Computational Modeling: Optimization

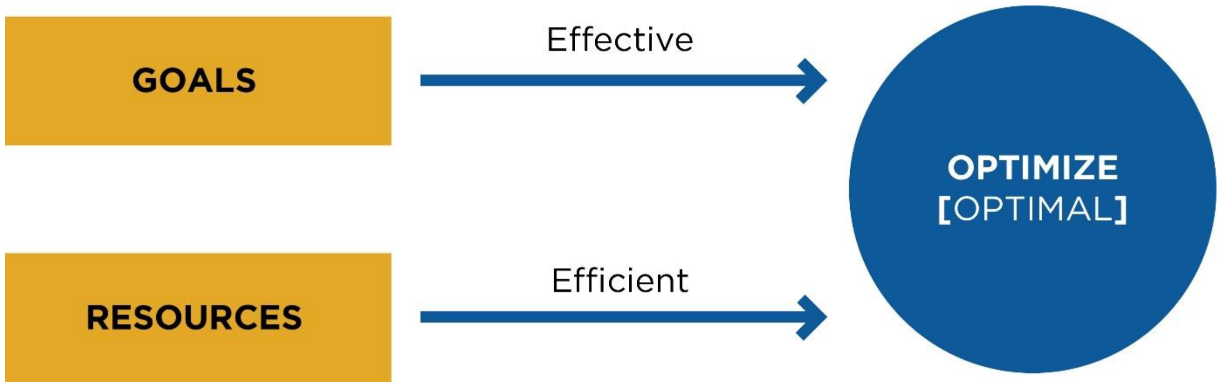
Agenda

- The Concept of Optimization
- Optimization Model
- Optimization in Business
  - Optimization Case 1
  - Optimization Case 2
- Optimization in Computing Context

The Concept of Optimization

- Management → efficient & effective
  - efficient → using resources (thrifty)
  - effective → goal attainment (at all cost, imperative)

- To achieve both efficient and effective goal respectively, use **OPTIMIZATION.**



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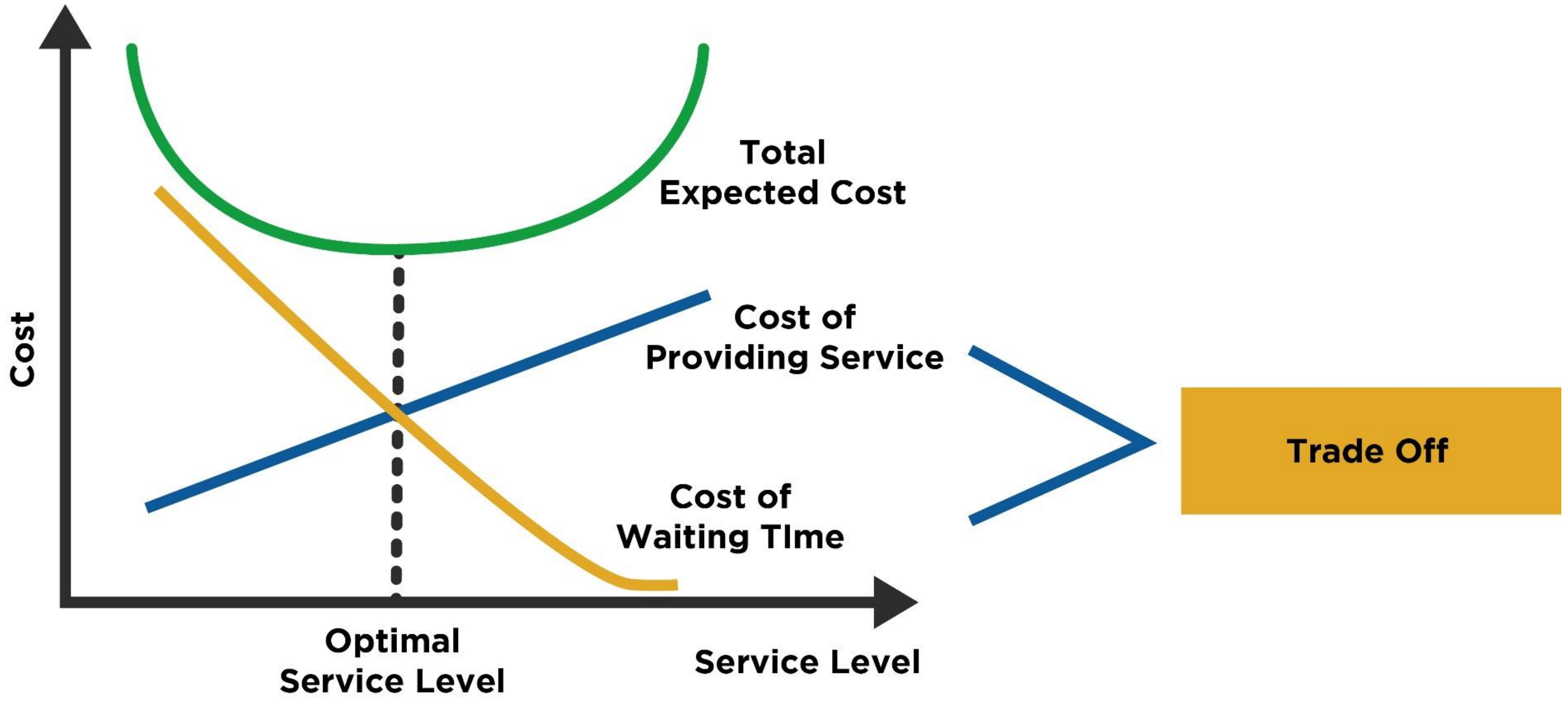
Optimization Model

Costs and service level



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Optimization
Case 1: Problem

- The manager of a Car manufacturer wants to determine the optimal number of teams of stevedores to employ each shift to obtain the minimum total expected cost which will increase profit in return.

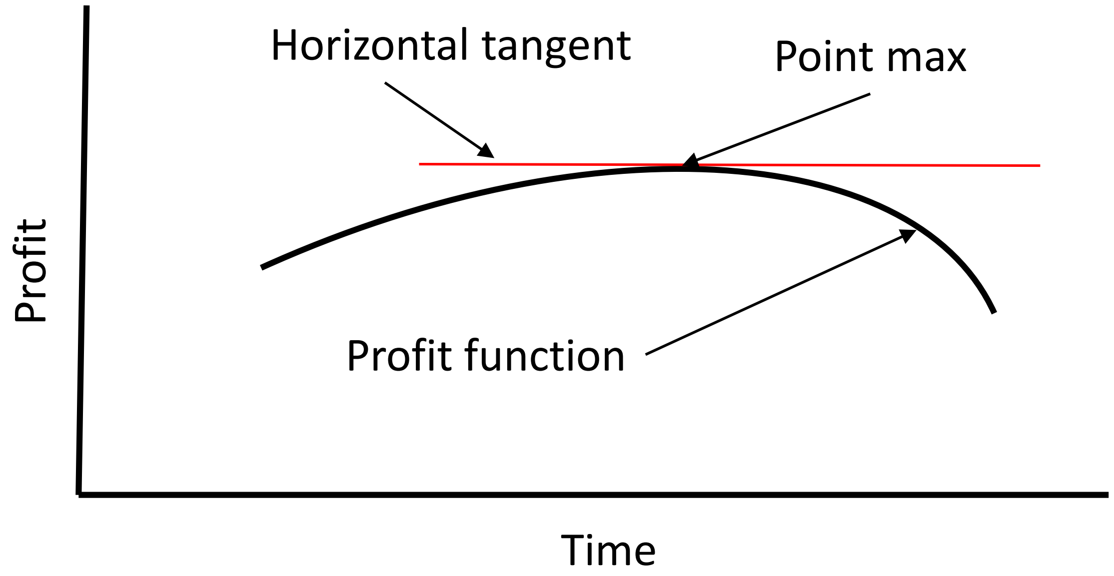
Optimization

Case 1: Problem Specification

- The Car manufacturer produces 2 types of car, A and B.
- Its factory runs 30 days per month.
- There are 1 robot, 2 engineers and 1 detailer available.
- The detailer only works for 21 days each month instead of 30.
- Car A provides Rp. 30,000,000 profit, whilst Car B offers Rp. 45,000,000 profit.
- At the moment, they produce 4 of each cars per month, for Rp. 300,000,000 profit. Not bad at all, but we think we can do better for them.
- Resources allocation for the cars:
  - Robot time: Car A – 3 days; Car B – 4 days.
  - Engineer time: Car A – 5 days; Car B – 6 days.
  - Detailer time: Car A – 1.5 days; Car B – 3 days.

Optimization
Case 1: Conceptual Model

- There must be a concave profit function (profit as a function of time) where the maximum profit can be deduced based on horizontal tangent of the curve.
- The horizontal tangent indicates optimal solution to the problem.



Optimization
Case 1: Mathematical Model

- The problem as objective function: $\textbf{\textit{Profit}}_{\text{for each month}} = 30{,}000{,}000 * A + 45{,}000{,}000 * B$
- Maximise objective function (profit),
- Subject to constraint:
  - A must be integer and $A \geq 0$
  - B must be integer and $B \geq 0$
  - Robot time in one month should not exceed 30 days: $3A + 4B \leq 30$
  - Engineer time in one month should not exceed 30 days: $5A + 6B \leq 60$
  - Detailer time in one month should not exceed 21 days: $1.5A + 3B \leq 21$

Optimization

Case 1: Computational Model

- See: https://colab.research.google.com/drive/1tKJrTnhLXc0c2msiPjws-LzjkMy7mAoN

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Optimization

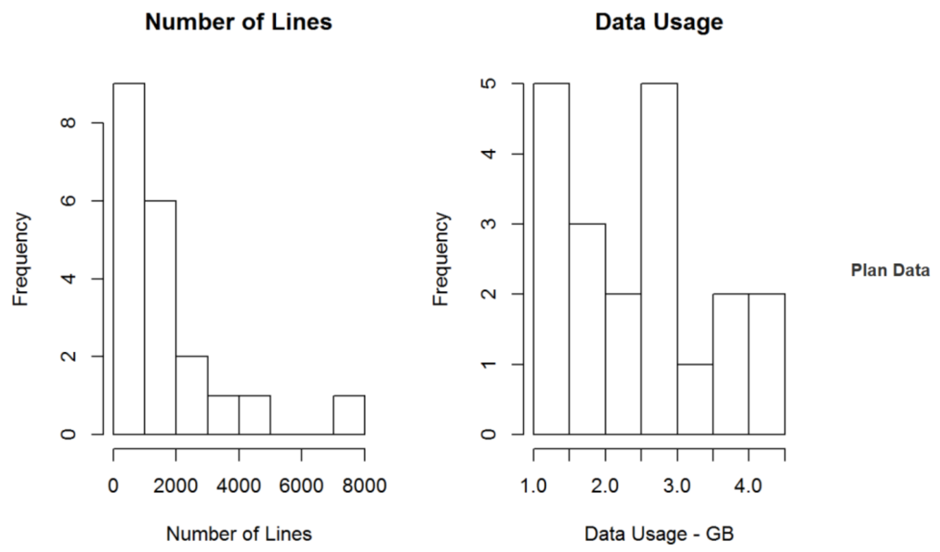
Case 2: Problem*

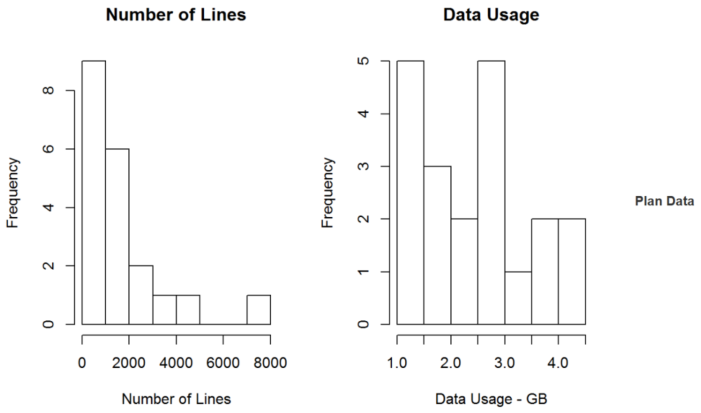
- A consulting individual (consultant) runs a projects where he helps his portfolio companies to select a wireless provider with the mix of data plans that can meet all their requirements (total number of lines and pooled data quantity) while cost them the least amount of money.

*https://rstudio-pubs-static.s3.amazonaws.com/254961_411bb6a0058647718830d3ead87f6cd7.html

Optimization
Case 2: Problem Specification

- The consultant has total 20 porfolio companies in the dataset with average number of lines and monthly data usage from past 3 months.
- The summary statistics and the histograms of the company data.
- Number of Lines : the average number of lines is around 1800 but most of companies have less than 2000 lines. Just one company is sort of outlier with more than 7000 lines.
- Data Usage : Average usage per line is around 2.5GB with range from 1GB to 4GB.





Optimization
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Optimization
Case 2: Problem Specification

- The consultant also see different level of data plan from AT&T and Verizon Wireless for us to choose from.
- The goal of this analysis is to choose the carrier with the mix of different data plans with the lowest total cost while satisfies the number of lines and total data requirements

	Wireless_Carrier	Data_GB	Plan_Rate
1	ATT	3	60
2	ATT	4	75
3	ATT	5	85
4	ATT	6	100
5	VZW	1	56
6	VZW	2	60

Optimization
Case 2: Problem specification

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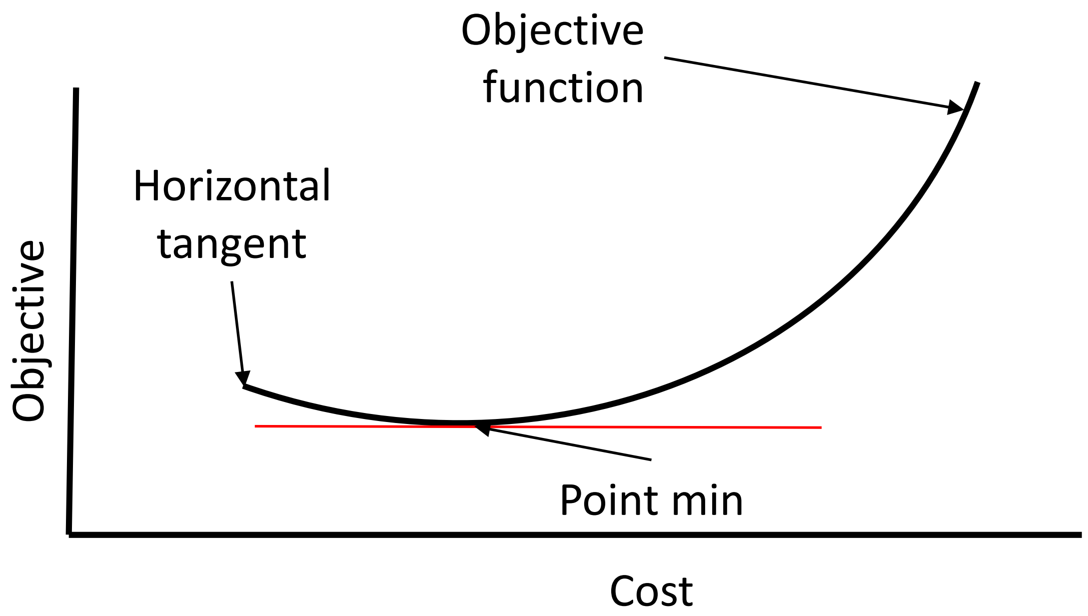
Optimization

Case 2: Conceptual Model

- There are two objective function: one for ATT and one for Verizon.
- There are two group of constraints: one for ATT and one for Verizon.
- There must be a mutual optimum (minimum combination of ATT and Verizon) for both objective function for each portfolio company



Optimization
Case 2: Mathematical Model

- The problem as objective function:
- objective: Each customer (A to T) has an objective to minimize the cost
  - ATT cost = (60x1 + 75x2 + 85x3 + 100x4)
  - Verizon cost = (56x1 + 60x2 + 70x3 + 80x4 + 90x5 + 100x6)
  - Xn : data plan
- Minimise objective function (cost),
- Subject to constraint: for each customer (A to T)

  ATT ->

  number of lines: x1 + x2 + x3 + x4 = number of lines for each data packet;

  data usage: 3x1 + 4x2 + 5x3 + 6x4 ≤ number of data usage per data packet;

  Verizon ->

  number of lines: x1 + x2 + x3 + x4 + x5 + x6 = number of lines for each data packet;

  data usage: x1 + 2x2 + 4x3 + 6x4 + 8x5 + 10x6 ≤ number of data usage per data packet.

Optimization
Case 2: Computational Model

- See: …

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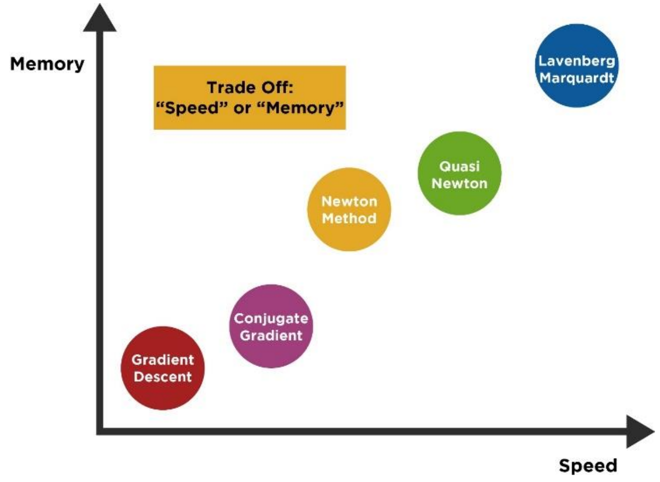
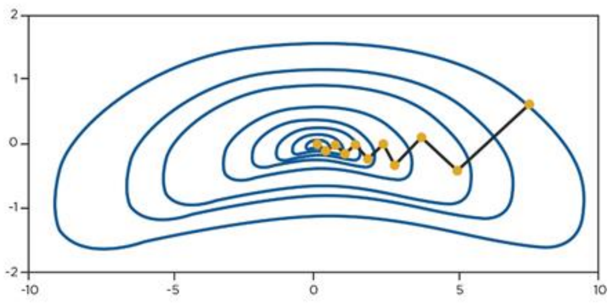
Optimization in Computing Context:
Trade-off in Processing Complex Data Analytics

- The two most common measures are:

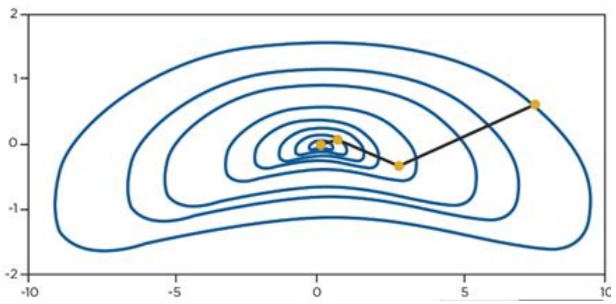
  1. Time: how long does the algorithm take to complete.

  2. Space: how much working memory (typically RAM) is needed by the algorithm. This has two aspects: the amount of memory needed by the code, and the amount of memory needed for the data on which the code operates.





Gradient descent

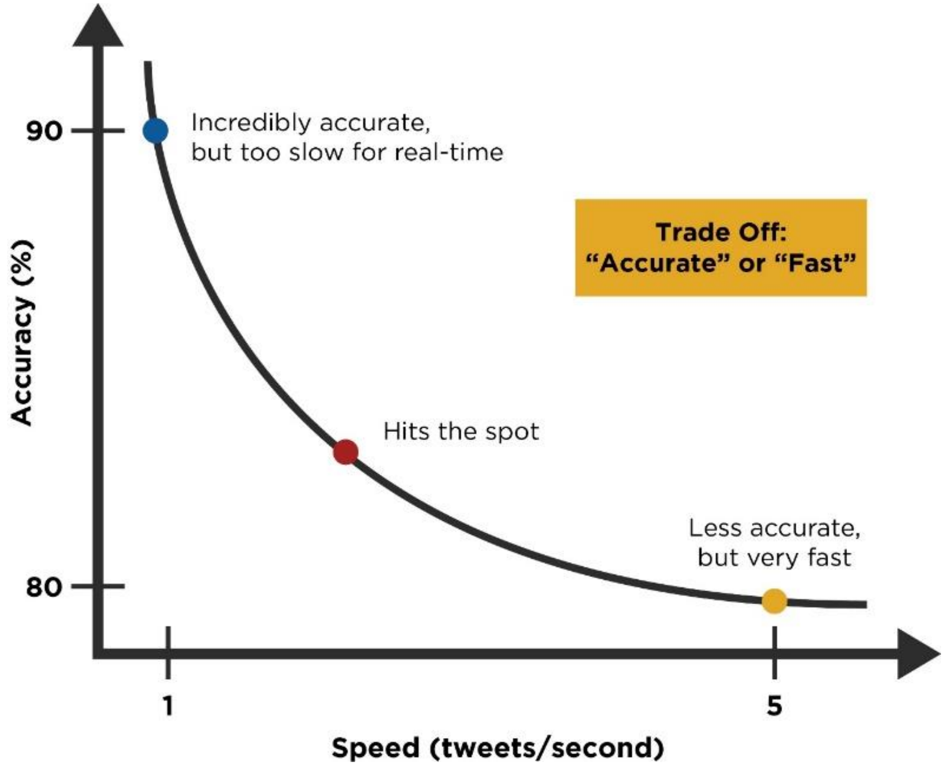


Newton’s method

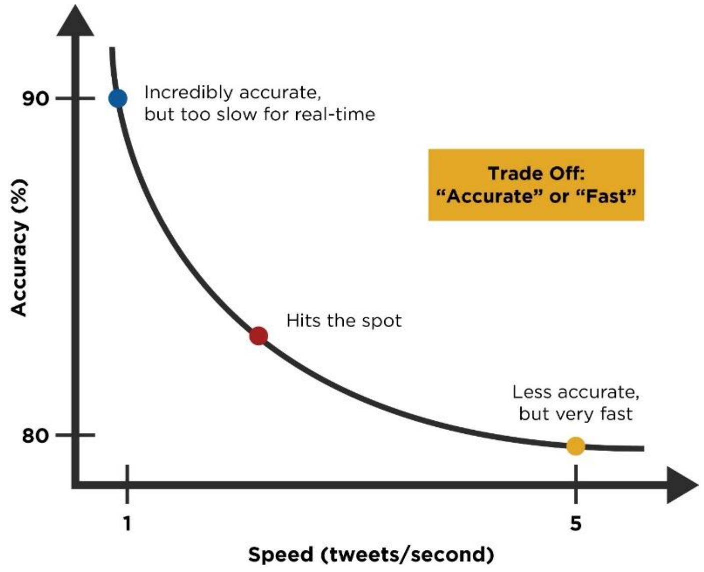
Newton's method requires less steps than gradient descent to find the minimum value of the loss function

Optimization in Computing Context:
Speed vs Accuracy in Machine Learning Model

- The two most common measures are:
  - Accuracy: incredibly accurate but too slow for real time.
  - Speed: very fast but less accurate.



Optimization in Computing Context: Speed vs Accuracy in Machine Learning Model



- The two most common measures are:
  1. *Accuracy*: incredibly accurate but too slow for real time.
  2. *Speed*: very fast but less accurate.

Optimization in Computing Context:
Code Optimization

- Computational tasks can be performed in several different ways with varying efficiency (optimizing computing resources).
- For example, consider the following C code snippet whose intention is to obtain the sum of all integers from 1 to N:

```
int i, sum = 0;
for (i = 1; i <= N; ++i)
{sum += i;}
printf("sum: %d\n", sum);
```

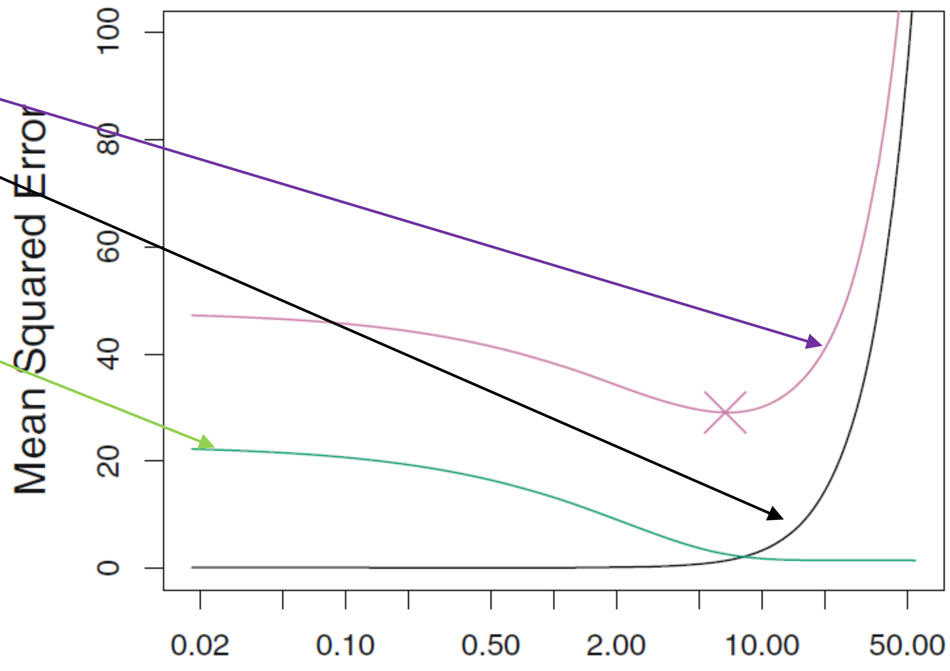
assuming no arithmetic overflow, can be rewritten using a mathematical formula like:

```
int sum = N * (1 + N) / 2;
printf("sum: %d\n", sum);
```

Michael Stueben, Good Habits for Great Coding, Improving Programming Skills with Examples in Python, Apress Media LLC, 2018

Optimization in Computing Context:
Bias Versus Variance Trade-Off

- Plots of squared bias (black), variance (green), and test MSE (purple) for the lasso.
- The cross indicates the lasso model for which the MSE is smallest.



Software
Demo

Cases demo using python software

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