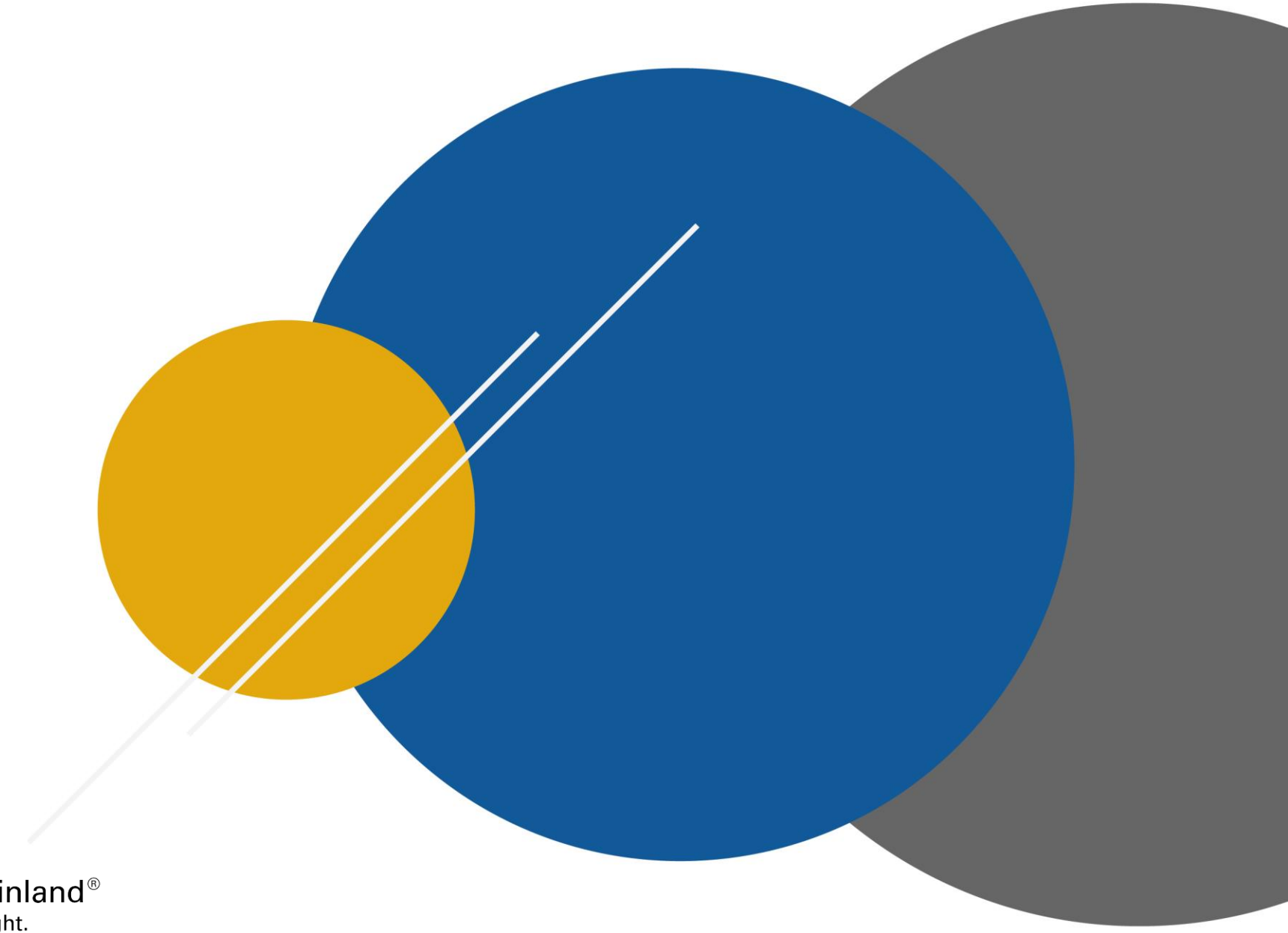
Social Network Analysis

Metrics and Measurements

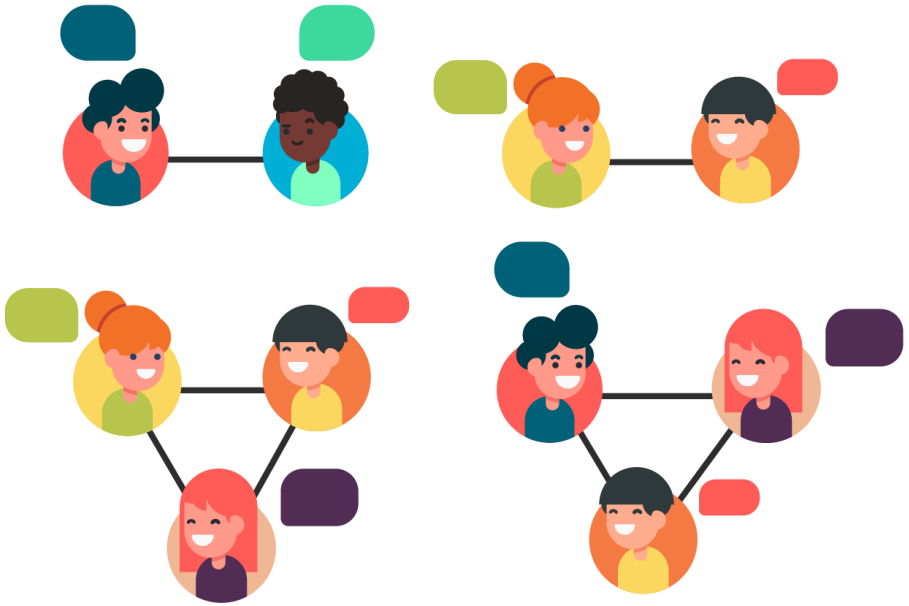


Agenda

- **Social Network Formation**
- Relationship Strength
- Homophily, Transitivity, and Bridge
- Centrality Metrics and Their Interpretation
- Modularity Metrics
- Core and Periphery Structure
- Small World
- Preferential Attachment

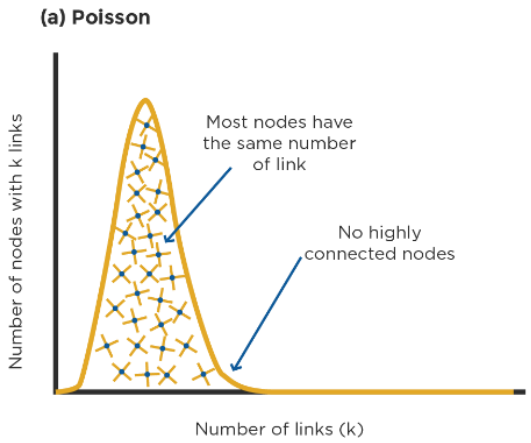
Social Network Formations

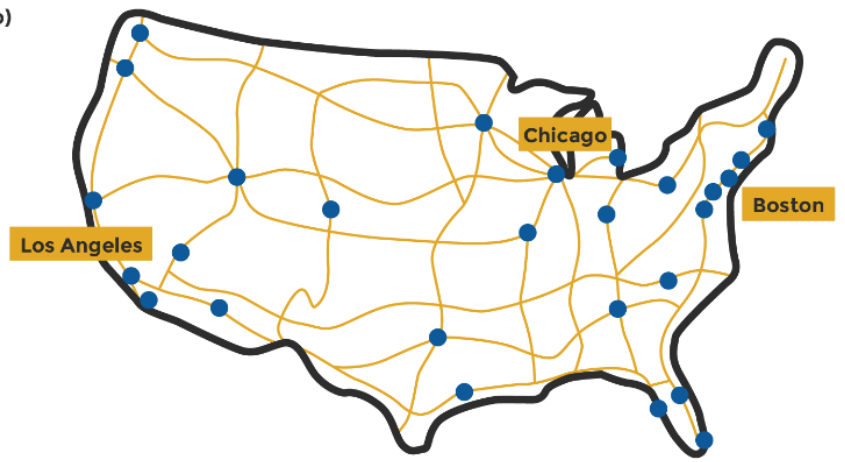
Early

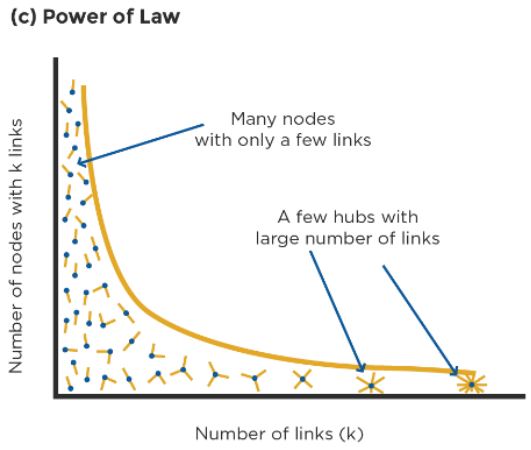


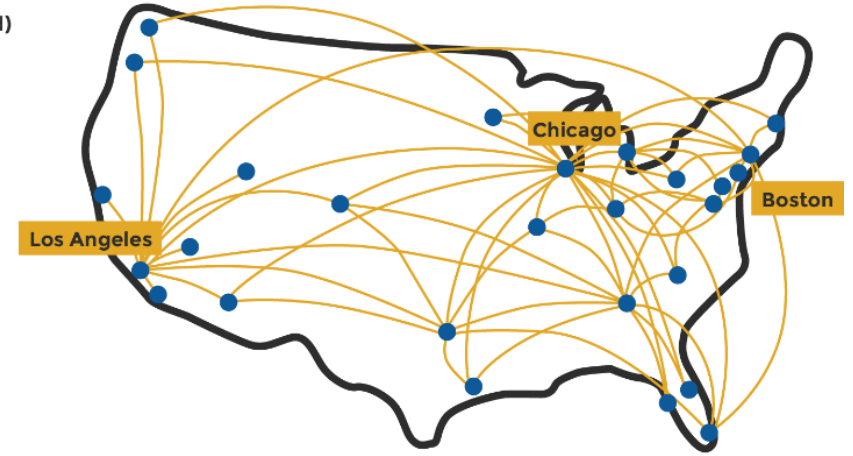
Later

Social Network
Characteristics









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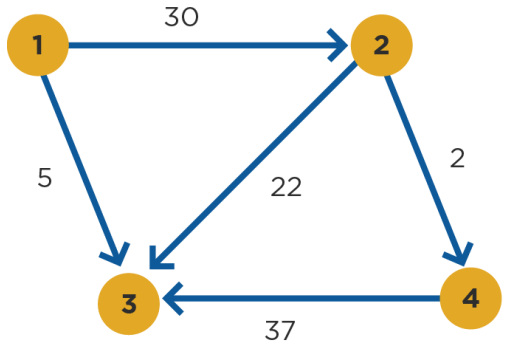
Tie Strength



Weight could be
- Frequency of interactions in period of observation
- Number of items exchanged in period
- Individual perceptions of strength of relationship
- Cost of communications or exchange, e.g. distance

Edge List

Vertex	Vertex	Weight
1	2	30
1	3	5
2	3	22
2	4	2
4	3	27

***Adjacency Matrix* (Weight)**

Vertex	1	2	3	4
1	-	30	5	0
2	30	-	22	2
3	5	22	-	37
4	0	2	37	-

Edge Weight as Relationship Strength

- Edges can represent interactions, flows of information or goods, similarities/affiliations, or social relations
- Specifically for social relations, a ‘proxy’ for the strength of a tie can be:
  - The frequency of interaction (communication) or the amount of flow (exchange)
  - Reciprocity in interaction or flow
  - The type of interaction or flow between the two parties (e.G., Intimate or not)
  - Other attributesof the nodes or ties (e.G., Kin relationships)
  - The structure of the nodes’ neighborhood (e.G. Many mutual ‘friends’)
- Surveys and interviews allows us to establish the existence of mutual or one-sided strength/affection with greater certainty, but proxies above are also useful

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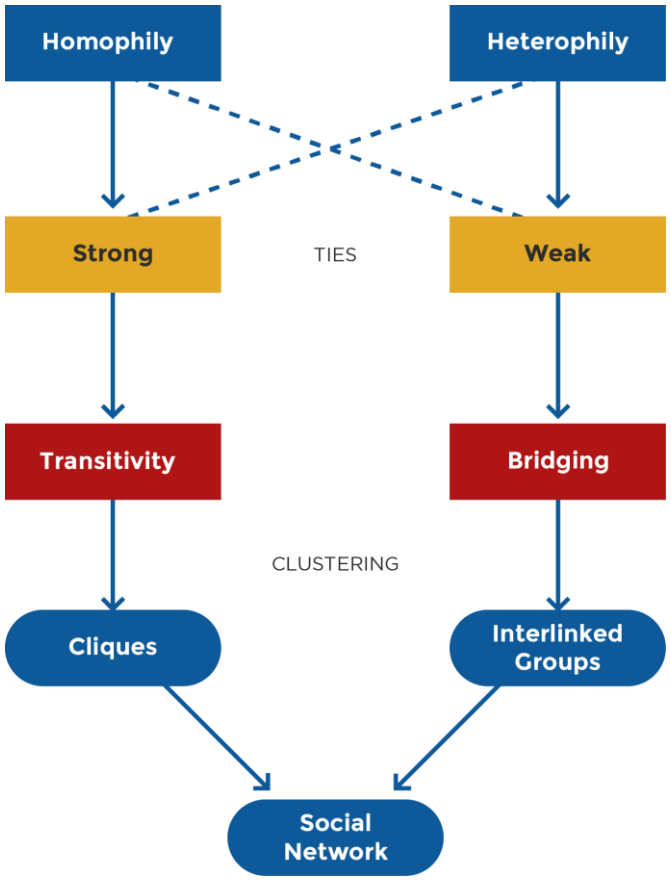
Homophily, Transitivity, and Bridge



- Homophily is the tendency to relate to people with similar characteristics (status, beliefs, etc.)
  - It leads to the formation of homogeneous groups (clusters) where forming relations is easier
  - Extreme homogenization can act counter to innovation and idea generation (heterophily is thus desirable in some contexts)
  - Homophilous ties can be strong or weak
- Transitivity in SNA is a property of ties: if there is a tie between A and B and one between B and C, then in a transitive network A and C will also be connected
  - Strong ties are more often transitive than weak ties; transitivity is therefore evidence for the existence of strong ties (but not a necessary or sufficient condition)
  - Transitivity and homophily together lead to the formation of cliques (fully connected clusters)
- Bridges are nodes and edges that connect across groups
  - Facilitate inter-group communication, increase social cohesion, and help spur innovation
  - They are usually weak ties, but not every weak tie is a bridge

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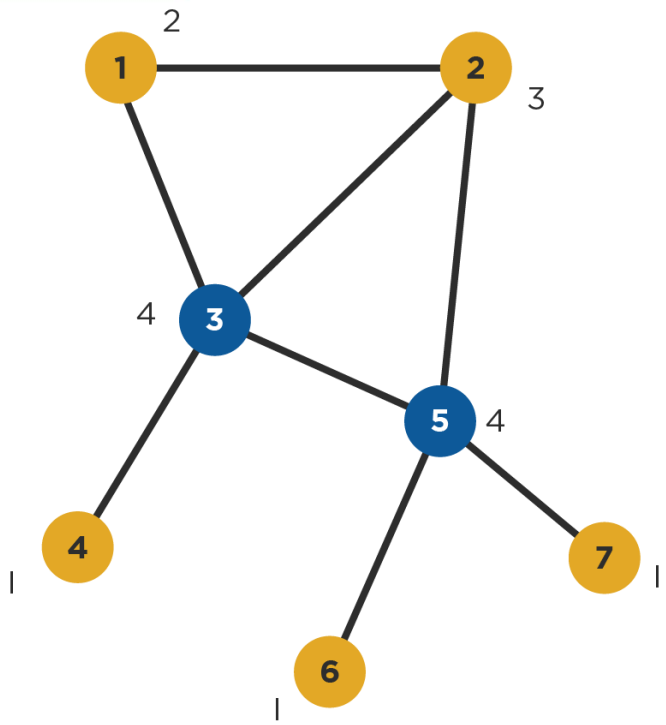
Degree
Centrality

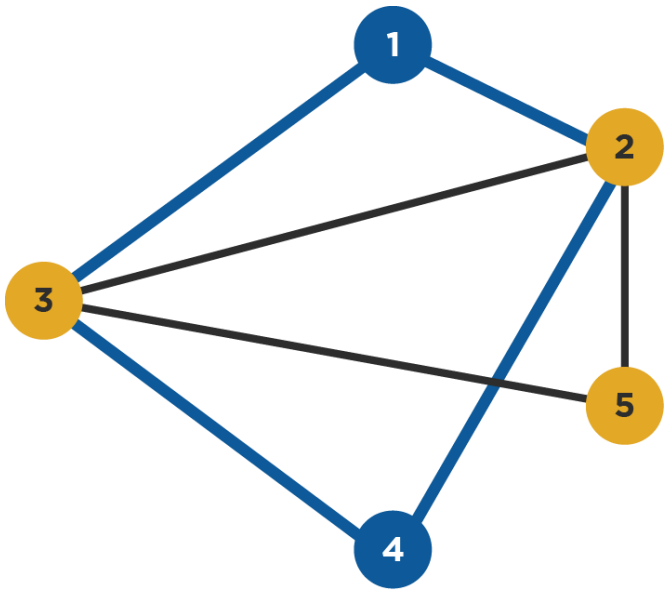


Nodes 3 and 5 have the highest degree (4)

- A node's (in-) or (out-) degree is the number of links that lead into or out of the node
- In an undirected graph they are of course identical
- Often used as measure of a node's degree of connectedness and hence also influence and/or popularity
- Useful in assessing which nodes are central with respect to spreading information and influencing others in their immediate 'neighborhood'

Values computed with the sna package in the R programming environment. Definitions of centrality measures may vary slightly in other software.

Paths and
Shortest Paths

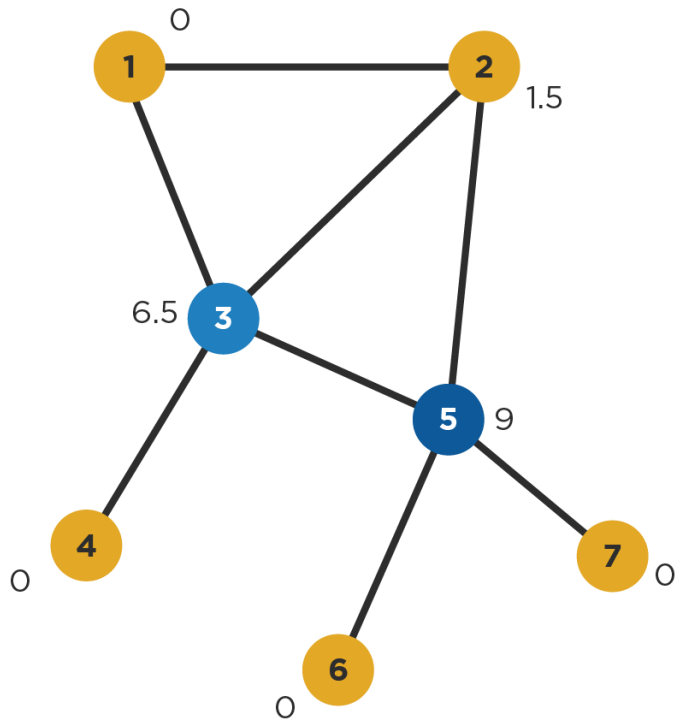


Hypothetical Graph

Shortest Path(s)

- A path between two nodes is any sequence of non-repeating nodes that connects the two nodes
- The shortest path between two nodes is the path that connects the two nodes with the shortest number of edges (also called the distance between the nodes)
- In the example to the right, between nodes 1 and 4 there are two shortest paths of length 2: {1,2,4} and {1,3,4}
- Other, longer paths between the two nodes are {1,2,3,4}, {1,3,2,4}, {1,2,5,3,4} and {1,3,5,2,4} (the longest paths)
- Shorter paths are desirable when speed of communication or exchange is desired (often the case in many studies, but sometimes not, e.g. in networks that spread disease)

Betweenness Centrality

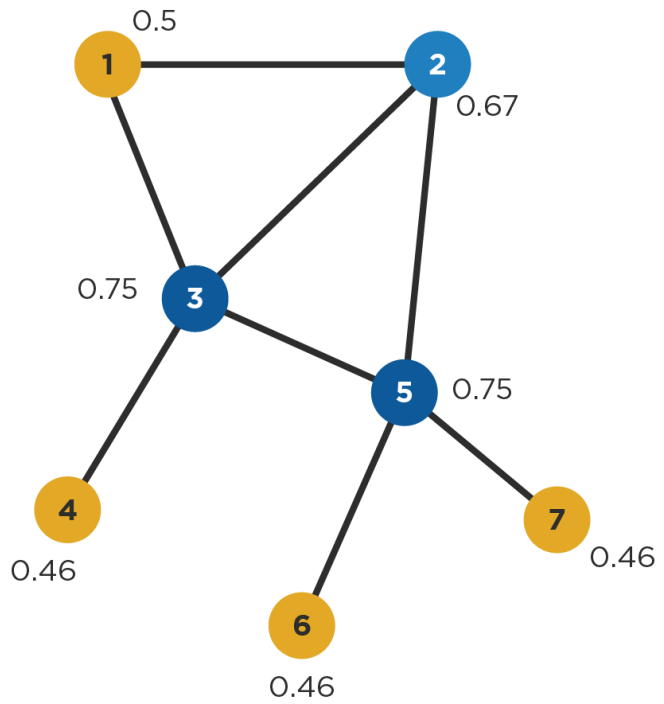


- For a given node v, calculate the number of shortest paths between nodes i and j that pass through v, and divide by all shortest paths between nodes i and j
- Sum the above values for all node pairs i,j
- Sometimes normalized such that the highest value is 1 or that the sum of all betweenness centralities in the network is 1
- Shows which nodes are more likely to be in communication paths between other nodes
- Also useful in determining points where the network would break apart (think who would be cut off if nodes 3 or 5 would disappear)

Node 5 has higher betweenness centrality than 3

Values computed with the sna package in the R programming environment. Definitions of centrality measures may vary slightly in other software.

Closeness
Centrality



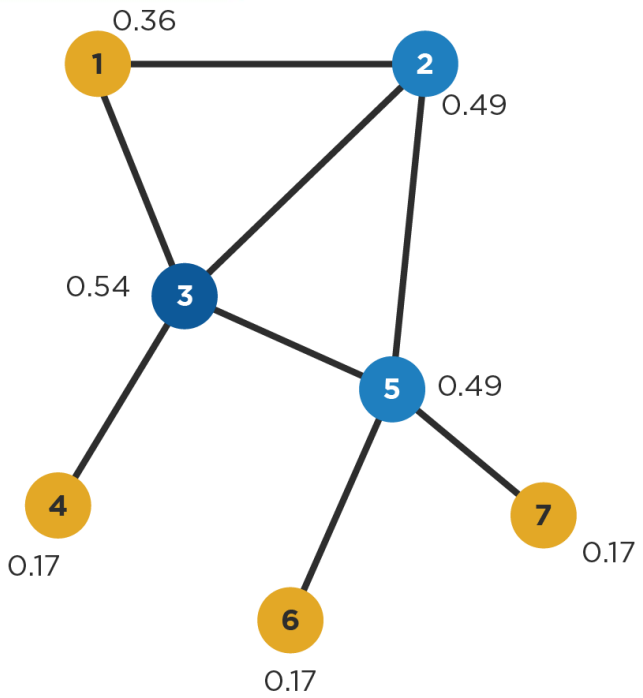
Note: Sometimes closeness is calculated without taking the reciprocal of the mean shortest path length. Then lower values are 'better'.

- Calculate the mean length of all shortest paths from a node to all other nodes in the network (i.e. how many hops on average it takes to reach every other node)
- Take the reciprocal of the above value so that higher values are 'better' (indicate higher closeness) like in other measures of centrality
- It is a measure of *reach*, i.e. the speed with which information can reach other nodes from a given starting node

Nodes 3 and 5 have the highest (i.e. best) closeness, while node 2 fares almost as well

Values computed with the sna package in the R programming environment. Definitions of centrality measures may vary slightly in other software.

Eigenvector Centrality



- A node's eigenvector centrality is proportional to the sum of the eigenvector centralities of all nodes directly connected to it
- In other words, a node with a high eigenvector centrality is connected to other nodes with high eigenvector centrality
- This is similar to how Google ranks web pages: links from highly linked-to pages count more
- Useful in determining who is connected to the most connected nodes

Note: The term 'eigenvector' comes from mathematics (matrix algebra), but it is not necessary for understanding how to interpret this measure

Node 3 has the highest eigenvector centrality, closely followed by 2 and 5

Values computed with the sna package in the R programming environment. Definitions of centrality measures may vary slightly in other software.

Interpretation of Measures (1)

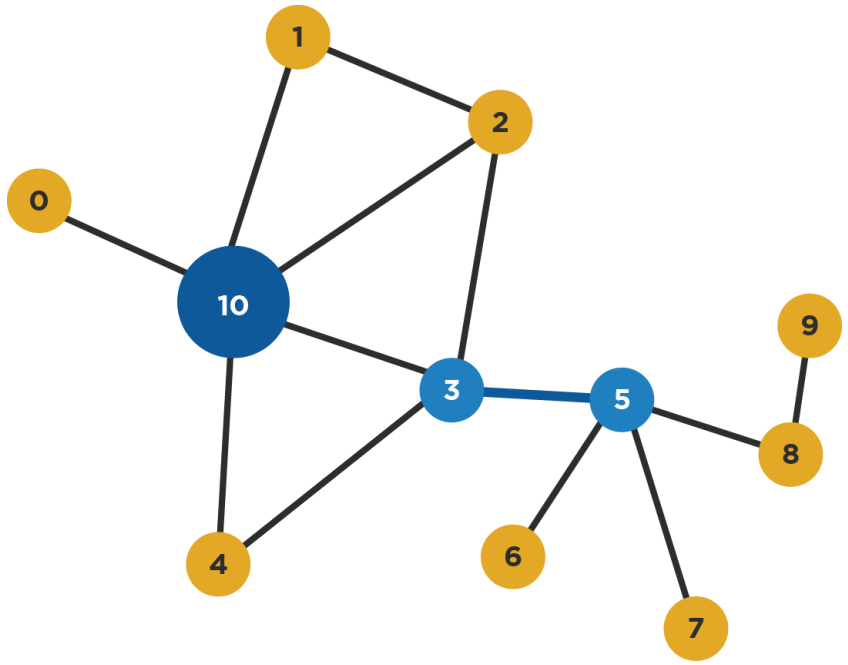
Centrality measure	Interpretation in social networks
Degree	How many people can this person reach directly?
Betweenness	How likely is this person to be the most direct route between two people in the network?
Closeness	How fast can this person reach everyone in the network?
Eigenvector	How well is this person connected to other well-connected people?

Interpretation of Measures (2)

Centrality measure	**Other possible interpretations…**
Degree	In network of music collaborations: how many people has this person collaborated with?
Betweenness	In network of spies: who is the spy though whom most of the confidential information is likely to flow? The JBs. [1]
Closeness	In network of sexual relations: how fast will an STD spread from this person to the rest of the network?
Eigenvector	In network of paper citations: who is the author that is most cited by other well-cited authors?

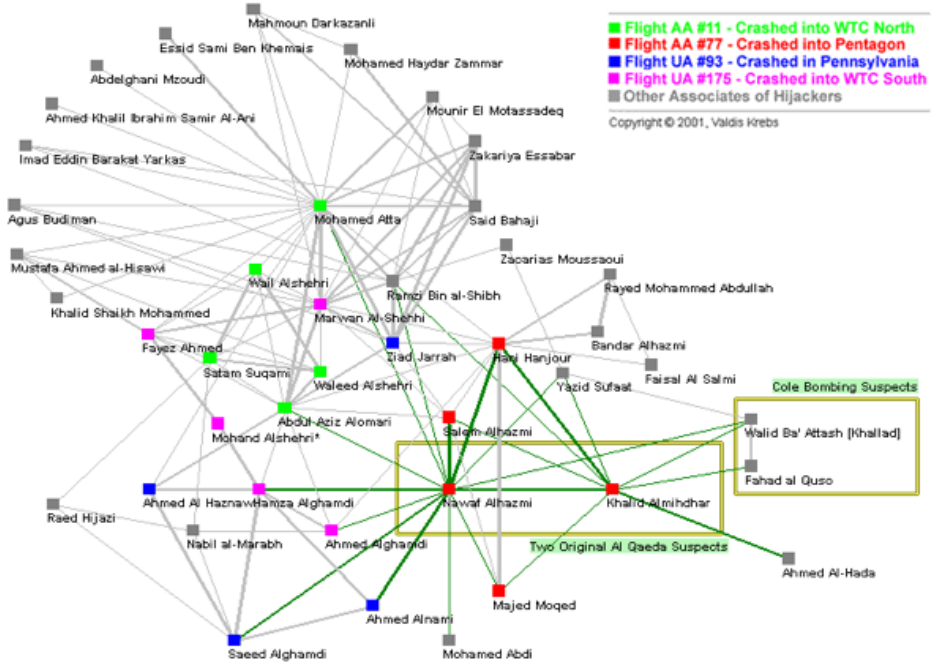
[1] **James Bond**. Dr. No (1962), Sean Connery; **Jason Bourne**. The Bourne Identity (2002), Matt Damon; **Jack Bauer**. 24 (2001 TV Series), Kiefer Sutherland; **Jack Bristow**. Alias (2001 TV Series), Victor Garber; **John Book**. Witness (1985), Harrison Ford; *Justin Bieber*. Maybe a Spy.

Identifying Sets of Key Players



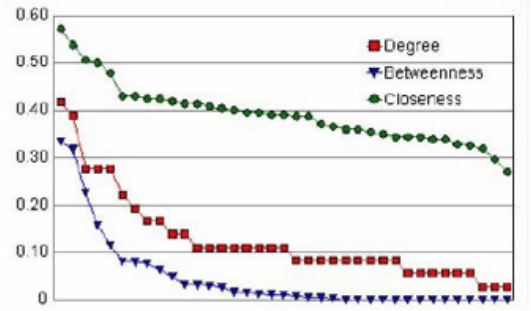
- In the network to the right, node 10 is the most central according to degree centrality
- But nodes 3 and 5 together will reach more nodes
- Moreover the tie between them is critical; if severed, the network will break into two isolated sub-networks
- It follows that other things being equal, players 3 and 5 together are more ‘key’ to this network than 10
- Thinking about sets of key players is helpful!

Example :
Finding Influencer



Group Size	37
Potential Ties	1332
Actual Ties	170
Density	13%

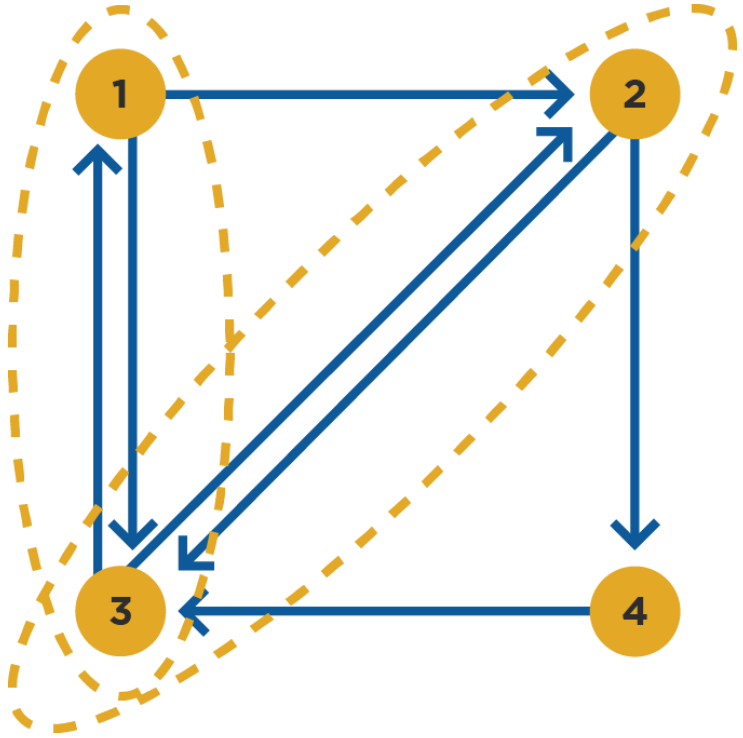
Geodesics	
length	#
1	170
2	626
3	982
4	558
5	136
6	0



Degrees		Betweenness		Closeness	
0.417	Mohamed Atta	0.334	Nawaf Alhazmi	0.571	Mohamed Atta
0.389	Marwan Al-Shehhi	0.318	Mohamed Atta	0.537	Nawaf Alhazmi
0.278	Hani Hanjour	0.227	Hani Hanjour	0.507	Hani Hanjour
0.278	Nawaf Alhazmi	0.158	Marwan Al-Shehhi	0.500	Marwan Al-Shehhi
0.278	Ziad Jarrah	0.116	Saeed Alghamdi*	0.480	Ziad Jarrah
0.222	Ramzi Bin al-Shibh	0.081	Hamza Alghamdi	0.429	Mustafa al-Hisawi

paper: Mapping Network of Terrorist Cells - V.E.Krebs (2002)

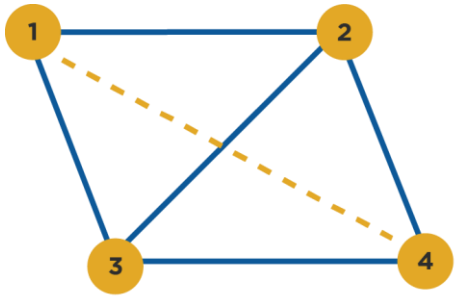
Reciprocity
(degree of)



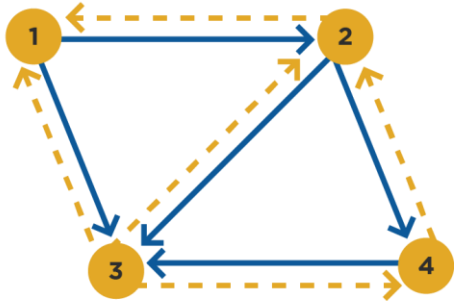
Reciprocity for network = 0.4

- The ratio of the number of relations which are reciprocated (i.e. there is an edge in both directions) over the total number of relations in the network
- …where two vertices are said to be related if there is at least one edge between them
- In the example to the right this would be 2/5=0.4 (whether this is considered high or low depends on the context)
- A useful indicator of the degree of mutuality and reciprocal exchange in a network, which relate to social cohesion
- Only makes sense in directed graphs

Density



Density → $\frac{5}{6} = 0.83$



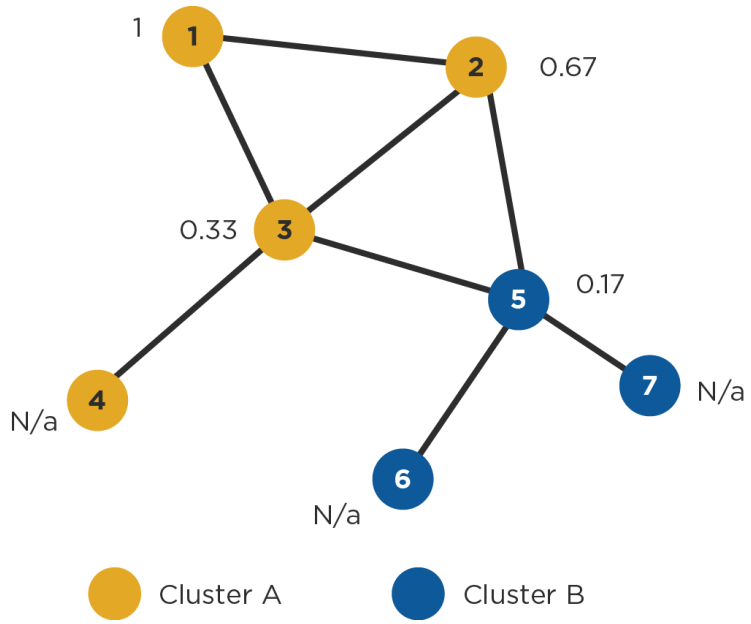
Density → $\frac{5}{12} = 0.42$

Edge present in network

Possible but not present

- A network's *density* is the ratio of the number of edges in the network over the total number of possible edges between all pairs of nodes (which is *n(n-1)/2*, where *n* is the number of vertices, for an undirected graph)
- In the example network to the right density=5/6=0.83 (i.e. it is a fairly *dense* network; opposite would be a *sparse* network)
- It is a common measure of how well connected a network is (in other words, how closely knit it is) – a perfectly connected network is called a *clique* and has density=1
- A directed graph will have half the density of its undirected equivalent, because there are twice as many possible edges, i.e. *n(n-1)*
- Density is useful in comparing networks against each other, or in doing the same for different regions within a single network

Clustering



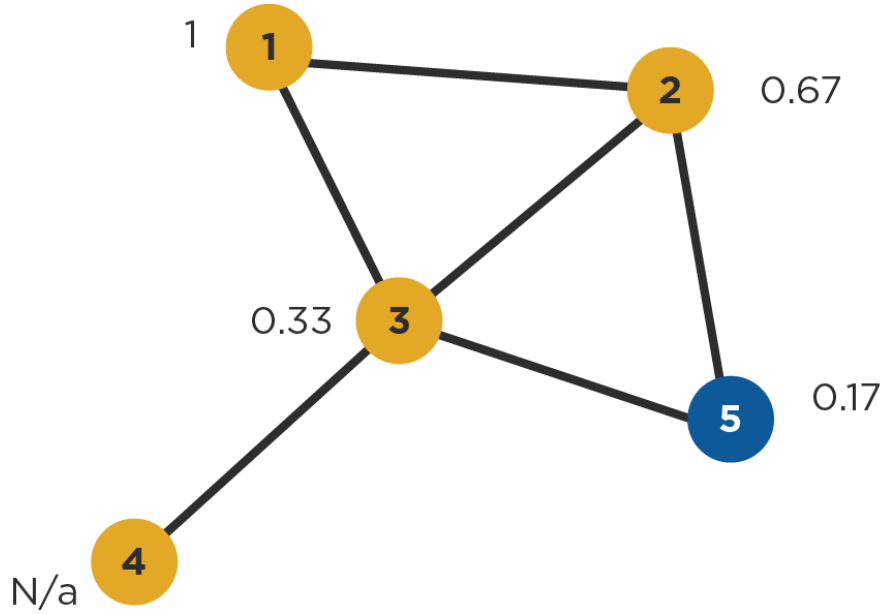
Network clustering coefficient = 0.375
(3 nodes in each triangle x 2 triangles = 6 closed triplets divided by 16 total)

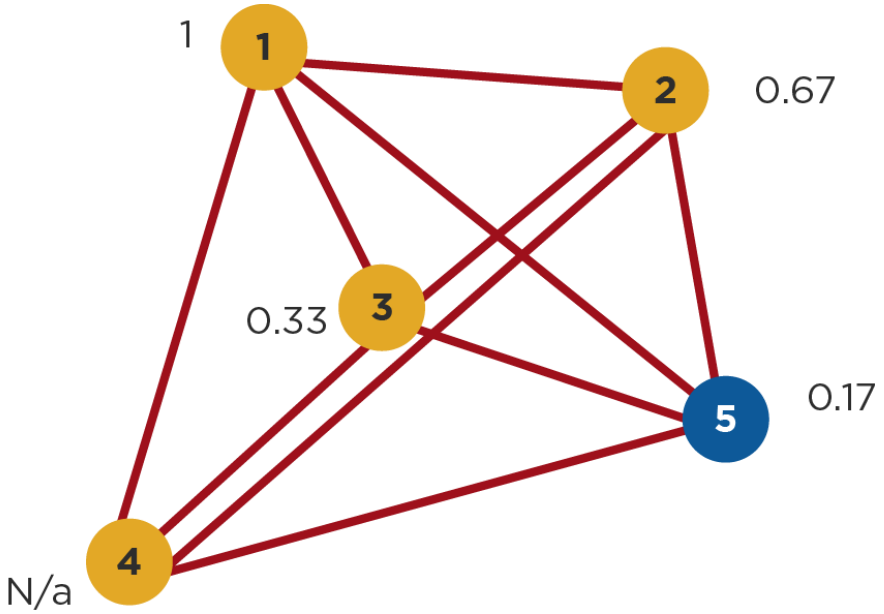
- A node's *clustering coefficient* is the number of closed triplets in the node's neighborhood over the total number of triplets in the neighborhood. It is also known as *transitivity*.
- E.g., node 1 to the right has a value of 1 because it is only connected to 2 and 3, and these nodes are also connected to one another (i.e. the only triplet in the neighborhood of 1 is closed). We say that nodes 1,2, and 3 form a *clique*.
- Clustering algorithms identify clusters or 'communities' within networks based on network structure and specific clustering criteria (example shown to the right with two clusters is based on *edge betweenness*, an equivalent for edges of the betweenness centrality presented earlier for nodes)

Values computed with the igraph package in the R programming environment. Definitions of centrality measures may vary slightly in other software.
NOTE: minimum number of nodes to form different clusters is 3. A node of each clusters can transit to another cluster.

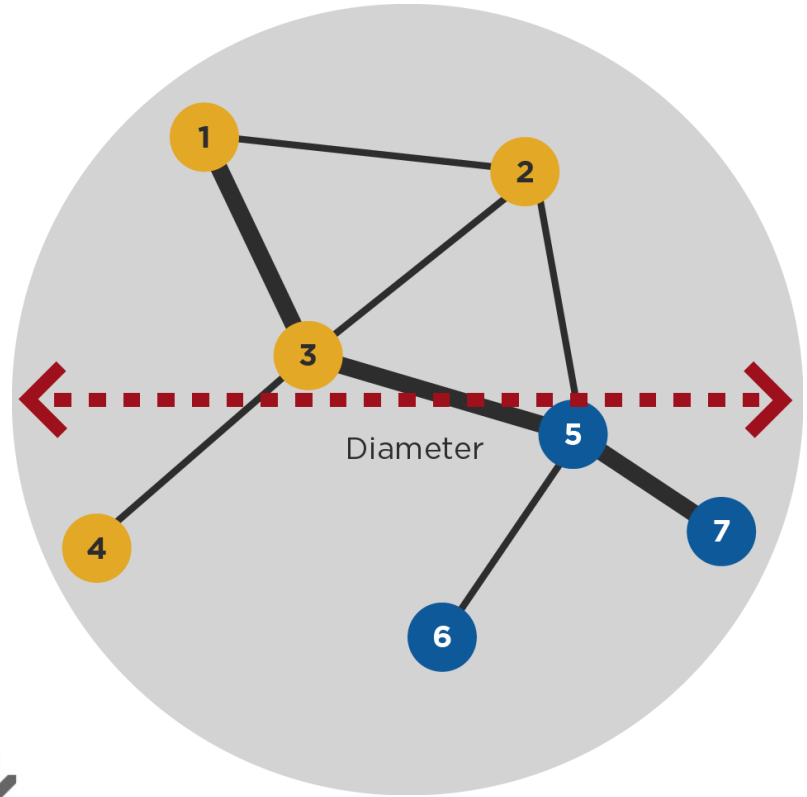
Triplets of
Node 3

Possible Number of Triplets





Average and
Longest Distance



- The longest shortest path (distance) between any two nodes in a network is called the network's *diameter*
- The diameter of the network on the right is 3; it is a useful measure of the *reach* of the network (as opposed to looking only at the total number of vertices or edges)
- It also indicates how long it will take at most to reach any node in the network (sparser networks will generally have greater diameters)
- The average of all shortest paths in a network is also interesting because it indicates how far apart any two nodes will be on average (average *distance*)

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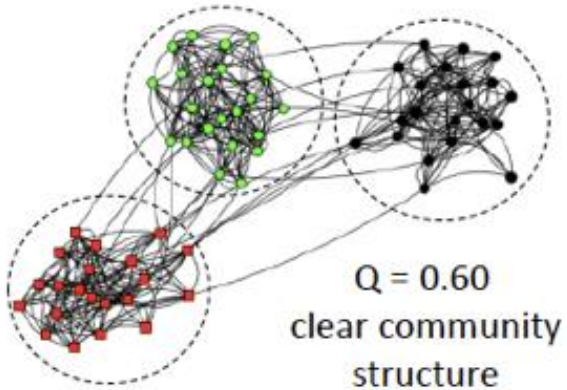
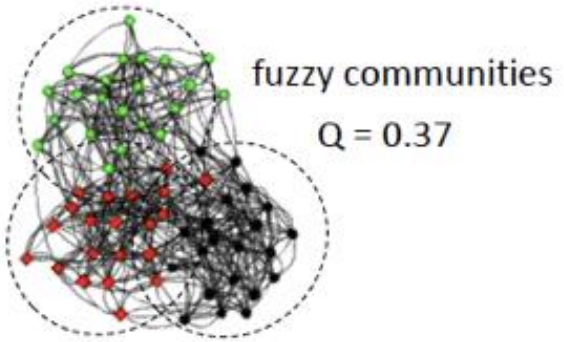
Modularity

$$Q = \frac{1}{2m}\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right)\delta(C_i, C_j)$$

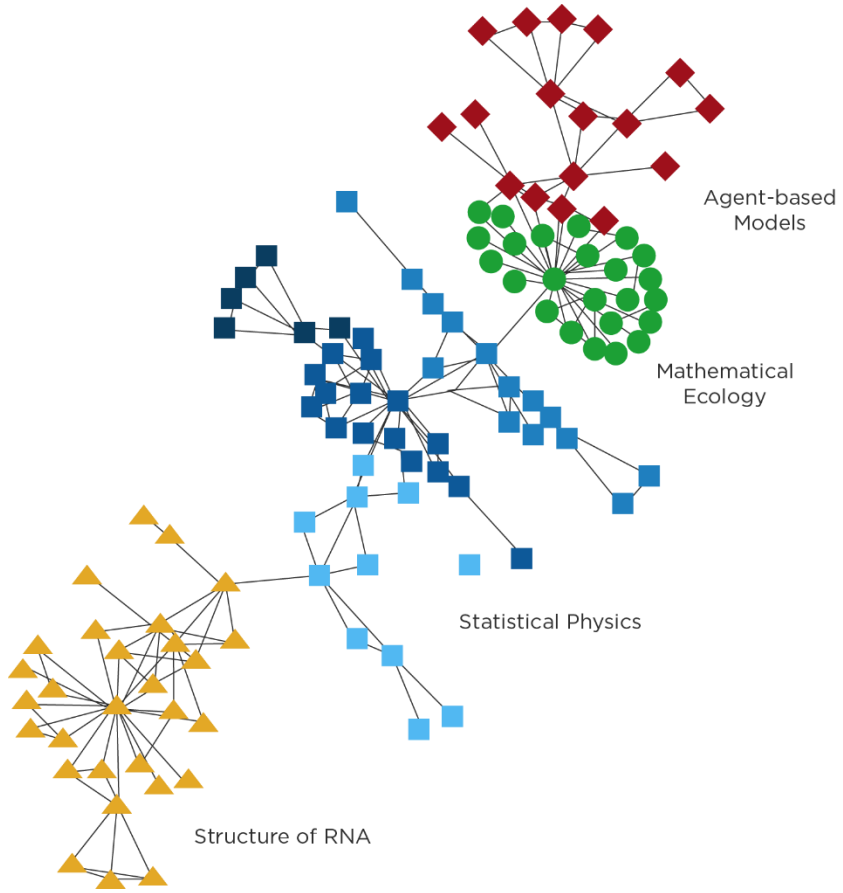
Edges inside the community

Expected number of edges if i,j places at random





Example :
Finding Community



- Collaboration network of scientist at Santa Fe Institut (Girvan & Nirwan)
- 27| scientist (vertices) / 1 | 8 nodes from largest component edge = scientist coauthor one of more publications
- Komunitas : kumpulan titik titik dimana jumlah hubungan internal antar titik lebih besar dari pada jumlah hubungan dengan titik eksternal

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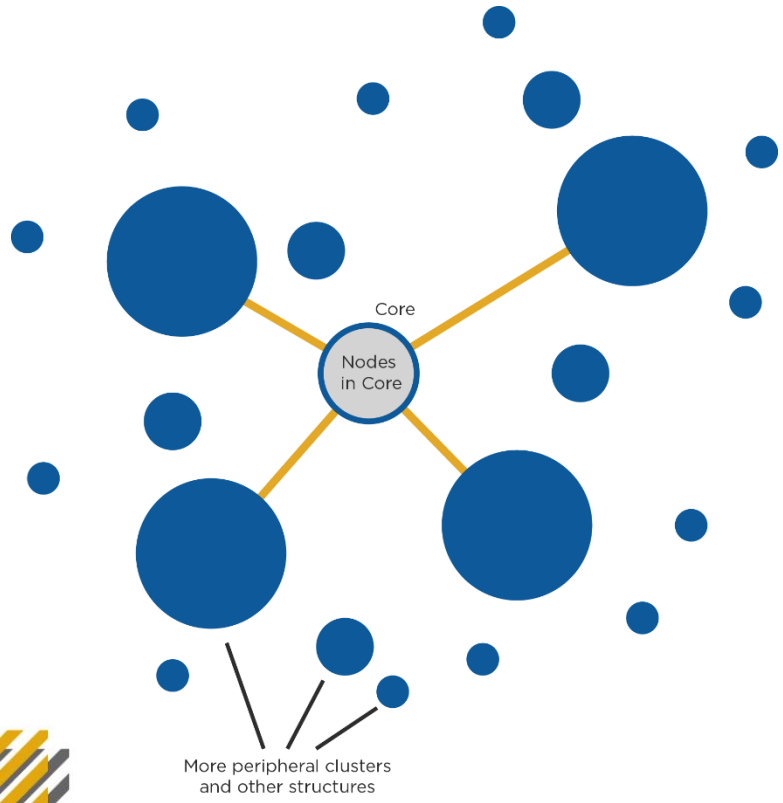
Core-Periphery
Structures



- A useful and relatively simple metric of the degree to which a social network is centralized or decentralized, is the centralization measure

  (usually normalized such that it takes values between 0 and 1)

  - It is based on calculating the differences in degrees between nodes; a network that greatly depends on 1-2 highly connected nodes (as a result for example of preferential attachment) will exhibit greater differences in degree centrality between nodes
  - Centralized structures can perform better at some tasks (like team-based problem-solving requiring coordination), but are more prone to failure if key players disconnect

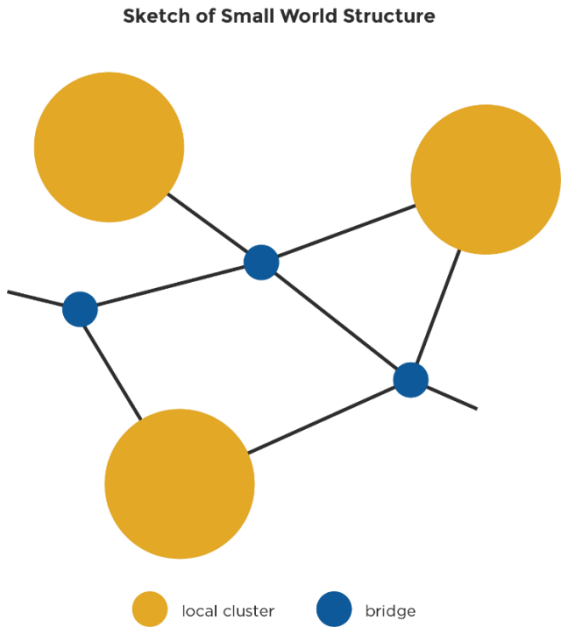
- In addition to centralization, many large groups and online communities have a core of densely connected users that are critical for connecting a much larger periphery

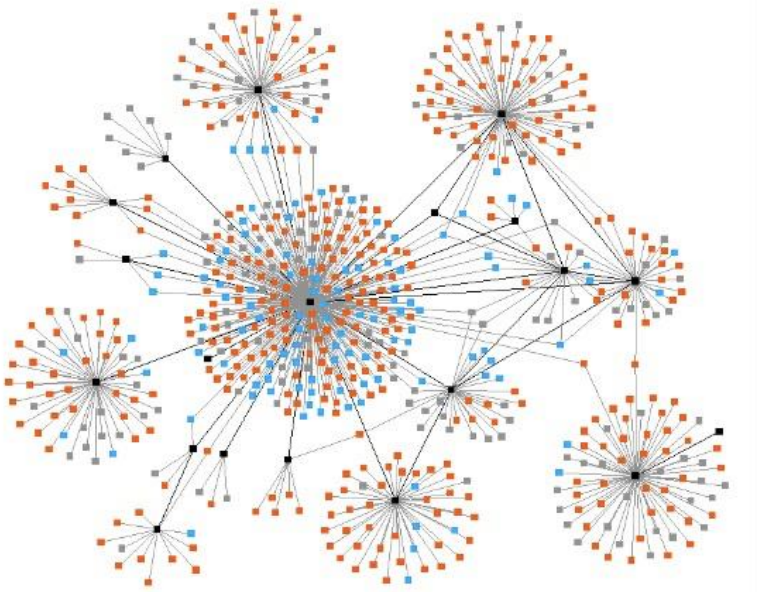
  - Cores can be identified visually, or by examining the location of high-degree nodes and their joint degree distributions (do high-degree nodes tend to connect to other high-degree nodes?)
  - Bow-tie analysis, famously used to analyze the structure of the Web, can also be used to distinguish between the core and other, more peripheral elements in a network (see earlier example here)

Agenda

- Social Network Formation
- Relationship Strength
- Homophily, Transitivity, and Bridge
- Centrality Metrics and Their Interpretation
- Modularity Metrics
- Core and Periphery Structure
- **Small World**
- Preferential Attachment

Small Worlds





- A small world is a network that looks almost random but exhibits a significantly high clustering coefficient(nodes tend to cluster locally) and a relatively short average path length (nodes can be reached in a few steps)
- It is a very common structure in social networks because of transitivity in strong social ties and the ability of weak ties to reach across clusters
- Such a network will have many clusters but also many bridges between clusters that help shorten the average distance between nodes

You may have heard of the famous “6 degrees” of separations

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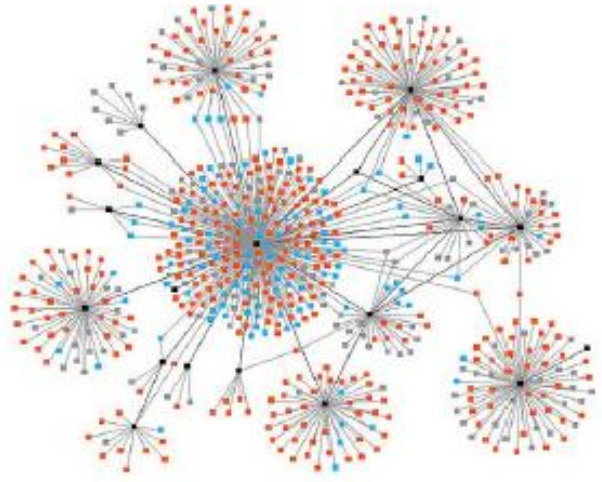
Preferential Attachment

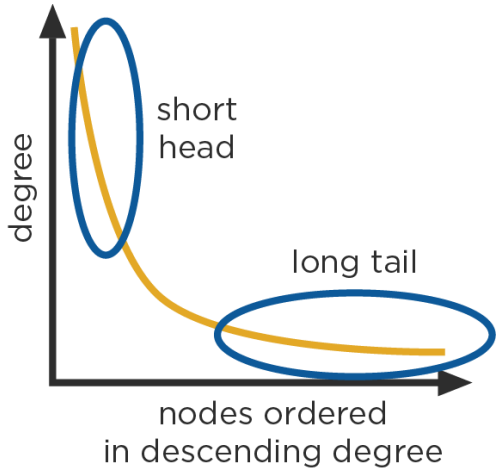
A property of some networks, where, during their evolution and growth in time, a the great majority of new edges are to nodes with an already high degree; the degree of these nodes thus increases disproportionately, compared to most other nodes in the network

- The result is a network with few very highly connected nodes and many nodes with a low degree
- Such networks are said to exhibit a long-tailed degree distribution
- And they tend to have a small-world structure!

(it turns out, transitivity and strong/weak tie characteristics are not neccessary to explain small world structures, but they are common and can also lead to such structures



Example of network with preferential attachment



Skectch of long-tailed degree distributions

Reasons for
Preferential Attachment

Popularity	Quality	Mixed Model
We want to be associated with popular people, ideas, items, thus further increasing their popularity, irrespective of any objective measurable characteristics	We evaluate people and everything else based on objective quality criteria, so higher quality nodes will naturally attract more attention faster	Among nodes of similar attributes, those that reach critical mass first will become 'star' with many friends and followers ('halo effect')
also known as 'the rich get richer'	*also known as 'the good get better'*	*may be impossible to predict who will become a star, even if quality matters*

Preferential Attachment
Process

New node join to a social network has higher probability connected to higher degree node in the network