Computational Modeling:
Simulation

Agenda

- Definition of Simulation
- Simulation Model
- Value of Simulation
- Applications
- Process of Simulation
- Monte Carlo Simulation
- Simulation Case

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Definition

- **Simulation** is the operation of a model, which is a representation of that system.
- A ***simulation*** uses a model to emulate the dynamic characteristics of a system
- The operation of the model can be studied, and then properties concerning the behavior of the actual system can be inferred.

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Simulation
Model

- General simulation model comprises n input variables (x_1 , x_2 ,..., x_n) and m output variables ($f_1(x)$,$f_2(x)$,...,$f_m(x)$) or (y_1,y_2,...,y_m)

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Value of Simulation

- Simulations are used instead of real systems for many reasons:
  - Cheaper
  - Optimize the resources
  - More configurable and controllable
  - Safer (reduce risk)
  - Faster (or slower)
  - More accessible – easier to collect data

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Application

- Evaluating H/W and S/W requirements for a computer system
- Determining ordering policies for an inventory system
- Analyzing financial or economic systems
- Evaluating designs for service organizations such as hospitals, post offices, or fast-food restaurants
- Designing and operating transportation facilities such as freeways, airports, subways, or ports

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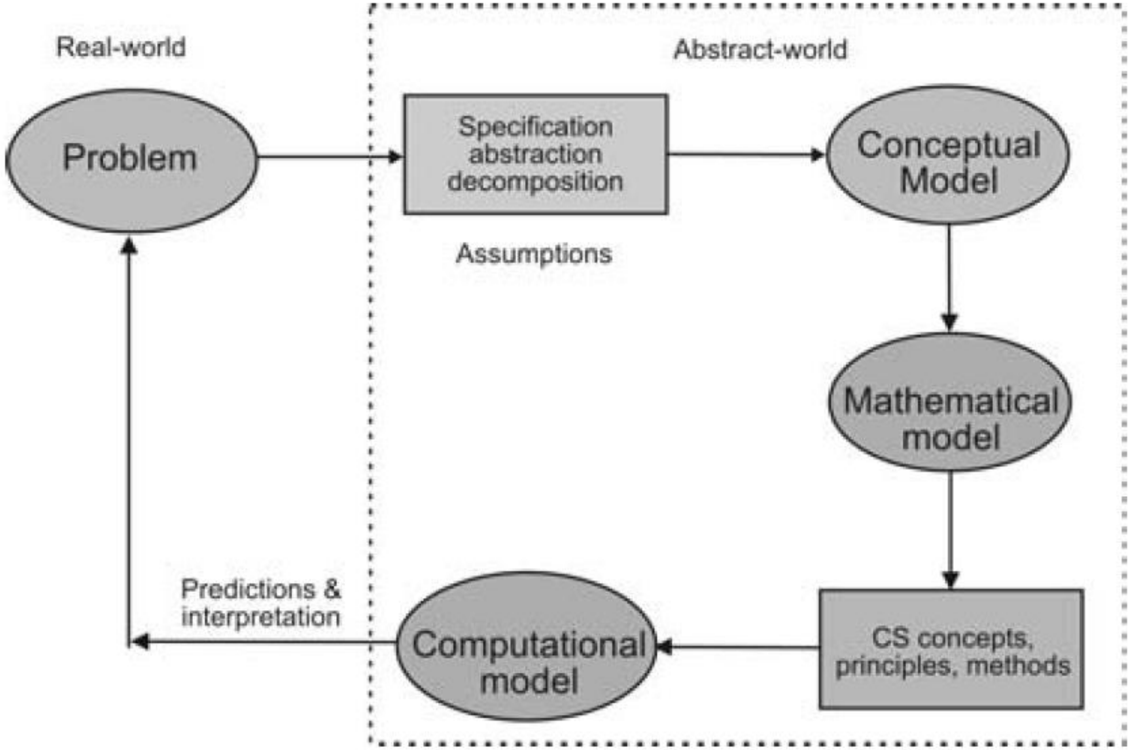
Process of Simulation



Simulation Steps and Criteria – University of Houston (1)

- **Problem Definition**
  The initial step involves defining the goals of the study and determining what needs to be solved. The problem is further defined through objective observations of the process to be studied. Care should be taken to determine if simulation is the appropriate tool for the problem under investigation.
- **Project Planning**
  The tasks for completing the project are broken down into work packages with a responsible party assigned to each package. Milestones are indicated for tracking progress. This schedule is necessary to determine if sufficient time and resources are available for completion.
- **System Definition**
  This step involves identifying the system components to be modelled and the performance measures to be analysed. Often the system is very complex, thus defining the system requires an experienced simulator who can find the appropriate level of detail and flexibility.
- **Model Formulation**
  Understanding how the actual system behaves and determining the basic requirements of the model are necessary in developing the right model. Creating a flow chart of how the system operates facilitates the understanding of what variables are involved and how these variables interact.
- **Input Data Collection & Analysis**
  After formulating the model, the type of data to collect is determined. New data is collected and/or existing data is gathered. Data is fitted to theoretical distributions. For example, the arrival rate of a specific part to the manufacturing plant may follow a normal distribution curve.

Simulation Steps and Criteria – University of Houston (2)

- **Model Translation**
  The model is translated into programming language. Choices range from general purpose languages such as fortran or simulation programs such as Arena.

- **Verification & Validation**
  Verification is the process of ensuring that the model behaves as intended, usually by debugging or through animation. Verification is necessary but not sufficient for validation, that is a model may be verified but not valid. Validation ensures that no significant difference exists between the model and the real system and that the model reflects reality. Validation can be achieved through statistical analysis. Additionally, face validity may be obtained by having the model reviewed and supported by an expert.

- **Experimentation & Analysis**
  Experimentation involves developing the alternative model(s), executing the simulation runs, and statistically comparing the alternative(s) system performance with that of the real system.

- **Documentation & Implementation**
  Documentation consists of the written report and/or presentation. The results and implications of the study are discussed. The best course of action is identified, recommended, and justified.

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Monte Carlo
Simulation

- **Monte Carlo** methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. – One of the popular algorithm -
- Their essential idea is using randomness to solve problems that might be deterministic in principle. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches.
- Monte Carlo methods are mainly used in three distinct problem classes: optimization, numerical integration, and generating draws from a probability distribution.

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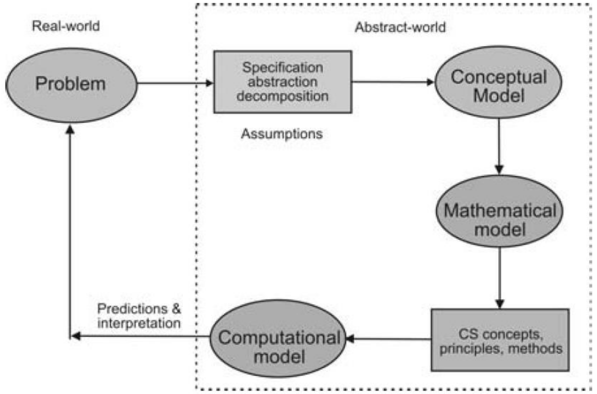
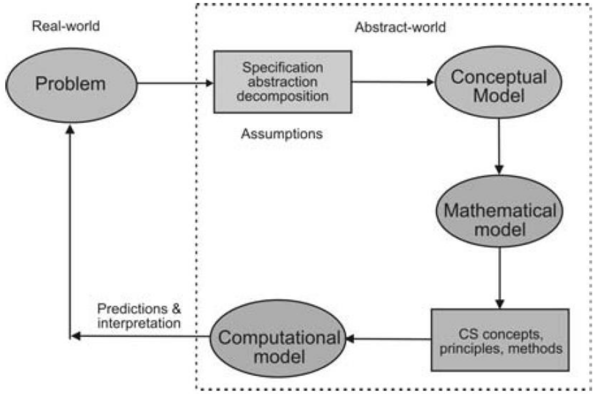
Simulation Case-1:
Problem Statement



- A popular radial tire accounts for a large portion of the sales at Bimo's Auto Tire.
- Bimo wishes to determine a policy for managing this inventory.
- He wants to simulate the daily demand for a number of days.

Simulation Case-1:
Abstraction



Step 1:
Establishing probability distributions

- One way to establish a probability distribution for a given variable is to examine historical outcomes
- Managerial estimates based on judgment and experience can also be used

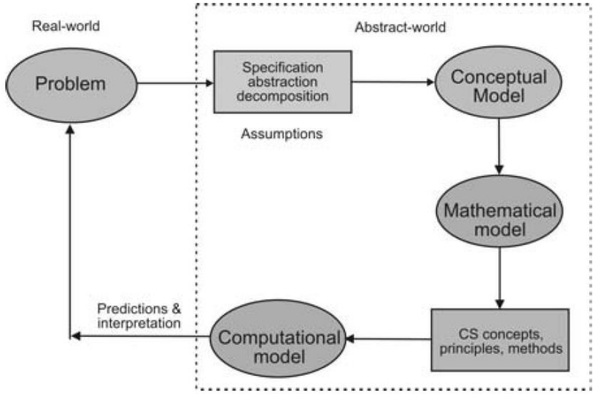
Demand for Tires	Frequency (Days)
0	10
1	20
2	40
3	60
4	40
5	30
	200

Historical daily demand for radial tires

Demand Variable	Probability of Occurrence
0	10/200 = 0.05
1	20/200 = 0.10
2	40/200 = 0.20
3	60/200 = 0.30
4	40/200 = 0.20
5	30/200 = 0.15
	200/200 = 1.00

Probability of demand for radial tires

Simulation Case-1: Abstraction



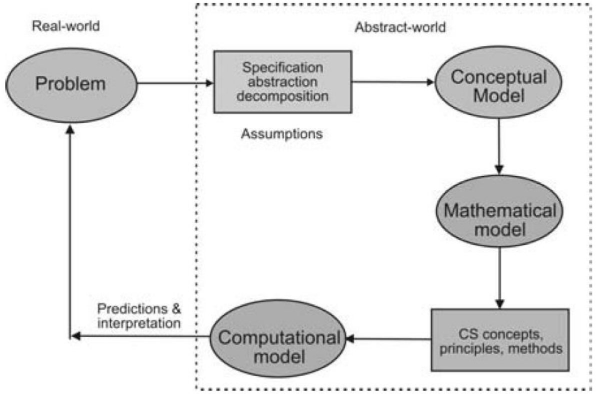
Step 2:
Building a cumulative probability distribution for each variable

- Converting from a regular probability to a cumulative distribution is an easy job
- A cumulative probability is the probability that a variable will be less than or equal to a particular value
- A cumulative distribution lists all of the possible values and the probabilities

Daily Demand	Probability	Cumulative Probability
0	0.05	0.05
1	0.10	0.15
2	0.20	0.35
3	0.30	0.65
4	0.20	0.85
5	0.15	1.00

Cumulative probability for radial tires

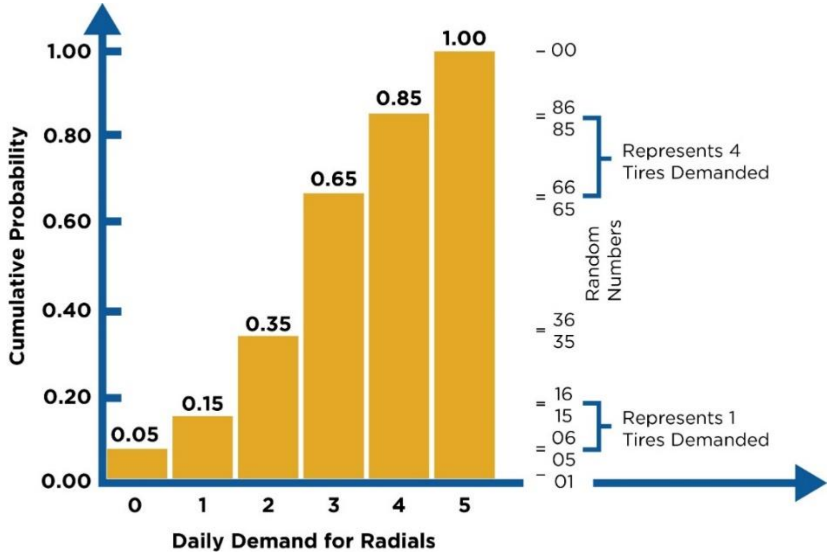
Simulation Case-1: Abstraction



Step 3:

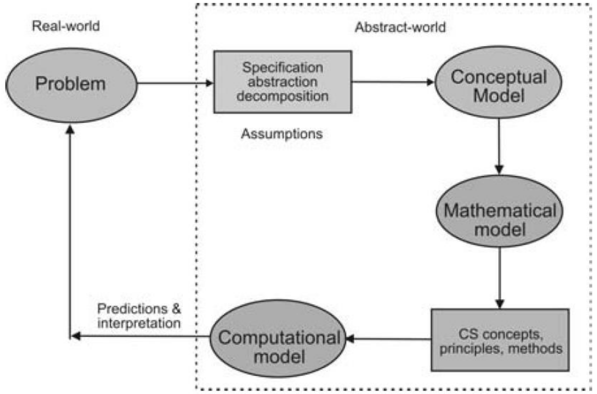
Setting random number intervals

- Assign a set of numbers to represent each possible value or outcome
- These are random number intervals
- A random number is a series of digits that have been selected by a totally random process
- The range of the random number intervals corresponds exactly to the probability of the outcomes as shown in the next figure



Graphical representation of the cumulative probability distribution for radial tires

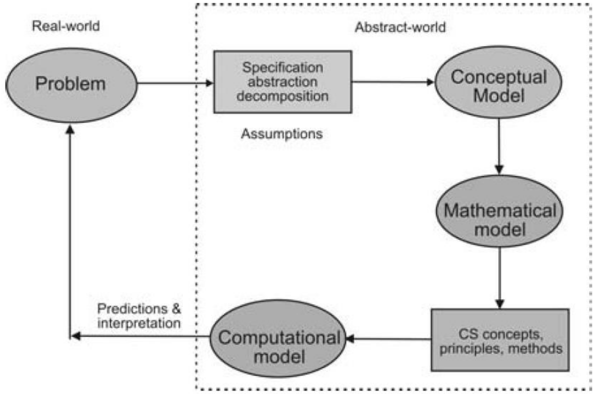
Simulation Case-1:
Abstraction



- Assignment of random number intervals for Harry's Auto Tire

Daily Demand	Probability	Cumulative Probability	Interval of Random Numbers
0	0.05	0.05	01 to 05
1	0.10	0.15	06 to 15
2	0.20	0.35	16 to 35
3	0.30	0.65	36 to 65
4	0.20	0.85	66 to 85
5	0.15	1.00	86 to 00

Simulation Case-1: Abstraction

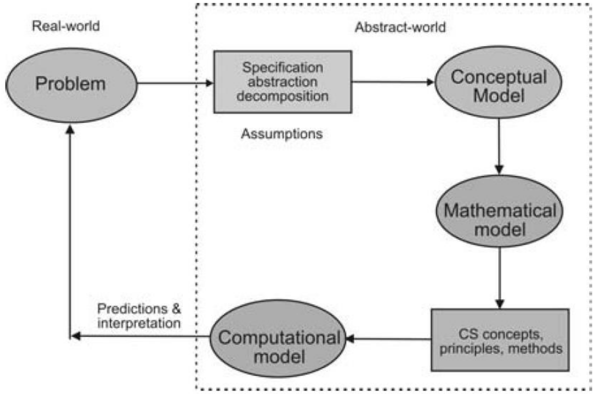


Step 4:

Generating random numbers

- Random numbers can be generated in several ways
- Large problems will use computer program to generate the needed random numbers
- For small problems, random processes like roulette wheels or pulling chips from a hat may be used
- The most common manual method is to use a random number table

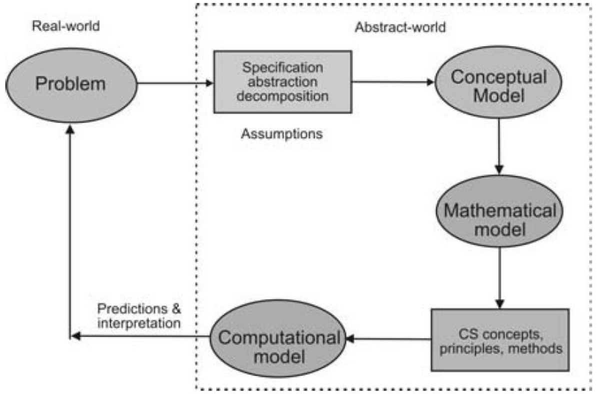
Simulation Case-1:
Abstraction



- Table of random numbers (partial)

52	06	50	88	53	30	10	47	99	37
37	63	28	02	74	35	24	03	29	60
82	57	68	28	05	94	03	11	27	79
69	02	36	49	71	99	32	10	75	21
98	94	90	36	06	78	23	67	89	85
96	52	62	87	49	56	59	23	78	71
33	69	27	21	11	60	95	89	68	48
50	33	50	95	13	44	34	62	64	39
88	32	18	50	62	57	34	56	62	31
90	30	36	24	69	82	51	74	30	35

Simulation Case-1: Abstraction

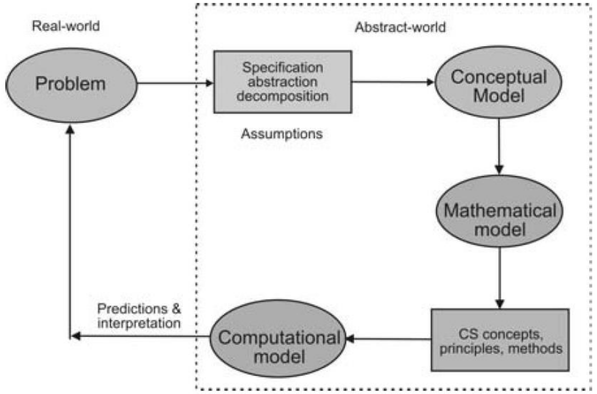


Step 5:

Simulating the experiment

- Select random numbers from random table
- The number selected will have a corresponding range by using the daily demand that corresponds to the probability range aligned with the random number

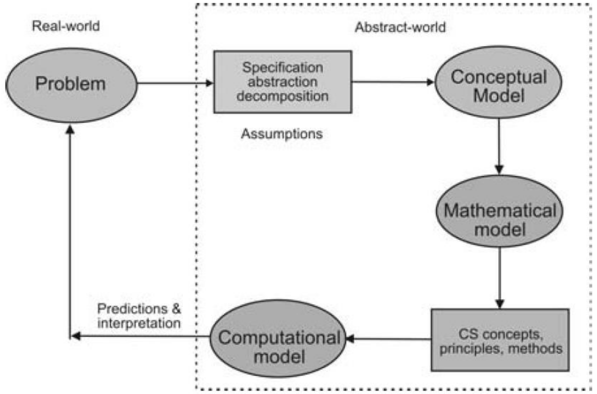
Simulation Case-1:
Abstraction



Ten-day simulation of demand for radial tires

Day	Random Number	Simulated Daily Demand
1	52	3
2	37	3
3	82	4
4	69	4
5	98	5
6	96	5
7	33	2
8	50	3
9	88	5
10	90	5
		39 = Total 10-day demand
		3.9 = Average daily demand for tires

Simulation Case-1:
Mathematical Model



- Note that the average demand from previous simulation (3.9 tires) is different from the expected daily demand

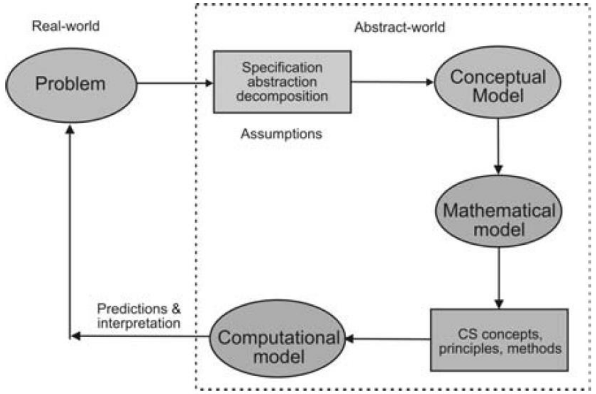
$$\text{Expected daily demand} = \sum_{i=0}^{5} (\text{Probability of } i \text{ tires})(\text{Demand of } i \text{ tires})$$

$$= (0.05)(0) + (0.10)(1) + (0.20)(2) + (0.30)(3) + (0.20)(4) + (0.15)(5)$$

$$= 2.95 \text{ tires}$$

- If this simulation were repeated hundreds or thousands of times it is much more likely the average simulated demand would be nearly the same as the expected demand

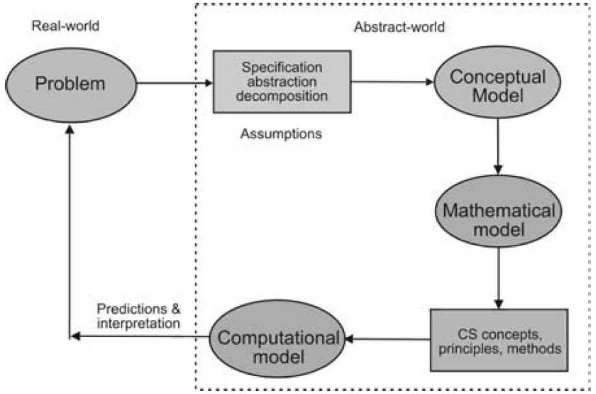
Simulation Case-2:
Problem Statement



- For this example, we will try to predict how much money we should budget for sales commissions for the next year. This problem is useful for modeling because we have a defined formula for calculating commissions and we likely have some experience with prior years' commissions payments.
- This problem is also important from a business perspective. Sales commissions can be a large selling expense and it is important to plan appropriately for this expense. In addition, the use of a Monte Carlo simulation is a relatively simple improvement that can be made to augment what is normally an unsophisticated estimation process.

https://pbpython.com/monte-carlo.html , February 18, 2019

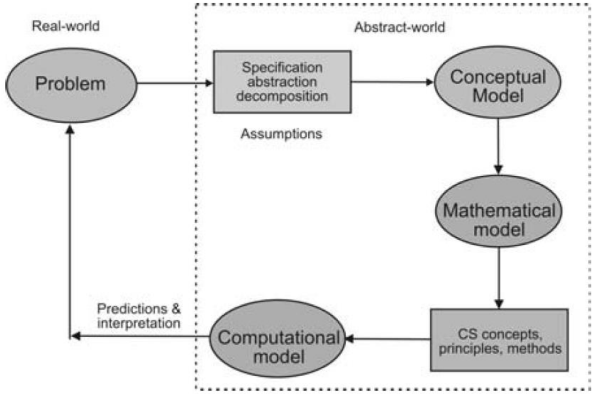
Simulation Case-2:
Problem Specification



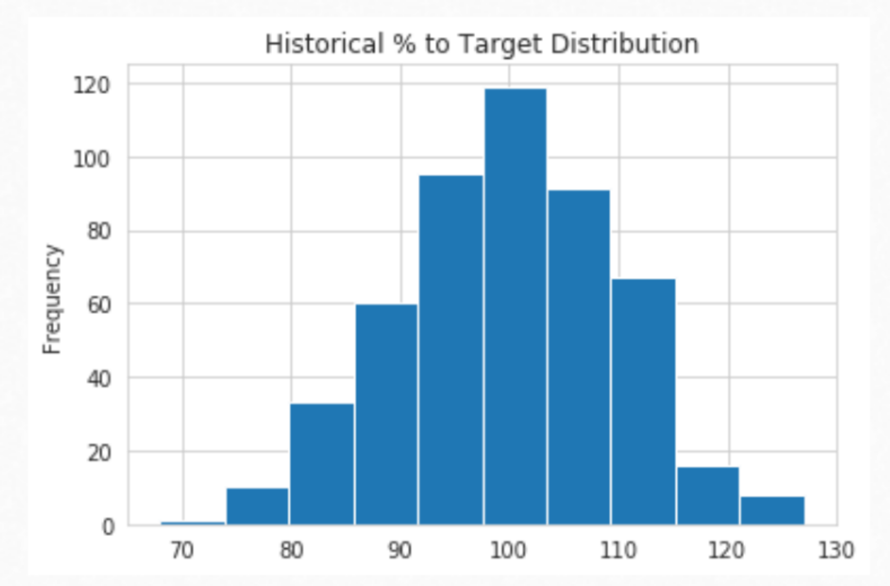
- In this example, the sample sales commission would look like this for a 5 person sales force:

Sales Rep	Sales Target	Actual Sales	Percent To Plan	Commission Rate	Commission Amount
1	$100,000	$88,000	88.0%	2.0%	$1,760
2	$200,000	$202,000	101.0%	4.0%	$8,080
3	$75,000	$90,000	120.0%	4.0%	$3,600
4	$400,000	$360,000	90.0%	0.0%	$0
5	$500,000	$350,000	70.0%	0.0%	$0
Total	$1,275,000	$1,090,000			$13,440

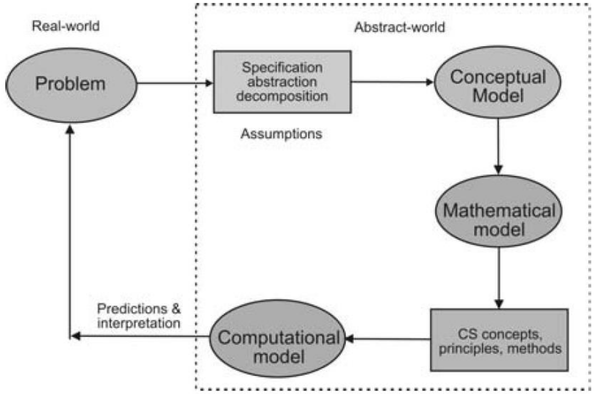
Simulation Case-2:
Problem Conceptualization



- Because we pay commissions every year, we understand our problem in a little more detail and can use that prior knowledge to build a more accurate model.
- Figure shows a typical historical distribution of percent to target.
- This distribution looks like a normal distribution with a mean of 100% and standard deviation of 10% -> we can model our input variable distribution so that it is similar to our real world experience.



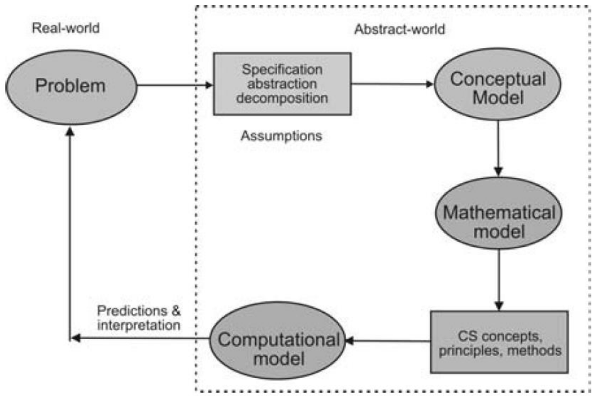
Simulation Case-2:
Mathematical Model



- Commission Amount = Actual Sales * Commission Rate
- The commission rate is based on this Percent To Plan table:

H	I
Rate Schedule	
0 – 90%	2.00%
91-99%	3.00%
>= 100	4.00%

Simulation Case-2:
Mathematical Model



- There are two components to running a Monte Carlo simulation:
  - The equation to evaluate:

    Commission Amount = Actual Sales * Commission Rate

    *)The commission rate is based on this Percent To Plan table:
  - The random variables for the input.

H	I
Rate Schedule	
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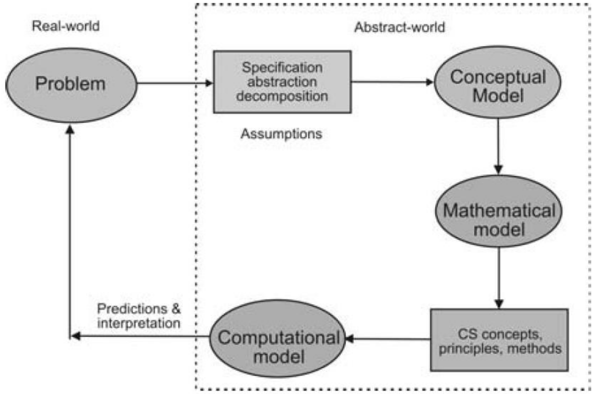
91-99%	3.00%
>= 100	4.00%

Simulation Case-2: Computational Model



- Python Model
- See: https://colab.research.google.com/drive/1yVtjwekyogMmdyMUMkBecwjSNVTteElh

Software Demo

- Monte Carlo Simulation using POM-QM
- Monte Carlo Simulation using RStudio

Discussion