HW1 - DATA 609

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Ex. 1 – Find the minimum of $f(x, y) = x^2 + xy + y^2 in(x, y) \in \mathrm{R}$

The minimum can be found by taking the first partial derivative of the formula and solving for zero.

$$\frac{\partial}{\partial x} f(x, y) = 2x + y$$

$$0 = 2x + y$$

$$x = \frac{-y}{2}$$

$$\frac{\partial}{\partial y} f(x, y) = x + 2y$$

$$0 = x + 2y$$

$$y = \frac{-x}{2}$$

The only values that satisfy these two differential equations are at the point (0,0).

To determine this is a minimum versus maximum, take the second derivative.

$$\frac{\partial^2}{\partial x \partial y} f(x, y) = 1$$

Since this value is greater than zero, the value is a minimum. There exists a minimum of f(x,y) at point (0,0).

Ex. 2 – For $f(x) = x^4 \in \mathrm{R}$, it has the global minimum at $x = 0$. Find its new minimum if a constraint $x^2 \geq 1$ is added.

Using the penalty method:

Let $g(x) = 1 - x^2$

$$\Pi(x) = f(x) + \mu[g(x)]^2 = x^4 + \mu(1 - x^2)^2$$

$$\Pi'(x) = 4x^3 + 2\mu(1 - x^2)(-2x) = 4x^3 + 4\mu x^3 - 4\mu x = 8\mu x^3 - 4\mu x = 4\mu x(x^2 - 1) = 0$$

So, $x_* = 0, \pm 1$.

The new minimum value of f(1) = f(-1) = 1.

Ex. 3 – Use a Lagrange multiplier to solve the optimization problem $min f(x, y) = x^2 + 2xy + y^2$, subject to $y = x^2 - 2$

Define f and h:

$$f = x^2 + 2xy + y^2, h = x^2 - y - 2$$

Substitute into formula:

$$\phi = f + \lambda h = x^2 + 2xy + y^2 + \lambda(x^2 - y - 2)$$

Differentiate with respect to x, y, λ, then set equal to zero:

$$\frac{\partial \phi}{\partial x} = 2x + 2y + 2\lambda x = (1 + \lambda)x + y = 0$$

$$\frac{\partial \phi}{\partial y} = 2y + 2x - \lambda = 0$$

$$\frac{\partial \phi}{\partial \lambda} = x^2 - y - 2 = 0$$

Substituting the second condition into the first:

$$\lambda + 2\lambda x = 0$$

We arrive at a value of x = -1/2

Substituting into the third condition:

$$(-1/2)^2 - y - 2 = 0$$

We arrive at a value of y = -7/4

Therefore, the optimality is located at (-1/2, -7/4) with a minimum value of f(x,y) = 81/16, or 5.0625.