\text{-I}_R}{A \rightarrow (A \vee \top)} \rightarrow\text{-I}_x$$

```
prop_or_true_b :: a -> Either a ()
prop_or_true_b = \x -> Right ()
```

$$\frac{\frac{\overline{A \vdash A}^x \quad \overline{A \vdash \top}^{\top\text{-I}}}{A \vdash A \wedge \top} \wedge\text{-I}}{A \rightarrow (A \wedge \top)} \rightarrow\text{-I}_x$$

```
prop_and_true :: a -> (a, ())
prop_and_true = \x -> (x, ())
```

$$\frac{\frac{\frac{\overline{A, (A \rightarrow \perp) \vdash A \rightarrow \perp}^f \quad \overline{A, (A \rightarrow \perp) \vdash A}^x}{A, (A \rightarrow \perp) \vdash \perp} \rightarrow\text{-E}}{A \vdash (A \rightarrow \perp) \rightarrow \perp} \rightarrow\text{-I}_f}{A \rightarrow (A \rightarrow \perp) \rightarrow \perp} \rightarrow\text{-I}_x$$

This is equivalent to: $A \rightarrow \neg\neg A$

```
prop_double_neg_intro :: a -> (a -> Void) -> Void
prop_double_neg_intro = \x -> \f -> f x
```

$$\frac{\frac{\frac{\overline{((A \rightarrow \perp) \rightarrow \perp) \rightarrow \perp}^f \quad \frac{\frac{\overline{A \rightarrow \perp}^g \quad \overline{A}^x}{\perp} \rightarrow\text{-E}}{(A \rightarrow \perp) \rightarrow \perp} \rightarrow\text{-I}_g}{\perp} \rightarrow\text{-E}}{A \rightarrow \perp} \rightarrow\text{-I}_x}{(((A \rightarrow \perp) \rightarrow \perp) \rightarrow \perp) \rightarrow A \rightarrow \perp} \rightarrow\text{-I}_f$$

This is equivalent to: $\neg\neg\neg A \rightarrow \neg A$

```
prop_triple_neg_elim :: ((a -> Void) -> Void) -> Void -> a -> Void
prop_triple_neg_elim = \f -> \x -> f (\g -> g x)
```