Self-Tuning Kalman Filter for Fault Estimation of Nonlinear Systems

Muhammad Hilmi, Agus Hasan, and Mary-Ann Lundteigen

Abstract— **This paper presents a methodology for joint state and fault estimation utilizing a self-tuning extended Kalman filter (EKF). The EKF algorithm builds upon the classical Kalman filter and recursive least squares (RLS) to facilitate simultaneous state and fault estimation. The self-tuning characteristic is incorporated through adaptive mechanisms that address the influence of faults by recursively estimating process and measurement noise covariances using innovation-based covariance matching, as well as by adaptively updating the fault profile matrix via a scaling algorithm. These adaptive strategies are employed to enhance the accuracy of the joint estimation. The effectiveness of the proposed methodology is validated through numerical simulations of a submersible injection pump in a subsea seawater injection system. The results demonstrate significant improvements in estimation performance through adaptive measures, ensuring convergence and robustness. These findings highlight the potential of the proposed approach for real-world fault diagnosis in nonlinear systems.**

I. Introduction

With the rapid advancement of technology, industries are striving to enhance operational efficiency to meet or even exceed economic objectives. However, this push for efficiency has led to increasingly complex systems, particularly as industries move toward distributed, autonomous, and digitalized operations, commonly referred to as cyber-physical systems (CPS). While these advancements offer numerous benefits, they also introduce new vulnerabilities, as faults become more frequent in interconnected systems. Additionally, unidentified faults may emerge due to varying operational scenarios and the integration of diverse technologies [1]. To address faults in operation, industries continuously seek new solutions to improve fault prevention and mitigation strategies. This is crucial not only for maintaining economic competitiveness, by maximizing profit and minimizing production losses and property damage, but also for ensuring safety and environmental protection [2]. Faults in industrial systems can lead to severe consequences, including operational disruptions, environmental contamination, and hazards that pose risks to human safety, particularly for operators working near the affected systems.

A notable example is the subsea production system in the oil and gas industry, which is currently undergoing a significant transition. Already inherently complex, these systems are becoming even more sophisticated with the implementation of autonomous control. This transition introduces numerous electronic components, increasing the number of potential faults within the system [3]. The resulting faults can lead to production downtime, causing substantial financial losses. Furthermore, faults in high-risk components can result in environmental contamination and safety incidents [4]. This paper focuses on one such critical component, which is the submersible injection pump in a subsea seawater injection system. Given its role in maintaining reservoir pressure, understanding and mitigating failures in this component is essential for ensuring the reliability and safety of subsea production systems.

Faults in this work are defined as any deviation in operation that is undesirable and caused by changes in system properties. In an effort to detect and estimate faults, many studies have employed dynamic state estimation (DSE), with the Kalman filter emerging as a popular choice due to its ability to handle uncertainties in both the process and measurement of a system. Significant efforts have been made to enhance the efficacy of Kalman filter, primarily by improving its adaptivity. This adaptivity is employed to dynamically determine the optimal parameters used during the filtering process, thereby improving performance. Adaptive filtering techniques have been explored early on through methods such as the Bayesian approach, maximum likelihood estimation (MLE), correlation properties, and covariance matching [5]. These adaptive techniques have been integrated into Kalman filter to dynamically estimate parameters that is usually quite complicated to obtained accurately from the system, particularly the noise covariance matrix, using approaches like the gradient algorithm [6], fuzzy logic (FL) [7], variational Bayesian inference [8], and Kalman smoothing [9]. However, existing studies on the adaptive adjustment of Kalman filter parameters have not specifically focused on process fault estimation, nor has adaptivity been utilized to characterize fault profiles affecting the system.

To address the limitations of existing methods in handling process faults in nonlinear systems, this paper proposes a self-tuning adaptive Kalman filter methodology. The approach employs the extended Kalman filter (EKF) to jointly estimate both the state and fault magnitudes while ensuring stability through sufficient conditions. Additionally, as a key contribution, a self-tuning approach is developed in the form of adaptive adjustment mechanisms to estimate the process and measurement noise covariance matrices within the EKF framework for faulted systems, utilizing innovation covariance matching. Furthermore, this adaptive measure is extended to estimate the fault profile matrix using a scaling algorithm, a parameter that is often assumed in traditional fault estimation methods. The specific technical contributions of this work are outlined in Section II-A.

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Unlike previous studies, this work focuses on designing an EKF methodology specifically for nonlinear faulted systems, integrating adaptive approaches to enhance its estimation performance. The effectiveness of the proposed method is demonstrated through its implementation in a subsea pump system, showcasing its potential for real-world applications.

The remainder of the paper is structured as follows: Section II presents the state-space model of a system under fault conditions and introduces the proposed self-tuning adaptive Kalman filter for joint estimation of states and faults. Section III, which derives the adaptive expressions for parameters of the adaptive filtering. Section IV demonstrates the efficacy of the proposed approach through numerical simulations of a subsea pump system, and finally, the paper is concluded in Section V.

II. Fault Estimation

This section formulate the general problem of system affected by fault, then designs the self-tuning Kalman filter to estimate both the state and fault in a nonlinear system while also providing stability conditions.

A. Problem Formulation

The nonlinear system under process faults is modeled in discrete-time state-space as follows:

$$\begin{aligned}\boldsymbol{x}(k) = \boldsymbol{A}\boldsymbol{x}(k-1) + \boldsymbol{f}(\boldsymbol{x}(k-1), \boldsymbol{u}(k)) \\ + \boldsymbol{\Phi}(k)\boldsymbol{\theta}(k) + \boldsymbol{w}(k)\end{aligned} \tag{1}$$

$$\boldsymbol{y}(k) = \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{v}(k). \tag{2}$$

Here, the system states and measured outputs are denoted by $\boldsymbol{x}(k) \in \mathbb{R}^n$ and $\boldsymbol{y}(k) \in \mathbb{R}^m$, respectively. These values are influenced by assumed white Gaussian noises, given by $\boldsymbol{w}(k) \sim \mathcal{N}(0, \boldsymbol{Q}(k)) \in \mathbb{R}^n$ and $\boldsymbol{v}(k) \sim \mathcal{N}(0, \boldsymbol{R}(k)) \in \mathbb{R}^m$, where $\boldsymbol{Q}(k) \in \mathbb{R}^{n\times n}$ and $\boldsymbol{R}(k) \in \mathbb{R}^{m\times m}$ represent their respective noise covariance matrices. The system is governed by $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, the state transition matrix, and $\boldsymbol{f} : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$, a nonlinear function. The control input is represented as $\boldsymbol{u}(k) \in \mathbb{R}^s$, while $\boldsymbol{C} \in \mathbb{R}^{m\times n}$ denotes the measurement matrix. The process faults are represented by $\boldsymbol{\theta}(k) \in \mathbb{R}^p$ with their corresponding fault profile matrix $\boldsymbol{\Phi}(k) \in \mathbb{R}^{n\times p}$, which define their impact on the system.

No specific assumption is imposed on the fault signals, except that process faults remain within actuator saturation limits and the feasible operational range. The faults are considered time-dependent but independent to each other and to the system dynamics, with no correlation. This lack of contextual information makes fault magnitude estimation more challenging, thus serving as a better demonstration of the proposed methodology's capability.

Based on the problem formulation, the primary contributions of this work are as follows:

- Develop an adaptive Kalman filter-based algorithm capable of accurately estimating both the state vector $\boldsymbol{x}(k)$ and the fault vector $\boldsymbol{\theta}(k)$.
- Enhance the self-sufficiency of the estimation method by incorporating adaptive measures to estimate parameters that are challenging to determine, particularly the process noise covariance $\boldsymbol{Q}(k)$ and the measurement noise covariance $\boldsymbol{R}(k)$, for system affected by process faults.
- Introduce a mechanism for adaptively estimating the fault profile matrix $\boldsymbol{\Phi}(k)$, improving fault characterization within the system.

These adaptive techniques constitute the self-tuning framework of the Kalman filter, enabling a more accurate representation of system dynamics and enhancing filtering performance.

B. Extended Kalman Filter

The EKF is preferred over general or linear Kalman filters because it can handle nonlinearities with second-order accuracy in the system approximation. The first step is to linearize the system around a certain operating point, resulting in the following representation of the nonlinear system:

$$\boldsymbol{x}(k) = \boldsymbol{F}(k)\boldsymbol{x}(k-1) + \boldsymbol{E}(k) + \boldsymbol{\Phi}(k)\boldsymbol{\theta}(k) + \boldsymbol{w}(k). \tag{3}$$

Here, $\boldsymbol{F}(k) \in \mathbb{R}^{n\times n}$ and $\boldsymbol{E}(k) \in \mathbb{R}^n$ represent the linearized terms of the system. In EKF implementation, linearization is performed around the a posteriori state estimate from the previous iteration, $\hat{\boldsymbol{x}}(k-1|k-1)$, using the first-order Taylor series expansion:

$$\boldsymbol{F}(k) = \boldsymbol{A} + \left.\frac{\partial \boldsymbol{f}(\boldsymbol{x}(k-1), \boldsymbol{u}(k))}{\partial \boldsymbol{x}(k-1)}\right|_{\hat{\boldsymbol{x}}(k-1|k-1)} \tag{4}$$

$$\begin{aligned}\boldsymbol{E}(k) = \boldsymbol{f}(\hat{\boldsymbol{x}}(k-1|k-1), \boldsymbol{u}(k)) \\ - (\boldsymbol{F}(k) - \boldsymbol{A})\hat{\boldsymbol{x}}(k-1|k-1).\end{aligned} \tag{5}$$

Obtaining the Jacobian matrix in (4)–(5) can be challenging. In cases where the system exhibits quasi-steady-state behavior, as in certain electric power systems [10], Holt's linear exponential smoothing can be employed for approximation as follows:

$$\boldsymbol{F}(k) = \boldsymbol{A} + \alpha(1+\beta)\boldsymbol{I}_l \tag{6}$$

$$\begin{aligned}\boldsymbol{E}(k) = (1-\alpha)(1+\beta)\hat{\boldsymbol{x}}(k|k-1) - \beta\boldsymbol{a}(k-1) \\ + (1-\beta)\boldsymbol{b}(k-1)\end{aligned} \tag{7}$$

$$\boldsymbol{a}(k) = \alpha\hat{\boldsymbol{x}}(k|k) + (1-\alpha)\hat{\boldsymbol{x}}(k|k-1) \tag{8}$$

$$\boldsymbol{b}(k) = \beta(\boldsymbol{a}(k) - \boldsymbol{a}(k-1)) + (1-\beta)\boldsymbol{b}(k-1). \tag{9}$$

This method incorporates both a priori and a posteriori estimates and utilizes two smoothing parameters, $\alpha, \beta \in [0, 1]$. Both approximation methods will be analyzed in this paper, as the investigated system, a subsea pump system, is considered a quasi-steady-state system.

The Kalman filter proposed in this work extends the approach in [11] to enable simultaneous state and fault estimation for nonlinear systems under process faults. This joint estimation method employs a recursive least squares (RLS) approach with an exponential forgetting factor for fault estimation, instead of the conventional random walk assumption. The recursive estimation of the a posteriori state covariance matrix, $\boldsymbol{P}(k|k) \in \mathbb{R}^{n\times n}$, and the innovation covariance matrix, $\boldsymbol{\Sigma}(k) \in \mathbb{R}^{m\times m}$, follows:

$$\boldsymbol{P}(k|k-1) = \boldsymbol{F}(k)\boldsymbol{P}(k-1|k-1)\boldsymbol{F}^{\intercal}(k) + \boldsymbol{Q}(k) \tag{10}$$

$$\boldsymbol{\Sigma}(k) = \boldsymbol{C}\boldsymbol{P}(k|k-1)\boldsymbol{C}^{\intercal} + \boldsymbol{R}(k) \tag{11}$$

$$\boldsymbol{K}(k) = \boldsymbol{P}(k|k-1)\boldsymbol{C}^{\intercal}\boldsymbol{\Sigma}^{-1}(k) \tag{12}$$

$$\boldsymbol{P}(k|k) = (\boldsymbol{I}_n - \boldsymbol{K}(k)\boldsymbol{C})\boldsymbol{P}(k|k-1) \tag{13}$$

where $\boldsymbol{K}(k) \in \mathbb{R}^{l\times m}$ is the Kalman gain for state estimation, obtained through the recursion algorithm. At this stage, the a priori prediction of the state and fault is performed based on the known model dynamics as follows:

$$\hat{\boldsymbol{\theta}}(k|k-1) = \hat{\boldsymbol{\theta}}(k-1|k-1) \tag{14}$$

$$\begin{aligned}\hat{\boldsymbol{x}}(k|k-1) = \boldsymbol{A}\hat{\boldsymbol{x}}(k-1|k-1) \\ + \boldsymbol{f}(\hat{\boldsymbol{x}}(k-1|k-1), \boldsymbol{u}(k)) \\ + \boldsymbol{\Phi}(k)\hat{\boldsymbol{\theta}}(k-1|k-1)\end{aligned} \tag{15}$$

The next step is to compute the fault estimation gain matrix, $\boldsymbol{\Gamma}(k) \in \mathbb{R}^{p\times m}$, which is obtained through another recursive computation based on the forgetting factor, $\lambda \in (0, 1)$, as follows:

$$\boldsymbol{\Omega}(k) = \boldsymbol{C}\boldsymbol{F}(k)\boldsymbol{\Upsilon}(k-1) + \boldsymbol{C}\boldsymbol{\Phi}(k) \tag{16}$$

$$\begin{aligned}\boldsymbol{\Upsilon}(k) = (\boldsymbol{I}_n - \boldsymbol{K}(k)\boldsymbol{C})\boldsymbol{F}(k)\boldsymbol{\Upsilon}(k-1) \\ + (\boldsymbol{I}_n - \boldsymbol{K}(k)\boldsymbol{C})\boldsymbol{\Phi}(k)\end{aligned} \tag{17}$$

$$\boldsymbol{\Lambda}(k) = (\lambda\boldsymbol{\Sigma}(k) + \boldsymbol{\Omega}(k)\boldsymbol{S}(k-1)\boldsymbol{\Omega}^{\intercal}(k))^{-1} \tag{18}$$

$$\boldsymbol{\Gamma}(k) = \boldsymbol{S}(k-1)\boldsymbol{\Omega}^{\intercal}(k)\boldsymbol{\Lambda}(k) \tag{19}$$

$$\begin{aligned}\boldsymbol{S}(k) = \lambda^{-1}(\boldsymbol{S}(k-1) \\ - \boldsymbol{S}(k-1)\boldsymbol{\Omega}^{\intercal}(k)\boldsymbol{\Lambda}(k)\boldsymbol{\Omega}(k)\boldsymbol{S}(k-1))\end{aligned} \tag{20}$$

where $\boldsymbol{\Omega}(k) \in \mathbb{R}^{m\times p}$, $\boldsymbol{\Upsilon}(k) \in \mathbb{R}^{n\times p}$, and $\boldsymbol{S}(k) \in \mathbb{R}^{p\times p}$ are auxiliary matrices, while $\boldsymbol{\Lambda}(k) \in \mathbb{R}^{m\times m}$ is a corrective matrix required for the RLS estimation. The innovation of the estimation is calculated as:

$$\tilde{\boldsymbol{y}}(k) = \boldsymbol{y}(k) - \boldsymbol{C}\hat{\boldsymbol{x}}(k|k-1) \tag{21}$$

which represents the difference between the measured and predicted states. Finally, the last step of the algorithm is the update stage, where the a posteriori state and fault estimations are refined using the gain matrices:

$$\hat{\boldsymbol{\theta}}(k|k) = \hat{\boldsymbol{\theta}}(k|k-1) + \boldsymbol{\Gamma}(k)\tilde{\boldsymbol{y}}(k) \tag{22}$$

$$\begin{aligned}\hat{\boldsymbol{x}}(k|k) = \hat{\boldsymbol{x}}(k|k-1) + \boldsymbol{K}(k)\tilde{\boldsymbol{y}}(k) \\ + \boldsymbol{\Upsilon}(k)(\hat{\boldsymbol{\theta}}(k|k) - \hat{\boldsymbol{\theta}}(k|k-1)).\end{aligned} \tag{23}$$

C. Estimation Stability

This section focuses on establishing stability conditions to ensure that the error dynamics of the estimation, which will be defined later, remain exponentially stable at the origin. To support the stability analysis, the following assumptions regarding the developed Kalman filter method are introduced:

Assumption 1. *The matrix pair* $[\boldsymbol{F}(k), \boldsymbol{Q}^{0.5}(k)]$ *satisfies the condition of uniform complete controllability, and the pair* $[\boldsymbol{F}(k), \boldsymbol{C}]$ *is uniformly completely observable.*

Assumption 2. *The persistence of excitation (PE) condition is defined as:*

$$\sum_{j=k}^{k+N-1} \boldsymbol{\Omega}^{\intercal}(j)\boldsymbol{\Sigma}^{-1}(j)\boldsymbol{\Omega}(j) \geq \sigma\boldsymbol{I}_p \tag{24}$$

and is satisfied with $\sigma > 0$ *for an iteration window of* $N > 0$.

As mentioned, the stability proof follows from the error dynamics of the state and fault estimations, which are defined by the following difference equation:

$$\tilde{\boldsymbol{x}}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k|k) \tag{25}$$

$$\tilde{\boldsymbol{\theta}}(k) = \boldsymbol{\theta}(k) - \hat{\boldsymbol{\theta}}(k|k) \tag{26}$$

The stability of the joint estimation is formalized in the following theorem:

Theorem 1. *Consider the error dynamics defined in* (25)–(26). *If Assumptions 1 and 2 hold. hold, then the expected value evolution of the error dynamics follows:*

$$\lim_{k\to\infty} \mathbb{E}(\tilde{\boldsymbol{x}}(k)) = 0 \tag{27}$$

$$\lim_{k\to\infty} \mathbb{E}(\tilde{\boldsymbol{\theta}}(k)) = 0 \tag{28}$$

exponentially.

Proof. Assumption 1 ensures the boundedness of all matrices involved in the recursive computations, while Assumption 2 provides the classical condition for fault estimation in system identification methodologies such as Kalman filter. Since the structure of the proposed estimation algorithm follows a similar formulation, the proof under these assumptions can be derived by following Proposition 3 in [11]. □

III. Adjustment to Adaptive Filtering

This section derives the adaptive methodology for self-tuning Kalman filter parameters, such as the noise covariance and fault profile matrices. It provides the motivation, derivation, and theoretical justifications for their implementation within the self-tuning Kalman filter framework.

A. Noise Covariance Matrix

This subsection discusses the adaptive estimation of the noise covariance matrices $\boldsymbol{Q}(k)$ and $\boldsymbol{R}(k)$ for systems subject to process faults. The primary motivation for adaptively estimating these matrices is to improve the accuracy of joint state and fault estimation. Prior research has primarily focused on adaptive filtering for standard systems, without explicitly addressing systems influenced by faults, despite the significant impact of faults on filtering performance. Typically, noise covariance is determined through observations of system operation and sensor data, however, this approach is often challenging and may not account for variations due to changing operating conditions. Furthermore, once obtained, a fixed noise covariance does not adapt to dynamic changes in the system. Another reason for implementing adaptivity is its role in as a "stabilizing noise", leading to more consistent estimation, as noted in [12]. However, in highly nonlinear systems, estimation bias can still persist. Additionally, adaptive noise modeling can aid in preserving the PE condition required for accurate estimation by introducing controlled random perturbations, as discussed in [13].

The process noise, $\boldsymbol{Q}(k)$, can be characterized by analyzing (3) and is expressed as follows:

$$\boldsymbol{w}(k) = \boldsymbol{x}(k) - (\boldsymbol{F}(k)\boldsymbol{x}(k-1) - \boldsymbol{E}(k) + \boldsymbol{\Phi}(k)\boldsymbol{\theta}(k)). \tag{29}$$

Similarly, the discrepancy between the predicted a priori and the updated a posteriori state estimates provides an indication of the estimated noise:

$$\begin{aligned}\hat{\boldsymbol{w}}(k) &= \hat{\boldsymbol{x}}(k|k) - \hat{\boldsymbol{x}}(k|k-1) \\ &= \boldsymbol{K}(k)\tilde{\boldsymbol{y}}(k) + \boldsymbol{\Upsilon}(k)(\hat{\boldsymbol{\theta}}(k|k) - \hat{\boldsymbol{\theta}}(k|k-1)).\end{aligned} \tag{30}$$

As observed, the correction term in the estimation process acts as a noise compensation mechanism, demonstrating the validity of this estimation approach. Referring to (21), the equation can be rewritten as:

$$\hat{\boldsymbol{w}}(k) = \Xi(k)\tilde{\boldsymbol{y}}(k) \tag{31}$$

where $\Xi(k) = \boldsymbol{K}(k) + \boldsymbol{\Upsilon}(k)\boldsymbol{\Gamma}(k)$ for brevity.

Given the estimated process noise, its noise covariance matrix can now be expressed as follows:

$$\begin{aligned}\hat{\boldsymbol{Q}}(k) &= \mathbb{E}(\hat{\boldsymbol{w}}(k)\hat{\boldsymbol{w}}^\intercal(k)) \\ &= \Xi(k)\mathbb{E}(\tilde{\boldsymbol{y}}(k)\tilde{\boldsymbol{y}}^\intercal(k))\Xi^\intercal(k).\end{aligned} \tag{32}$$

The expression in (32) can be reformulated as:

$$\hat{\boldsymbol{Q}}(k) = \Xi(k)\boldsymbol{\Sigma}(k)\Xi^\intercal(k). \tag{33}$$

This formulation is often referred to as the innovation-based adaptive method [5], but here, it is specifically tailored for fault estimation. However, direct computation of (32) at each iteration is impractical for real-time applications. Instead, an exponential moving average (EMA) is used to iteratively estimate the process noise covariance adaptively, as follows [14]:

$$\boldsymbol{Q}(k) = \delta\boldsymbol{Q}(k-1) + (1-\delta)\Xi(k)\tilde{\boldsymbol{y}}(k)\tilde{\boldsymbol{y}}^\intercal(k)\Xi^\intercal(k) \tag{34}$$

where $\delta \in (0, 1)$ is the forgetting factor.

To adaptively adjust the measurement noise covariance $\boldsymbol{R}(k)$, we also utilize an innovation-based adaptive approach specifically tailored for fault estimation scenario. Intuitively, similar to the previous method, the measurement noise can be estimated as the residual between the posteriori estimate and the actual measurement, as follows:

$$\hat{\boldsymbol{v}}(k) = \tilde{\boldsymbol{y}}(k) - \boldsymbol{C}\hat{\boldsymbol{x}}(k|k). \tag{35}$$

Substituting the relationships derived in (22)–(23) into (35) yields:

$$\hat{\boldsymbol{v}}(k) = (\boldsymbol{I}_m - \boldsymbol{C}\Xi(k))\tilde{\boldsymbol{y}}(k). \tag{36}$$

Taking the covariance of the estimated measurement noise results in:

$$\hat{\boldsymbol{R}}(k) = (\boldsymbol{I}_m - \boldsymbol{C}\Xi(k))\mathbb{E}(\tilde{\boldsymbol{y}}(k)\tilde{\boldsymbol{y}}^\intercal(k))(\boldsymbol{I}_m - \boldsymbol{C}\Xi(k))^\intercal. \tag{37}$$

For a more practical and smoother adjustment, an EMA is employed again for estimating the measurement noise covariance with a forgetting factor $\varepsilon \in (0, 1)$, as follows:

$$\boldsymbol{R}(k) = \varepsilon\boldsymbol{R}(k-1) + (1-\varepsilon)\boldsymbol{\Psi}(k)\tilde{\boldsymbol{y}}(k)\tilde{\boldsymbol{y}}^\intercal(k)\boldsymbol{\Psi}(k)^\intercal \tag{38}$$

where the weighting matrix $\boldsymbol{\Psi}(k) = (\boldsymbol{I}_m - \boldsymbol{C}\Xi(k))$.

To ensure the validity of this adaptive noise covariance estimations, the following theorem establishes the boundedness of the resulting matrix:

Theorem 2. *If $\boldsymbol{Q}(0)$ is symmetric positive semidefinite and $\boldsymbol{R}(0)$ is symmetric positive definite, then the adaptive update expression in (34) and (38) ensures that $\boldsymbol{Q}(k)$ remains symmetric positive semidefinite and $\boldsymbol{R}(k)$ remains symmetric positive definite and upper-bounded for all k.*

Proof. To prove this theorem, we roll forward the dynamics of the adaptive expression in (34), which results in:

$$\begin{aligned}\boldsymbol{Q}(k) &= \delta^k\boldsymbol{Q}(0) \\ &\quad + (1-\delta)\sum_{j=1}^{k}\delta^{k-j}\Xi(j)\tilde{\boldsymbol{y}}(j)\tilde{\boldsymbol{y}}^\intercal(j)\Xi^\intercal(j).\end{aligned} \tag{39}$$

It is straightforward to observe that the right-hand side (RHS) of (39) always yields a symmetric positive semidefinite matrix, given that $\boldsymbol{Q}(0)$ is initialized accordingly and that $0 < \delta < 1$. Furthermore, since the term $\Xi(j)\tilde{\boldsymbol{y}}(j)\tilde{\boldsymbol{y}}^\intercal(j)\Xi^\intercal(j)$ is symmetric and positive semidefinite, the updated noise covariance matrix preserves these properties. The boundedness of $\boldsymbol{Q}(k)$ follows from the exponential stability of the estimation process, as established in Theorem 1. Additionally, Proposition 1 in [11] has demonstrated the boundedness of $\Xi(j)$, ensuring that $\boldsymbol{Q}(k)$ has a finite upper bound. A similar argument applies to $\boldsymbol{R}(k)$, given that $\boldsymbol{R}(0)$ is initialized as a symmetric positive definite matrix and $0 < \varepsilon < 1$ ☐

B. Fault Profile Matrix

The motivation for adaptively estimating the fault profile matrix $\boldsymbol{\Phi}(k)$ lies in its critical role in enhancing joint state and fault estimation for systems affected by process faults. Defining an accurate fault profile matrix is inherently challenging, particularly in the absence of prior knowledge regarding the fault characteristics. Even when the nature of the fault is known, modeling it precisely remains a complex task. By employing an adaptive estimation approach, the proposed methodology seeks to improve the robustness and reliability of fault characterization within the system. Another advantage of adaptively updating this matrix throughout the estimation iterations is that it helps maintain the PE, as $\boldsymbol{\Phi}(k)$ significantly contributes to the term in (24), providing sufficient signals for accurate estimation. Typically, the fault profile matrix is assumed to represent actuator gain losses, and is defined as:

$$\boldsymbol{\Phi}(k) = -\boldsymbol{B}\text{diag}(\boldsymbol{u}(k)). \tag{40}$$

Here, $\boldsymbol{B} \in \mathbb{R}^{n\times s}$ is the input matrix for linear association. However, this assumption holds only when $s = p$ which is not always the case, particularly in systems like the subsea pump system modeled in the following section. As such, the assumption also may not adequately reflect the main source of the fault. To address this, we adaptively update the fault profile matrix using a scaling algorithm inspired by the work in [15], as follows:

$$\mathbf{\Phi}(k) = \mathbf{\Phi}(k-1)\sqrt{\gamma(k)}. \tag{41}$$

Here, $\gamma \in \mathbb{R}$ is the scaling factor, and the square root provides a smoothing effect for the adjustment.

This adjustment is based on the estimation of the fault, focusing on obtaining the most accurate corrective matrix for the RLS estimator, as shown below:

$$\mathbf{\Lambda}^{-1}(k) = \lambda \mathbf{\Sigma}(k) + \mathbf{\Omega}(k)\boldsymbol{S}(k-1)\mathbf{\Omega}^{\intercal}(k). \tag{42}$$

The prediction of $\hat{\mathbf{\Omega}}(k)$ contributes to ensuring that the predicted corrective matrix closely matches the actual one obtained through the recursive update process. Consequently, the scaling factor is chosen as:

$$\gamma(k) = \frac{\mathrm{tr}(\hat{\mathbf{\Omega}}(k)\boldsymbol{S}(k-1)\hat{\mathbf{\Omega}}^{\intercal}(k))}{\mathrm{tr}(\mathbf{\Omega}(k)\boldsymbol{S}(k-1)\mathbf{\Omega}^{\intercal}(k))}. \tag{43}$$

The numerator in (43) can be estimated by referring to (42) as follows:

$$\gamma(k) = \frac{\mathrm{tr}(\mathbf{\Lambda}^{-1}(k) - \frac{\lambda}{M}\sum_{j=0}^{M-1}\tilde{\boldsymbol{y}}(k-j)\tilde{\boldsymbol{y}}^{\intercal}(k-j))}{\mathrm{tr}(\mathbf{\Omega}(k)\boldsymbol{S}(k-1)\mathbf{\Omega}^{\intercal}(k))}. \tag{44}$$

In this expression, M is the moving window of the expected value calculation. However, directly calculating the expected value in (44) is impractical, and directly estimating $\hat{\mathbf{\Omega}}(k)$ in (43) may cause inconsistencies in the adaptive filtering process. Since the goal is to adaptively update the fault profile matrix, the prediction of $\hat{\mathbf{\Phi}}(k)$ could becomes critical when determining the scaling factor for adjustment. Based on (16), the fault profile matrix has the following impact on the scaling factor:

$$\hat{\mathbf{\Omega}}(k) = \boldsymbol{C}\boldsymbol{F}(k)\mathbf{\Upsilon}(k-1) + \boldsymbol{C}\hat{\mathbf{\Phi}}(k). \tag{45}$$

The estimation of $\hat{\mathbf{\Phi}}(k)$ can be derived from (16)–(17), assuming that the recursive filtering process in the Kalman filter has reached an optimal steady-state gain. However, this requires that $(\boldsymbol{I}_n - \boldsymbol{K}(k)\boldsymbol{C})$ remains non-singular, ensuring numerical stability in the adaptive update process.

IV. Result and Validation

This section demonstrates the capabilities and validates the efficacy of the proposed self-tuning Kalman filter method. We begin by describing the model used as the platform for the study, which is a submersible injection pump in a subsea seawater injection system. Then, we present numerical simulations based on this model and analyze the performance of the proposed methodology.

A. Submersible Injection Pump

The subsea pump system, in the form of a submersible injection pump, is part of the subsea seawater injection system. It is responsible for injecting seawater to maintain water pressure within the desired range and enhance oil recovery from the reservoir. The model is illustrated in Fig. 1. The seawater enters the system at the suction pump at pressure $p_1(k)$, which is then pumped to a specific head $h(k)$, determined by the designed rotational speed of the pump, $\omega(k)$. The discharge of the pump results in pressure $p_2(k)$, which acts as the upstream of the venturi tube, facilitating the calculation of the volumetric flow rate $q(k)$ through the differential pressure to the downstream $p_3(k)$. The system sensors include pressure transmitters for each state of pressure difference.

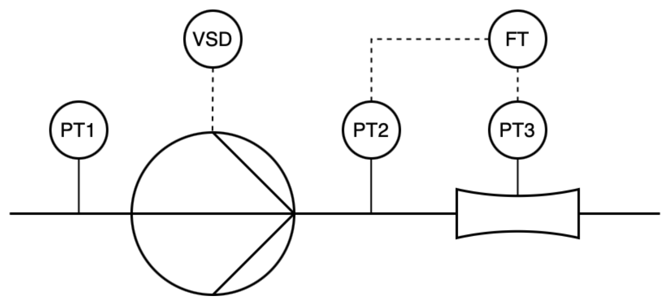

Fig. 1. Simple schematic of the submersible injection pump.

The model used for this system is based on the work in [16], assuming a quasi-steady state ($x(k) \approx x(k-1)$) for the pump dynamics and flow rate. Additionally, the suction pump pressure and downstream venturi pressure are assumed to drop at a constant rate. The system, without fault, is modeled as follows:

$$p_1(k) = p_1(k-1) + (\Delta t)\phi_p + w_1(k) \tag{46}$$

$$p_2(k) = p_1(k) + \rho g h(k) + w_2(k) \tag{47}$$

$$p_3(k) = p_3(k-1) + (\Delta t)\phi_p + w_3(k) \tag{48}$$

$$q(k) = c_v\sqrt{\frac{p_2(k) - p_3(k)}{\rho}} + w_4(k). \tag{49}$$

Here, the system states are defined, with the head generated by the pump following the approximated pump curve:

$$h(k) = \phi_h^{(0)}\omega(k) + \phi_h^{(1)}q(k)\omega(k) + \phi_h^{(2)}q^2(k). \tag{50}$$

In these equations, the parameters are defined as follows: ϕ_p denotes the pressure drop, ρ is the density of the seawater (assumed incompressible), g is the gravitational acceleration, c_v is the constant for the venturi tube, and $\phi_h^{(0)}$, $\phi_h^{(1)}$, and $\phi_h^{(2)}$ are the coefficients of the pump-curve approximation. As there are three pressure transmitters, the measurement matrix is defined as $\boldsymbol{C} = \boldsymbol{I}_4$.

B. Numerical Simulation

To validate the proposed self-tuning Kalman filter, we conduct numerical simulations on the submersible injection pump. The system parameters are defined as follows: $\phi_p = -0.5$ bar, $\rho = 1020$ kgm^{-3}, $g = 9.8$ ms^{-2}, $c_v = 30.875$ $\mathrm{m}^3\mathrm{h}^{-1}$, $\phi_h^{(0)} = 3.51 \times 10^{-5}$, $\phi_h^{(1)} = 3.29 \times 10^{-4}$, and $\phi_h^{(2)} = -0.01$. For the estimation methodology, the following parameters are used: the Holt's approximation constants $\alpha = 0.1$ and $\beta = 0.7$, the forgetting factors $\lambda, \delta, \varepsilon = 0.999$, the noise covariance initializations $\boldsymbol{Q}(0) = 0.001\boldsymbol{I}_4$ and $\boldsymbol{R}(0) = 0.1\boldsymbol{I}_4$, and the fault profile matrix initialization $\boldsymbol{\Phi}(k) = \boldsymbol{I}_4$.

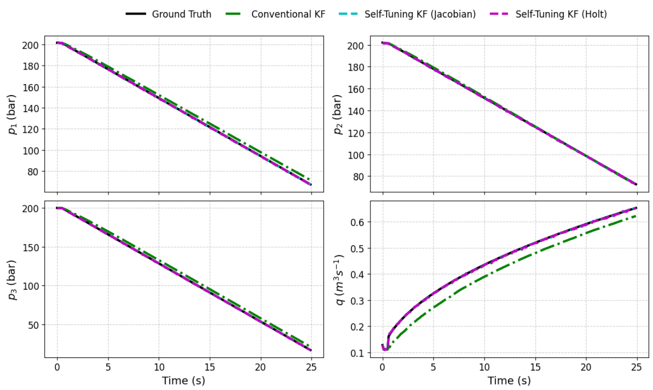

Fig. 2. Estimation of pump's state using conventional and self-tuning Kalman filter.

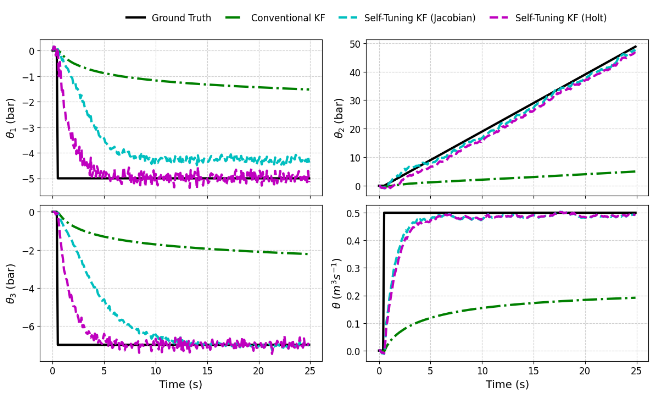

Fig. 3. Fault estimation of the pump using conventional and self-tuning Kalman filter.

For state estimation, the adaptive approaches (Jacobian and Holt approximations) outperform the conventional method, with marginal improvement observed using the Jacobian approximation. However, in fault estimation, the adaptive approach shows a clear advantage over the conventional Kalman filter. Furthermore, Holt's approximation achieves better convergence than the Jacobian-based method. While Holt's constants were carefully chosen for high performance in this simulation, tuning them experimentally may be challenging. Future work could explore an adaptive adjustment for these constants. Nevertheless, Holt's approximation presents a viable alternative when the Jacobian form of the system is difficult to derive and the system is not highly dynamic, such as in the case of this subsea pump system. One notable observation is that fault estimation results exhibit some noise, likely due to the innovation covariance matching approach, which relies on noisy measurements. The overall performance comparison of each estimation methodology is further quantified in Table I, which presents the root mean square errors (RMSEs) of the estimations.

TABLE I
RMSEs OF STOCHASTIC JOINT STATE AND FAULT ESTIMATION BY KALMAN.

Algorithm	p_1 (bar)	p_2 (bar)	p_3 (bar)	q ($\mathrm{m}^3\mathrm{s}^{-1}$)
		State Estimation		
Conventional	3.07	0.75	3.91	4.32×10^{-2}
Self-Tuning (Jacobian)	0.30	0.30	0.29	2.02×10^{-3}
Self-Tuning (Holt)	0.31	0.30	0.30	1.98×10^{-3}
		Fault Estimation		
Conventional	3.81	24.98	5.25	0.35
Self-Tuning (Jacobian)	1.57	1.15	2.04	0.07
Self-Tuning (Holt)	0.75	2.10	1.04	0.08

To further validate the efficacy of the proposed methodology, we conducted a Monte Carlo simulation by varying the initial values of the noise covariance matrices $\boldsymbol{Q}(0) = c_1\boldsymbol{I}_4$ and $\boldsymbol{R}(0) = c_2\boldsymbol{I}_4$ where the constants were varied in the range $c_1, c_2 \in [10^{-5}, 10]$. Values beyond this range were deemed unrealistic that could lead to backflow, which is not feasible for (49). The results, presented in Fig. 4, illustrate the norm of the state and fault vectors from both the actual system and the self-tuning Kalman filter estimates using the Jacobian approximation. These results demonstrate the accuracy and robustness of the proposed methodology.

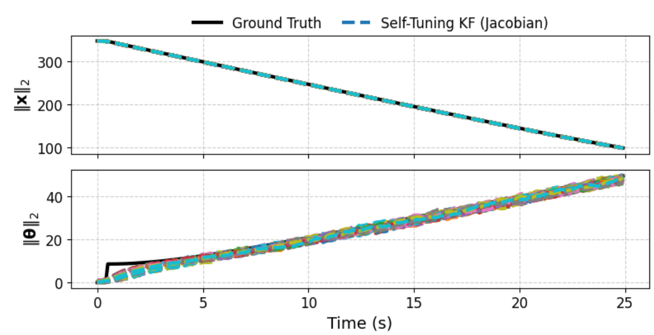

Fig. 4. Robustness test of the self-tuning Kalman filter methodology.

V. Conclusion

Numerous studies have explored adaptive measures for dynamic state estimation (DSE) to achieve a more accurate representation of system dynamics, thereby enhancing estimation performance. However, most existing research has not focused on systems affected by faults. In this paper, we proposed a self-tuning Kalman filter designed for joint state and fault estimation, incorporating an exponential forgetting factor to enhance estimation accuracy. Additionally, we introduced an adaptive adjustment approach to refine the estimation of both the noise covariance matrices and the fault profile matrix through innovation covariance matching and a scaling process, respectively. The numerical simulation on a subsea pump system demonstrated the superior performance of the self-tuning Kalman filter in both state and fault estimation, as well as its robustness against variations in initial conditions. Future research will focus on extending the methodology to estimate faults in the measurement process, introducing condition monitoring for a more comprehensive fault diagnosis, addressing the resilience control framework to mitigate the impact of faults on the system, and implementing the proposed approach in real-world experimental setups.

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Data Availability

To support this paper, we provide the code used, available in the GitHub repository at the following link: `https://github.com/hilmi-ica/AEKF-Pump.git`.

AI-Assisted Writing Disclosure

During the preparation of this work, the authors used GPT-4o to enhance readability and language. The authors reviewed and checked the content carefully. The use of AI did not extend to generating any aspect of the research itself.

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