MATHEMATICS MAA EXPLORATION

DECLARATION OF AUTHENTICITY

Session Number: 000592-0069

Title: Investigating the best distance between a pair of speakers to be put in a large room mathematically

With this I declare that this work is my own.

………………………………………………………

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Criterion	**Achievement level**						
Criterion A: Communication – assesses the organization and coherence of the exploration	0	1	2	3	4		
Criterion B: Mathematical presentation – assesses to what extent the student is able to use appropriate math language and multiple forms of math representation including the use of appropriate ICT tools	0	1	2	3	4		
Criterion C: Personal engagement – assesses the extent to which the student engages with the exploration and makes it his or her own	0	1	2	3			
Criterion D: Reflection – assesses how the student reviews, analyses and evaluates the exploration	0	1	2	3			
Criterion E: Use of Mathematics – assesses to what extent and how well students use mathematics in the exploration	0	1	2	3	4	5	6
TOTAL MARKS	15 / 20						

A - clear and well organised

B - consistennt math communication

C - significant personal engagement - running experiment

D - eviednece of meaningful reflection - comparing differrent approach

E - HL level and good understanding

<u>**Investigating the best distance between a pair of speakers to be put in a large room mathematically**</u>

1. INTRODUCTION

aim

This investigation aims to find the most preferable distance between a pair of speakers, specifically wireless ones, to be set up in a large room mathematically. The topic is of significance because we need to find the best distance of a pair of speakers so that every single member of the audience can experience the best quality of sound and music, especially when the audience is very large.

It is very significant because in any schools in Malaysia, school prefects are responsible to place the school's wireless speaker at the best place in a few large rooms. Some rooms at school are not equipped with a built sound system, therefore wireless speakers are used. Sometimes, when they do place the wireless speakers at the usual place, some of the members in the audience complained about the poor quality of the sound system. This is because of a few main factors. Firstly, it is because they were too far from the audio source. Other than that, audience members who were affected were positioned at the very side of the room. Next, there were also audience members who complained that they did not hear the bass of the melody as well as some listeners at other positions. It was difficult for the teachers and other students to teach or give information through the audio system during that time. Audience members were also affected, and it can be dangerous if the audio system was not at the best quality for listening practices and tests.

These people did have the absolute difficulty to understand the true best distance to set up the wireless speakers in a large room, such as the school's main hall and the main auditorium. This is because the teachers did not point out the best location for the speakers to be set up. Therefore, the best position for a pair of wireless speakers to be set up in a large room of any dimension is then determined to cover most of the area of spectators in the said large room.

2. EXPERIMENTAL METHODOLOGY

In this part of the mathematical exploration, the area of coverage of the aforementioned wireless speakers is observed by calculating the area in an actual large hall. The place that is chosen is the Dewan Orang Ramai Bukit Changgang near the college. The venue is chosen because it is easy to go there on the weekends when there are no classes to attend to so that I can pursue with my experiment.

personal engagement

The length and the width of the large hall is firstly measured to figure out the maximum area of coverage that can be reached by the wireless speakers. The wireless speakers that are used for this mathematical exploration are a pair of the Jumia RS-8880 Bluetooth Loudspeakers (see Figure 1). It is determined that the dimensions of the hall are 12.00×20.45 metres. This implies that the assumed area of the hall is approximately $254.4\ m^2$. This area value however is not the actual area of the hall because there were a lot of inaccuracies during the measurement of the hall. For example, measuring a very long distance may result in the unaligned therefore this is just an assumed area of the hall.

Figure 1: Jumia RS-8880 Bluetooth Loudspeakers[1]

Next, a few sets of measurements are then taken on the alignment of the width of the hall according to the planned measurements. Figure 2 shows the planned graphical representation of Dewan Besar Bukit Changgang. Figure 3 shows the zoomed out version of Figure 2, focusing towards the 'X' points with their respective coordinates.

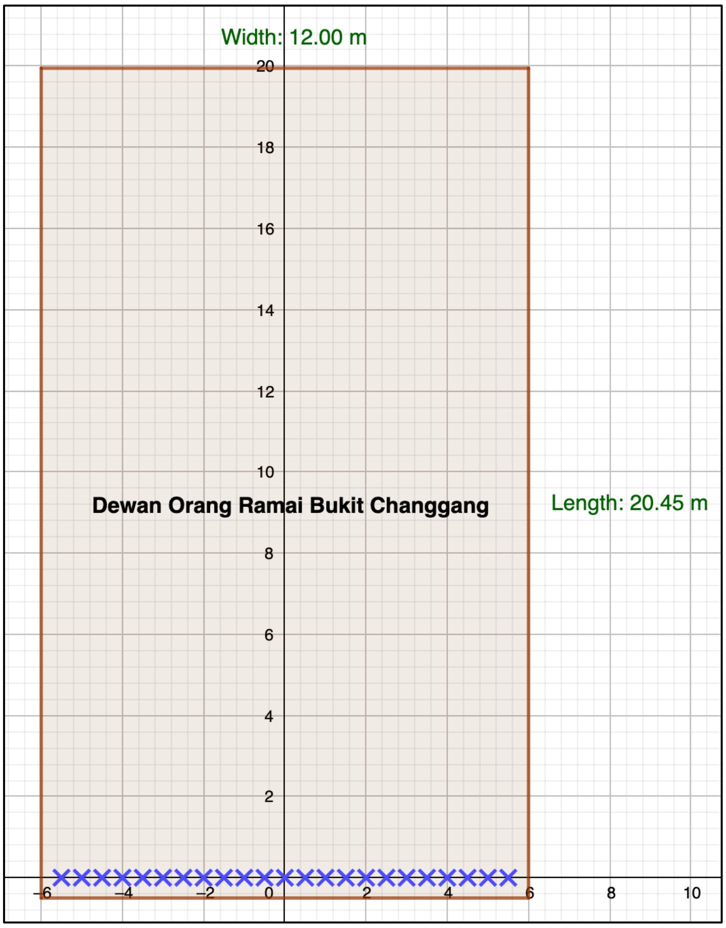

Figure 2: Planned graphical representation of Dewan Orang Ramai Bukit Changgang

[1] 'Classic Mic Supporting Bluetooth Loudspeaker Rs-8880 Price from Jumia in Nigeria - Yaoota!' <https://yaoota.com/en-ng/product/classic-mic-supporting-bluetooth-loudspeaker-rs-8880-price-from-jumia-nigeria> [accessed 16 September 2020].

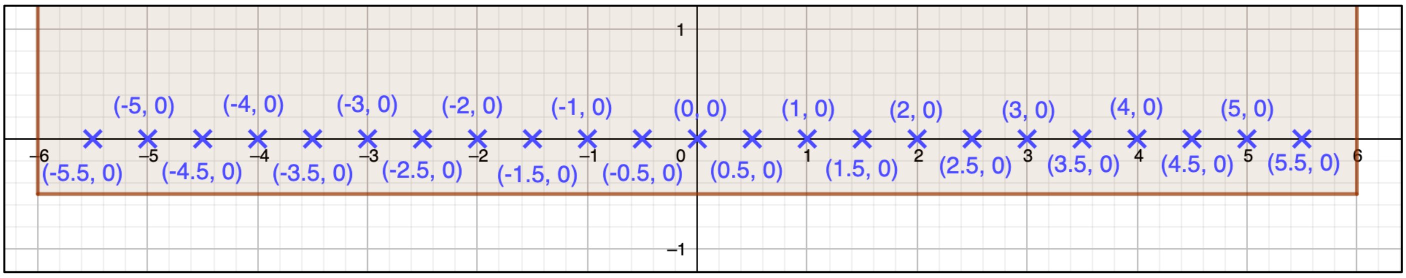

Figure 3: Marked x-coordinates, x_0 for the experiment.

From Figure 3, the centre of the hall is marked as the reference point for the experiment with coordinates $(0, 0)$. Then, 0.5 metres is then measured from the reference point. The left side of the measurement of 0.5 metres is considered as coordinates $(-0.5, 0)$ and the right side will be considered as coordinates $(0.5, 0)$. It is done continuously until it is measured until -5.5 metres and 5.5 metres. These measurements are considered as x-coordinates, x_0, $-5.5 \leq x_0 \leq 5.5$. These x-coordinates measurement will be used to measure the maximum position that can be heard by a person relative to their respective x-coordinate, p_{max} when distance of speakers, d_s, $0 \leq d_s \leq 11$. The distance of p_{max} will be assumed as the y-value of the x-coordinates that has been indicated at Figure 3 and it will be represented in the graphical representation of Dewan Orang Ramai Bukit Changgang as coordinates.

From the data, GeoGebra will be used to undergo nonlinear regression analysis to model the approximate area of coverage of speakers. The reason of using GeoGebra is because other mathematical simulation applications is difficult for the experimenter to undergo regression analysis for area modelling of the simulation. Figure 4 shows the simulation of the experiment in which the experiment begins with measuring the maximum position that can be heard by a person at their respective x-coordinate, p_{max}, when the distance between the wireless speakers, $d_s = 1$ metre. A podcast audio is played to simulate a noisy environment in the hall.

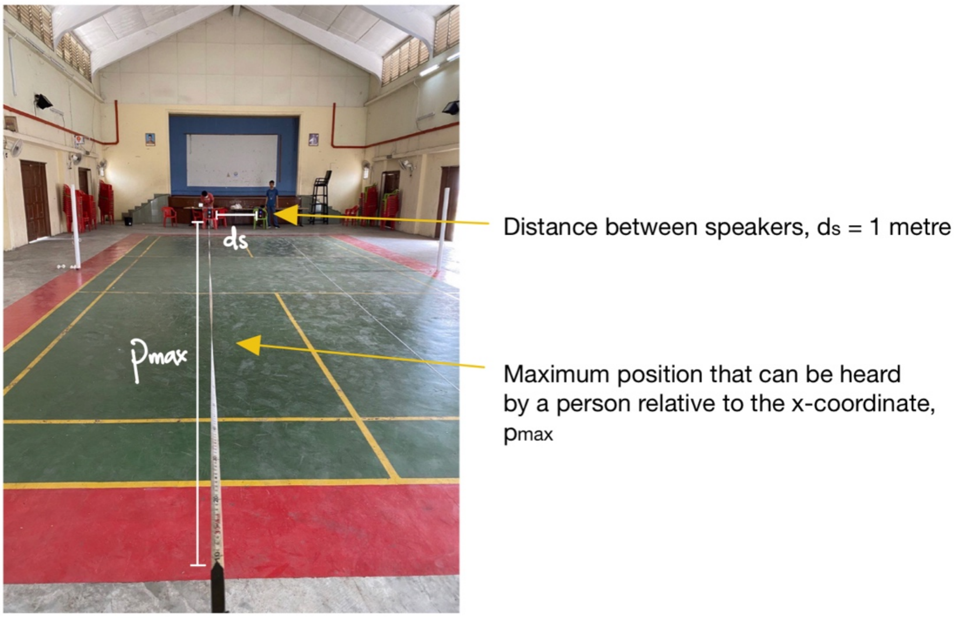

Figure 4: The simulation of the experiment

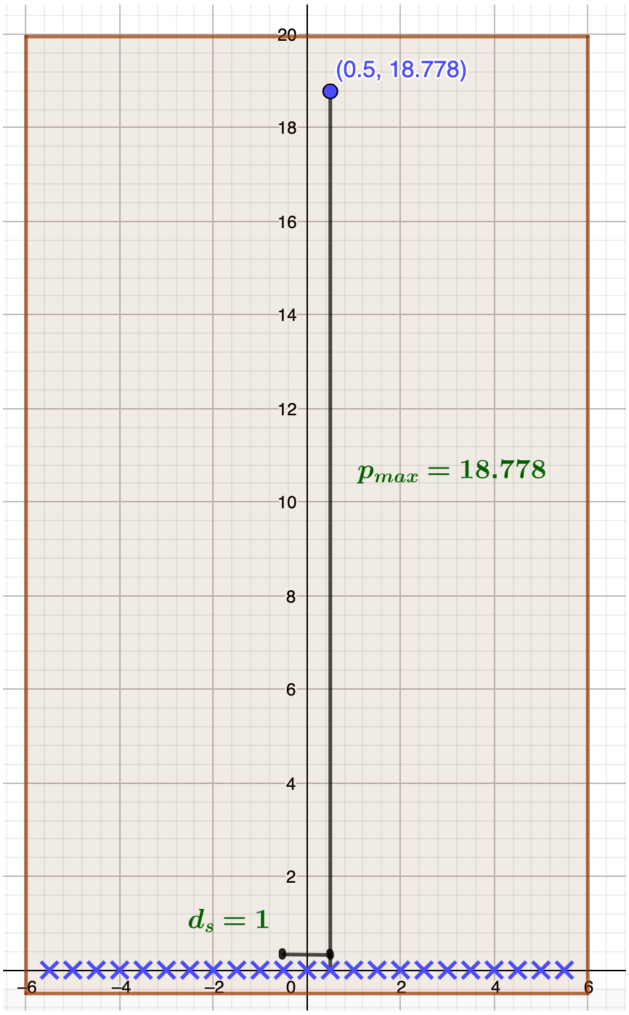

Figure 5: The coordinates (0.5, 18.778) means that the maximum position that can be heard by a person relative to the x-coordinate = 0.5, p_{max}, is 18.778 metres, when the distance between the pair of speakers, - d_s is 1 metre.

Then, maximum position that can be heard by a person relative to their respective x-coordinate, p_{max} when the distance between speakers, $d_s = 1$ metre is measured. In this methodology, due to limited time and resources, the data can only be collected once when in theory, the experiment should be repeated at least three times to determine the normality of the data.[2] In this case, the data collected in this experiment are not truly accurate and invalid therefore the data is assumed to be normal, which is one of the limitations of this experiment. This would influence the confidence in data collecting which will impact the outcome of the analysis of this experiment.

3. EXPERIMENTAL ANALYSIS

Next, the data of measurements (from Appendix 1) of the maximum position from the starting point that be heard by a person when the distance between speakers, $d_s = 1$, is plotted to a graph. Figure 6 shows the plotted data in a graph.

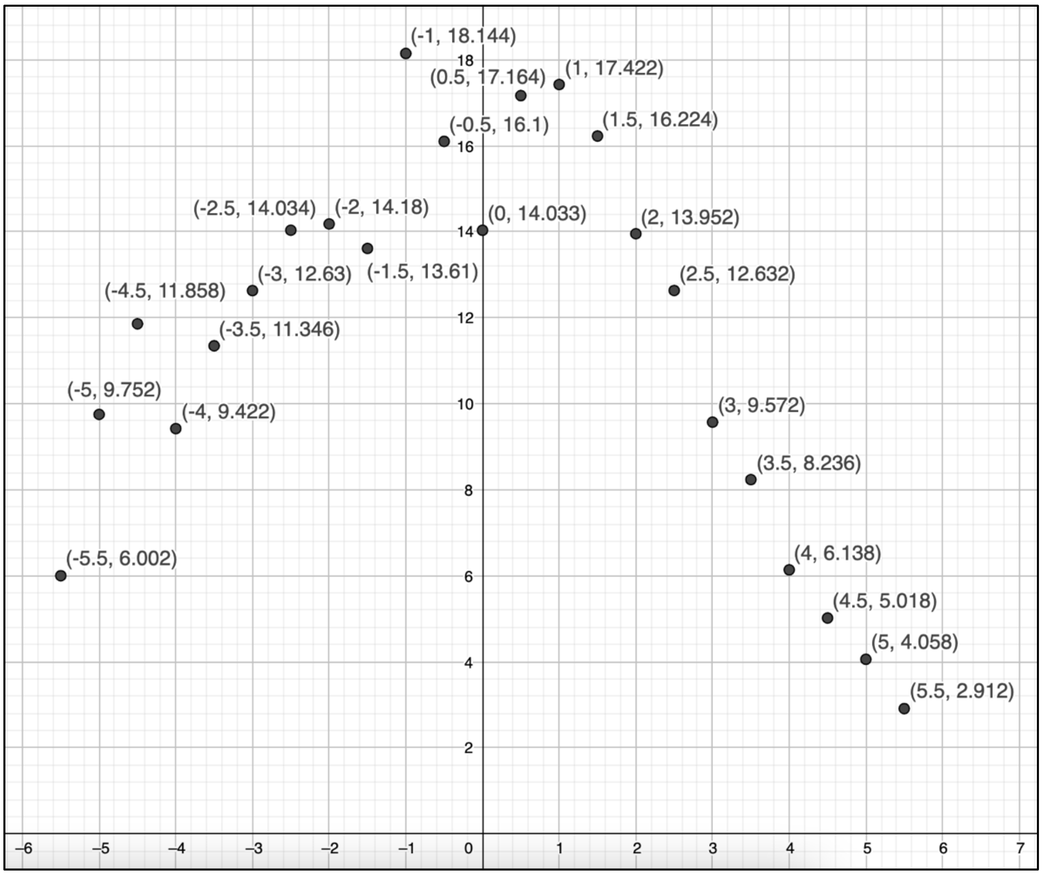

Figure 6: Graph plotted with data of maximum positions that can be heard from the speakers relative to its x-coordinate, p_{max}, when the distance between speakers, $d_s = 1$

It is then used to fit the suitable polynomial function which will depict the speaker coverage area. The data is fitted with polynomial functions with the largest coefficient of x of 7, 8 and 9 and the R^2 of the functions are compared to get the best fit of the polynomial function. The reason of choosing polynomial curves as the suitable curve to fit the non-linear data is that the data collected contains multiple turning or maximum points

[2] 'Increasing the Ability of an Experiment to Measure an Effect' <https://www.sciencebuddies.org/science-fair-projects/competitions/experimental-design-increasing-signal-to-noise> [accessed 23 January 2021].

which can be shown in Figure 6.[3] However, the polynomial models cannot provide the most precise shape of the area coverage since GeoGebra offers the polynomial model with the maximum degree of the model of 9. In this case with the 9th degree of the polynomial model, it can be assumed as the most accurate structure to imply the coverage area of the speakers.

Table 1: R^2 against the three possible best fit functions for the area of speaker coverage when the distance of speakers, $d_s = 1$ metre

Largest degree of polynomial	**Best fit curve**	**Coefficient of determination, R^2**
7	$q_1(x) = 0.00001x^7 - 0.00046x^6 + 0.00219x^5 + 0.02853x^4 - 0.10178x^3 - 0.84074x^2 + 0.39818\,x + 16.91264$	0.9430
8	$q_2(x) = -0.00009x^8 + 0.00001x^7 + 0.00498x^6 + 0.00219x^5 - 0.07127x^4 - 0.10178x^3 - 0.26588x^2 + 0.39818x + 16.40824$	0.9527
9	$q_3(x) = 0.00001x^9 - 0.00009x^8 - 0.00096x^7 + 0.00498x^6 + 0.02364x^5 - 0.07127x^4 - 0.27409x^3 - 0.26588x^2 + 0.76513x + 16.40824$	0.9544

reflection

Comparing the three R^2 of the respective polynomial curve, it is observed that $R^2\, for\, q_1(x) < R^2\, for\, q_2(x) < R^2\, for\, q_3(x)$ and the R^2 for $q_3(x)$ is seen to be approaching to 1 than the two polynomial fit curves. Therefore, it can be implied that the function $q_3(x)$ is the best fit for the polynomial function for the coverage of the wireless speakers when the distance between them, $d_s = 1$. After that, integral calculus is applied on the function $q_3(x)$ is used to approximate the area of speaker coverage. The boundary of x-value of the integral is between -5.5 and 5.5 as the maximum distance from reference point, x is 5.5 metres. From Figure 7, the area can be determined by using GeoGebra.

[3] '4.8.1.1. Polynomial Functions' <https://www.itl.nist.gov/div898/handbook/pmd/section8/pmd811.htm> [accessed 23 January 2021].

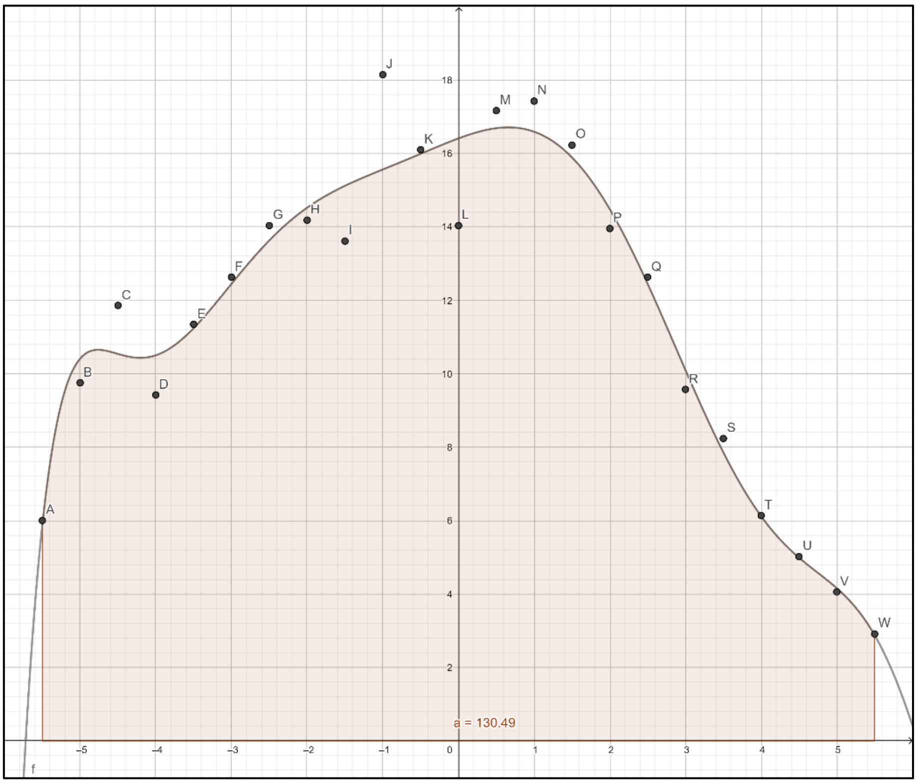

Figure 7: The integral of $q_3(x)$ from $x = -5.5$ to $x = -5.5$

$$\text{Area of speaker coverage} = \int_{5.5}^{-5.5} q_3(x)\,dx$$

$$= \int_{5.5}^{-5.5} (0.00001x^9 - 0.00009x^8 - 0.00096x^7 + 0.00498x^6 + 0.02364x^5 - 0.07127x^4 - 0.27409x^3 - 0.26588x^2 + 0.76513x + 16.40824)\,dx$$

$$\text{Area of speaker coverage} = 130.49\ \text{m}^2 \text{ or } 51.29\%$$

From this calculation above, it is shown that the area of speaker coverage when the distance between speakers, $d_s = 1$, is 130.49 m^2. By comparing with the area of the hall, the percentage of coverage for when the distance between speakers, d_s is 1 metre is 51.29%. This implies that more than half of the audience can hear the audio presented in front of the hall. However, it does not cover the whole audience therefore the distance between speakers could be altered to optimize the coverage percentage in the hall area.

The experiment is then repeated with the measure of distance between the pair of wireless speakers, $d_s = 2, 3, 4, \dots, 11$ metres (Refer Figure 4 for illustration). In this case, 11 metres would be the maximum measurement of the distance of between speakers since the maximum width of the hall is 12 metres. Measuring the distance between speakers of more than 11 metres would imply that the speakers would be outside of the hall, therefore not suitable to depict the coverage area of the speakers. The speaker coverage area for $d_s = 0$ is also determined as the reference measurement of area for when only one speaker is used for (See appendix 1 and 2).

Table 2: Approximate area of speaker coverage

Distance between wireless speakers, d_s	Approximate area of speaker coverage
0	115.73
1	130.49
2	143.24
3	149.13
4	153.98
5	173.28
6	195.63
7	165.86
8	182.2
9	169.89
10	138.82
11	135.66

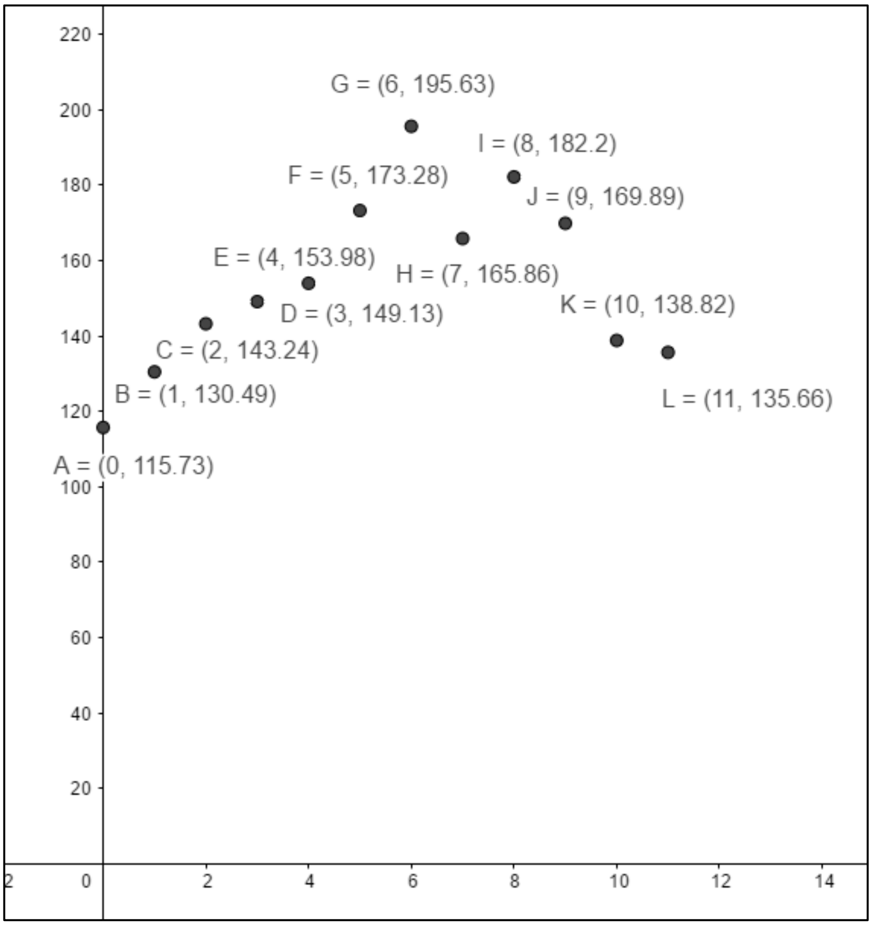

Figure 8: Plotted graph of approximate experimental value of coverage area for a pair of speakers relative to the distance between wireless speakers

From Table 2, it is observed that the value of approximate experimental value of coverage area for a pair of speakers increases until it reaches a certain distance between the pair of speakers, d_s. To determine the x-value that holds the largest value of coverage area of speakers, all the information in Table 1 will be plotted and fitted with the best curve to depict the trend of speaker coverage area. By looking at the plotted data in Figure 8, it is observed that the trend of the data shown is increasing and decreasing. This means that there is only one maximum point in the expected model. In this case, the best possible functions that can be fitted is the polynomial model with the largest x coefficient of 2, 3, and also 4. Their coefficient of determination R^2 is compared to determine the best possible curve that can represent the trend.

Table 3: R^2 against the three possible best fit functions for the area of speaker coverage relative to the distance of speakers

Largest degree of polynomial	**Best fit curve**	**Coefficient of determination, R^2**
2	$g_1(x) = -1.7352x^2 + 21.5484x + 109.1415,$	0.8123
3	$g_3(x) = -0.1471\,x^3 + 0.69194\,x^2 + 11.32524\,x + 116.42267$	0.8537
4	$g_3(x) = 0.02009\,x^4 - 0.58899\,x^3 + 3.73639\,x^2 + 4.57062x + 118.69526$	0.8603

Comparing the three R^2 of the respective curve, it is observed that $R^2\ for\ g_2(x) < R^2\ for\ g_1(x) < R^2\ for\ g_3(x)$ and the R^2 for $g_3(x)$ is seen to be the nearest to be approaching to 1 than the two other fit curves. The R^2 shows that the accuracy of the possible best fit curve that is regressed. Therefore, it can be implied that the function $g_3(x)$ is the best fit for the function that represents the trend of area of speaker coverage in Dewan Orang Ramai Bukit Changgang.

By using GeoGebra, Figure 9 shows that the maximum value for graph $g_3(x)$, when the graph is restricted to the boundary value of x between 0 and 11 is $x = 6.6$. This implies that the maximum area of speaker coverage that can be achieved by the wireless speakers are 180.41 metres^2. This occurs when the distance between the wireless speakers is $d_s = 6\ metres$.

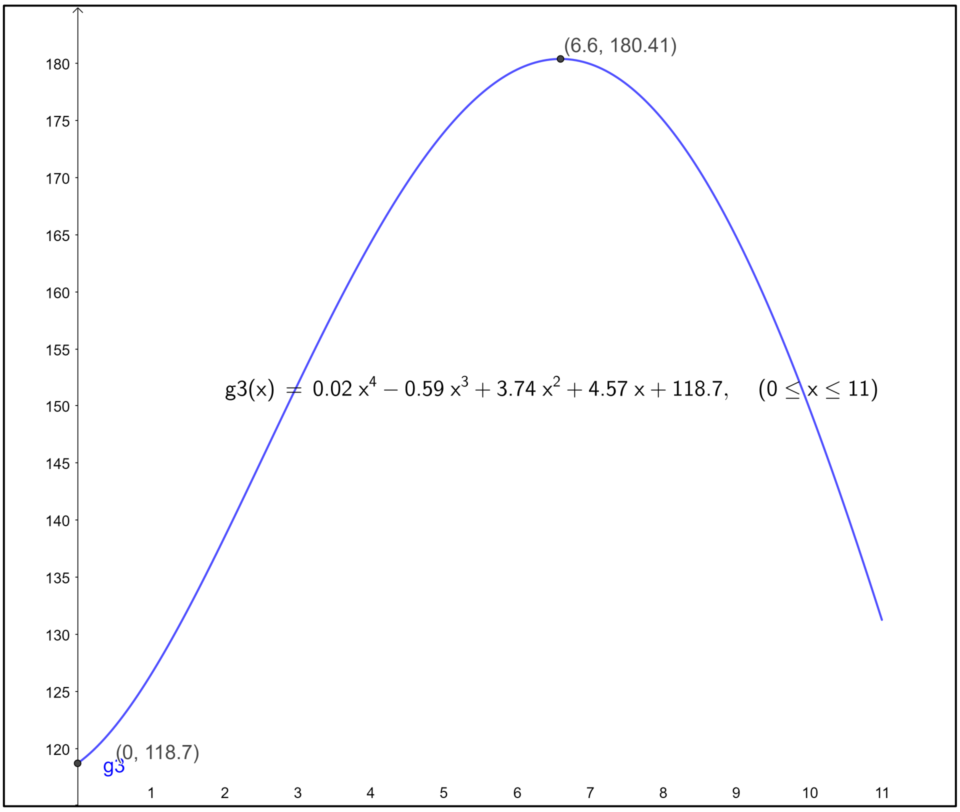

Figure 9: The graph of the maximum area of speaker coverage relative to its distance between speakers, d_s with maximum point $(6.6, 180.41)$

4. THEORETICAL ANALYSIS

In this section, the area of speaker coverage is explored in a theoretical matter. Generally, there are two types of audio coverage, which are nominal area coverage and effective area coverage.

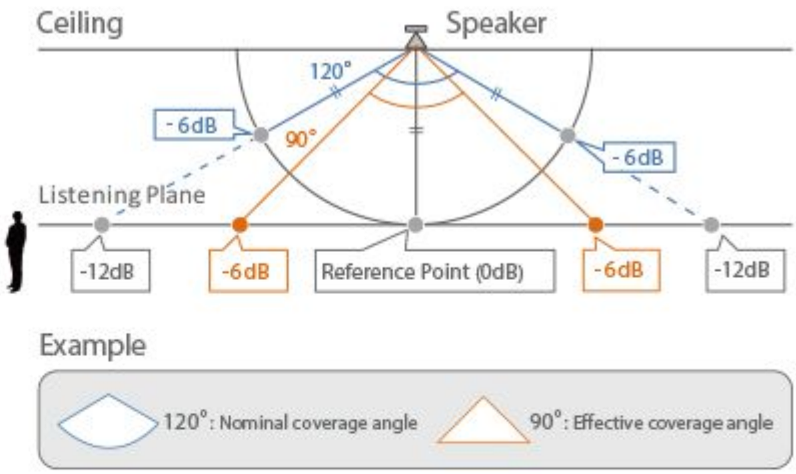

Figure 10: Nominal and effective speakers' coverage[4]

[4] 'Commercial Installation Solutions Speaker Calculator (CISSCA) - Overview - Software - Professional Audio - Products - Yamaha - United States' <https://usa.yamaha.com/products/proaudio/software/commercial_installation_solutions_speaker_calculator/index.html> [accessed 17 September 2020].

In this context, the nominal area coverage, A_N, is the area coverage of the speakers with the angle of 120° or $\frac{2}{3}\pi$ radians. The effective area coverage, A_E, is defined as the area that is covered by the audio range of the speakers with the angle of 60° or $\frac{\pi}{3}$ radians.[5]

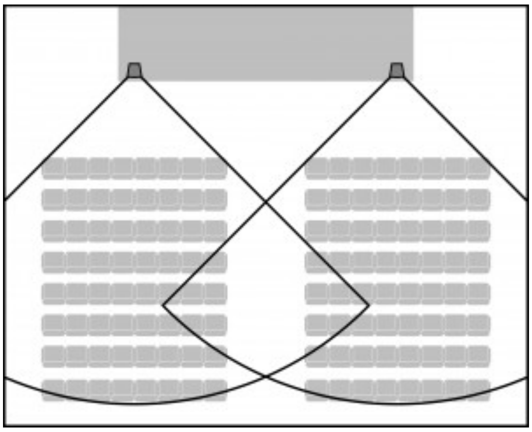

Figure 11: Horizontal nominal speaker coverage[6]

For this exploration, the same pair of personal wireless speakers is used (as shown in Figure 1), which have the maximum input power of 10 watts and a sensitivity rating of 80 decibels (dB) per 1 watt per 1 metre.

From the information, the volume pressure level of audio output of the wireless speakers can be determined. From the context, when the input power of the wireless speakers is doubled, the volume pressure level of audio output is increased by 3 decibels (dB).[7] Table 4 shows the trend of the volume of output audio versus the input power that has supplied to both wireless speakers.

Table 4: Output audio volume vs. Input power

Input power (watt)	Output audio volume (dB)
1	80
2	83
4	86
8	89
16	92
32	95
64	98

[5] 'Commercial Installation Solutions Speaker Calculator (CISSCA) - Overview - Software - Professional Audio - Products - Yamaha - Other European Countries' <https://europe.yamaha.com/en/products/proaudio/software/commercial_installation_solutions_speaker_calculator/index.html> [accessed 7 July 2020].
[6] 'A Dummy's Guide to Speaker Coverage | Sweetwater' <https://www.sweetwater.com/insync/pa-speaker-coverage/> [accessed 7 July 2020].
[7] 'Speaker Power and Distance - PUI Audio | A Projects Unlimited Company Located in Dayton, Ohio' <http://www.puiaudio.com/resources-white-papers-speaker-power.aspx> [accessed 8 July 2020].

128	101
256	104

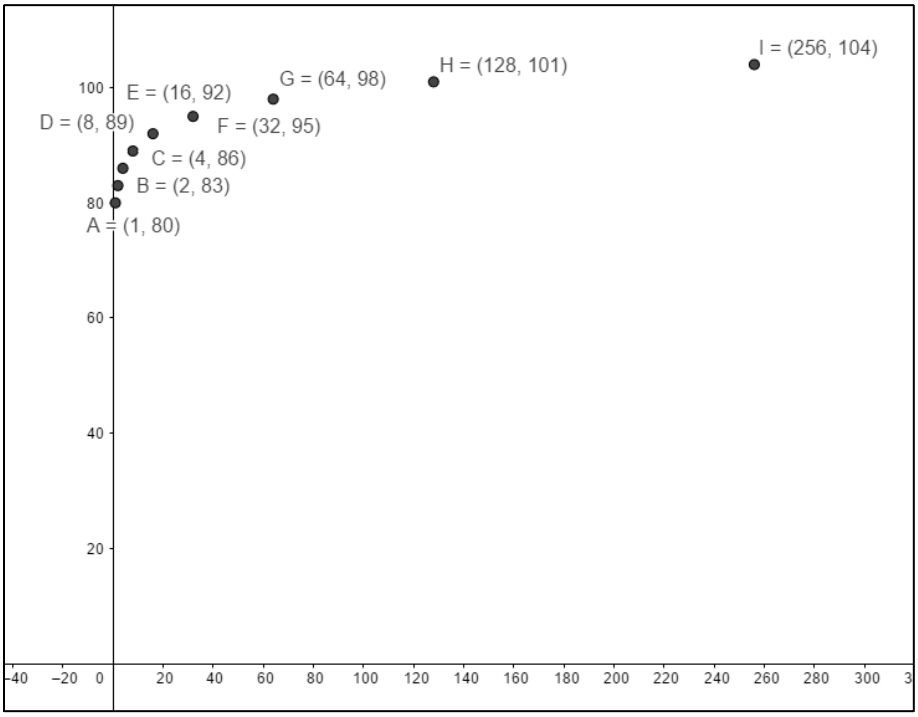

Figure 12: Plotted data of output audio volume against input power

From Table 4, the information is plotted onto a Cartesian plane and the points are then regressed by using GeoGebra as shown in Figure 11 to fit a function to find out the output audio volume when the speakers reach the maximum input power of 10 watts. Three possible best fit functions are compared with its R^2 as shown in Table 5.

Table 5: R^2 against the three possible best fit functions for output audio volume vs. input power

Possible Best Fit Functions, $f_n(x), n = 1, 2, 3$	Coefficient of determination, R^2
$f_1(x) = \dfrac{101.58347}{1 + 0.24008\, e^{-0.04511\, x}}$	0.9514
$f_2(x) = 80 + 4.32809 \ln(x)$	1
$f_3(x) = 80.41788\, x^{0.04724}$	0.9986

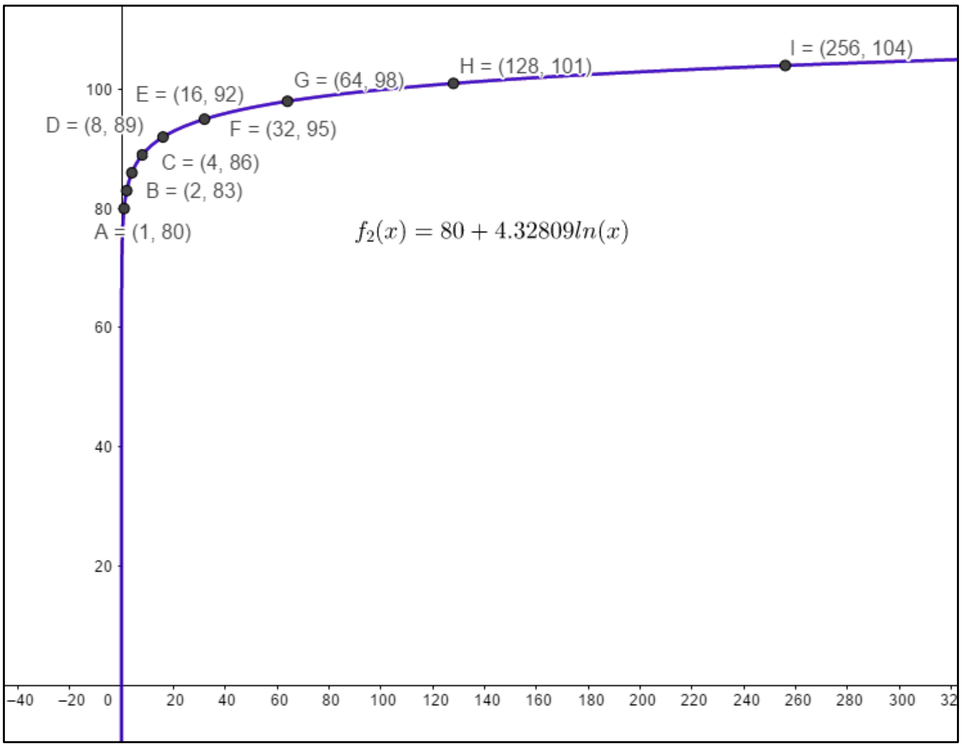

Figure 13: Best fit model of output audio volume against input power, $f_2(x)$

Therefore, the best fit function for the trend of the output audio volume against input power is $f_2(x)$ with the graphical representation shown in Figure 13. This is due to the comparison of R^2 in which we can observe that the value of R^2 of $f_2(x)$ has approached to 1 which indicates that the function is the best fit to represent the trend of the volume of output audio against the input power. Hence, approximate value of the output audio volume when the input power is 10 can be traced by using the symbolic evaluation function in GeoGebra.

From the function, it is found that $f_2(10) \approx 90$. This implies that the maximum output audio volume is approximately 90 dB. Next, the maximum radius of coverage is determined for a wireless speaker. According to the inverse square law, when the distance of a spectator is doubled, the strength of the audio output will be decreased by 50%.[8]

Table 6: Output audio volume vs. Distance of spectator from speaker

Distance of spectator from speaker (m)	Output audio volume (dB)
1	90
2	45
4	22.5
8	11.25
16	5.625
32	2.8125
64	1.40625

[8] Ibid.

128	0.703125

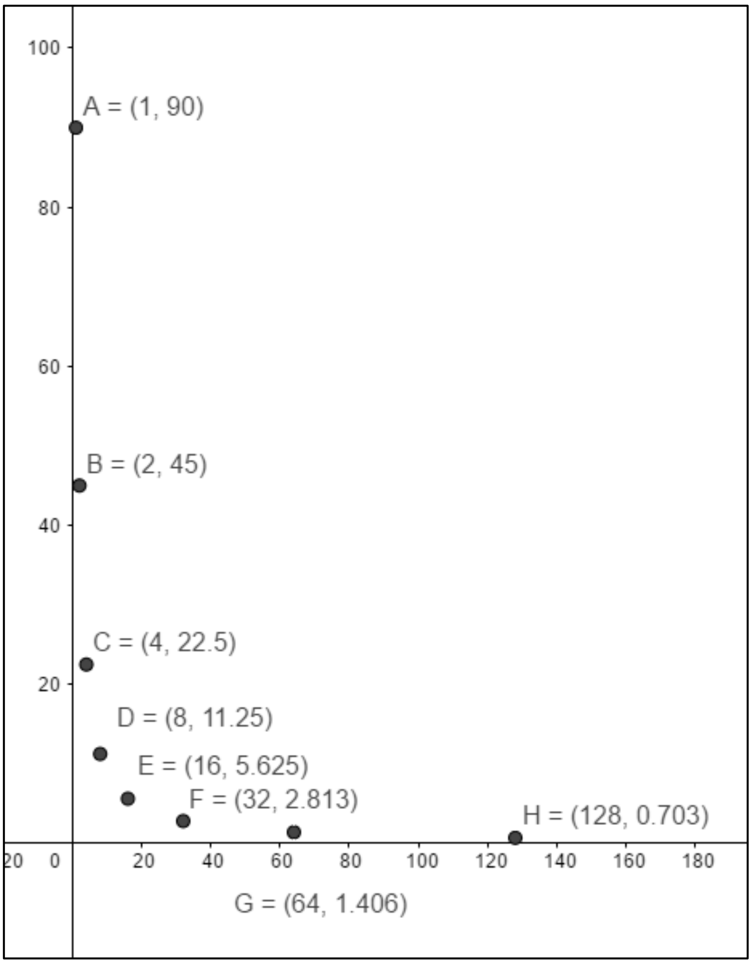

Figure 14: Plotted data of output audio volume against distance of audience from speaker

From Table 6, the information is plotted onto a Cartesian plane and the points are then regressed by using GeoGebra as shown in Figure 14 to fit a function to find out the output audio volume when the speakers reach the maximum input power of 10 watts. Three possible best fit functions are compared with its R^2 as shown in Table 7.

Table 7: Comparison of R^2 of the three possible best fit functions for the trend of output audio volume against distance

Possible Best Fit Functions, $f_n(x), n = 1, 2, 3$	Coefficient of Determination, R^2
$f_1(x) = 22.5\, e^{-0.03262\, x}$	0.2098
$f_2(x) = \frac{90}{x}, x > 0$	1
$f_3(x) = 60.23438 - 15.59028\, ln(x)$	0.7232

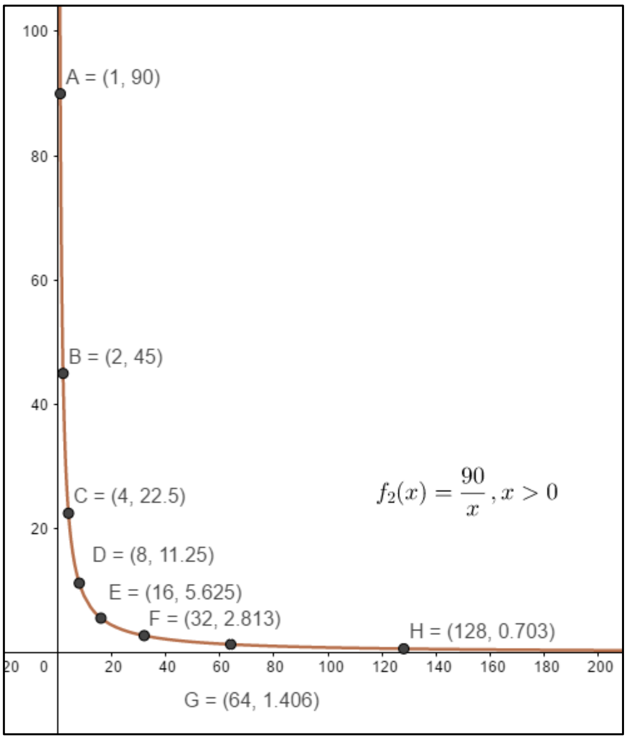

Figure 15: Best fit model of output audio volume against distance of audience from speaker, $f_2(x)$

Therefore, the best fit function for the trend of the output audio volume against input power is $f_2(x)$ as shown in Figure 15. This is due to the comparison of R^2 in which we can observe that the value of R^2 of $f_2(x)$ has approached to 1 which indicates that the function is the best fit to represent the trend of the volume of output audio against the input power. From the function, it is found that $f_2(10) = 9$. This implies that the maximum distance that can be heard by a person from the speaker is 9 metres.

Now, a diagram of nominal speaker coverage is sketched onto the GeoGebra application. In the diagram below, the coverage has been set out with 120° angle. The radius, r, of the diagram is assumed as 11.0 metres, which is the maximum radius of coverage that can be heard by a person from the speakers.

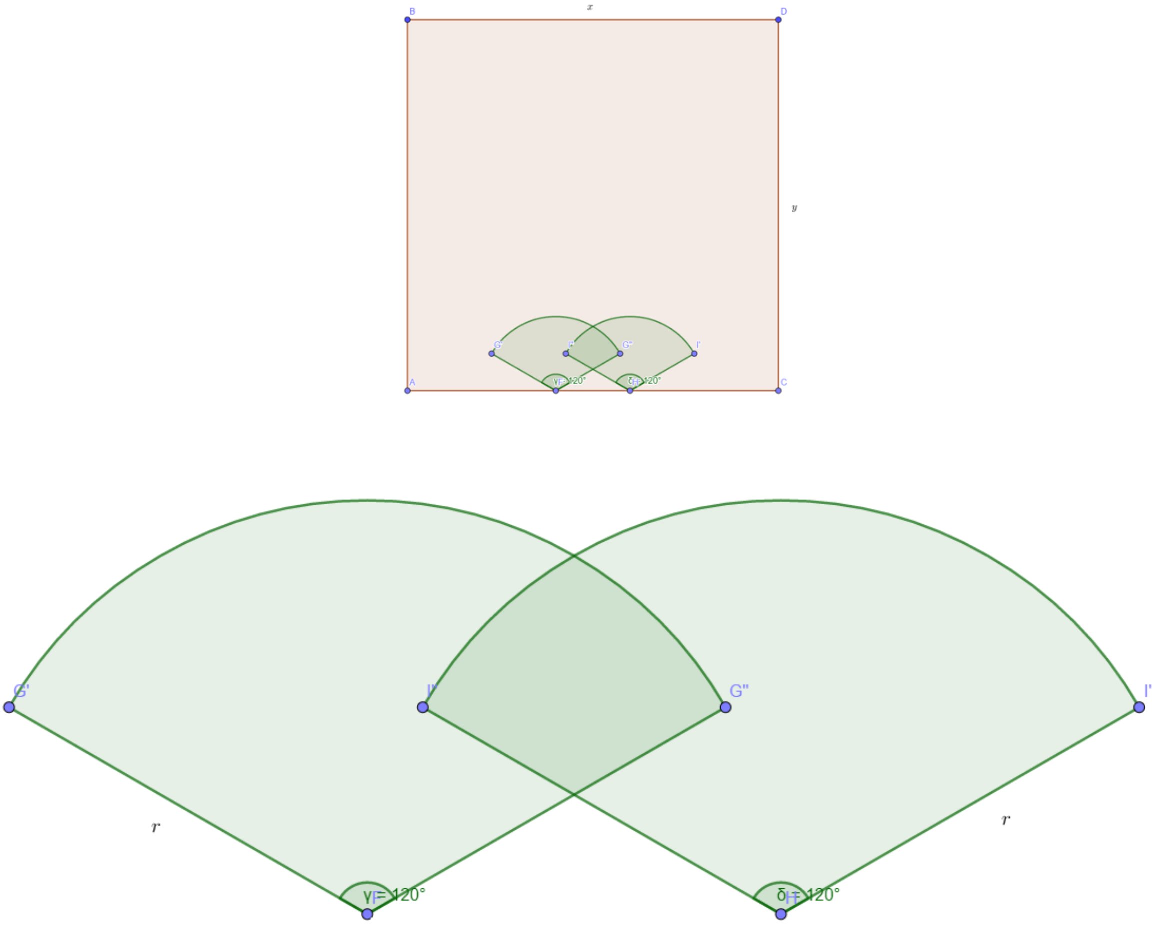

Figure 16: Diagram of nominal coverage with radius r *metre*

Firstly, a graphical representation of the nominal speaker coverage when only one wireless speaker is sketched as shown in Figure 16. This is to identify the intersection of coordinates which will be the boundary of the mathematical equations that has come up from the aforementioned mathematical representation.

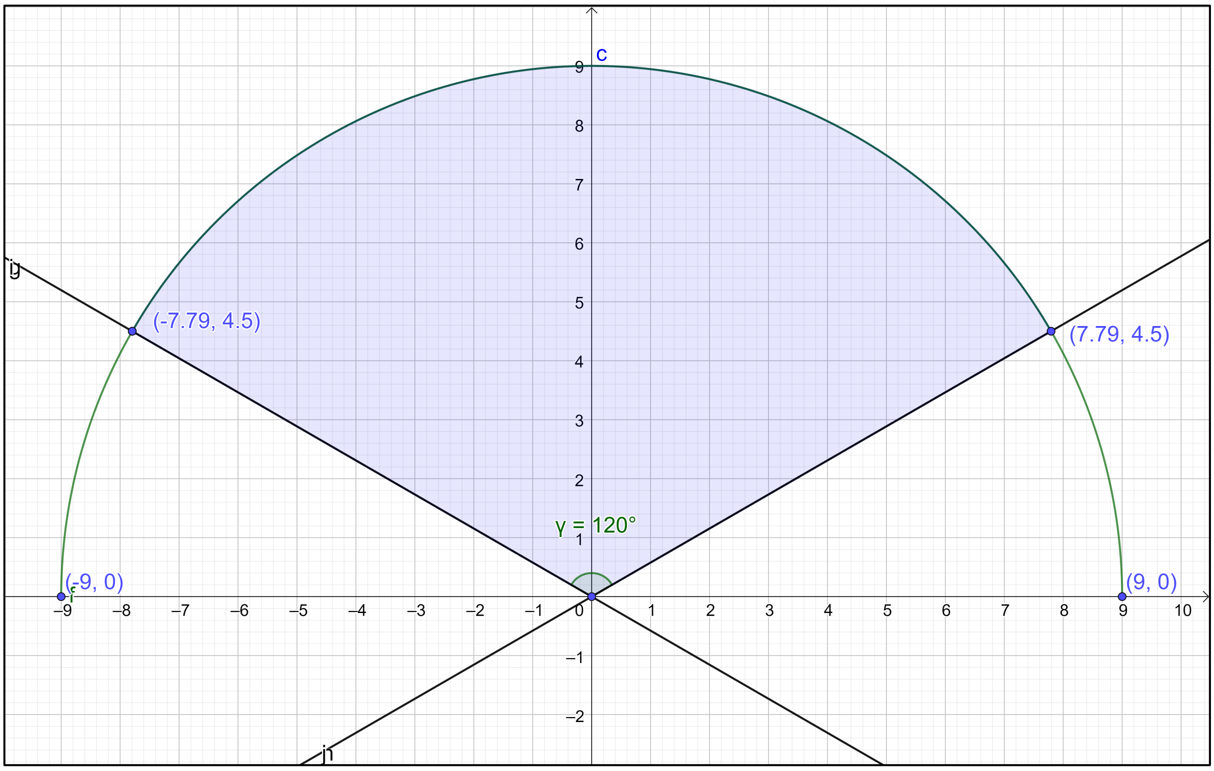

Figure 17: Graphical representation of nominal area coverage of one speaker

The functions that can represent the area of nominal coverage of speakers can be formed. The shape of the curve as shown in Figure 17 can be represented with a parabolic shape. This implies that the representation of the shape is represented by the general formula of parabolic equation,

$$x^2 + y^2 = r^2.$$

Since $r = 9$, the equation that can represent the shape is as follows,

$$x^2 + y^2 = 9^2$$

In this case, in order for the equation to accurate corresponds to the shape of the curve, a parabolic function is required to represent it. Hence, letting $y = C(x)$, the parabolic function of the representation of the curve shape of the coverage area is as follows,

$$C(x) = \sqrt{9^2 - x^2}$$

The edges of the speaker coverage clearly show a straight line. In Figure 16, by obtaining two points $(0,0)$ and $(7.79, 4.5)$, the gradient of the linear function can be obtained by calculating $m = \frac{4.5-0}{7.79-0}$. The shape of the area is depicted as a linear function with a reflected graph on the negative part of the function, therefore the linear function, $D(x)$, obtained would have a modulus sign to signify the shape as shown in Figure 16,

$$D(x) = |0.5771x|$$

The points of intersection that intersect $C(x)$ and $D(x)$ (as shown in Figure 17) are as follows,

$$(7.79, 4.5) \text{ and } (-7.79, 4.5)$$

Therefore, the x-values that bound the necessary function for the area of the nominal coverage are the x-values of the points of intersection of $C(x)$ and $D(x)$. Hence, the refined functions which defines the nominal speaker coverage when one speaker is used are as follows,

$$C(x) = \sqrt{81 - x^2}, \qquad -7.79 \leq x \leq 7.79$$
$$D(x) = |0.5771x|, \qquad -7.79 \leq x \leq 7.79$$

Now, it is observed that the wireless speakers are moved to the left and to the right, hence it can be represented through the movement of the centre of the semicircle function by α units to the left and to the right. In this context, the function of $C(x)$ and $D(x)$ can be moved by α units to the left and to the right. The mathematical definition of the statement is as follows,

$$C(x \pm a) = \sqrt{81 - (x \pm a)^2}, \qquad -7.79 < x \pm a < 7.79$$
$$\boldsymbol{C(x \pm a) = \sqrt{81 - (x \pm a)^2}, \qquad -7.79 \mp \alpha < x < 7.79 \mp \alpha} \tag{1}$$

and,

$$D(x \pm a) = |0.5771(x \pm a)|, \qquad -7.79 < x \pm a < 7.79$$
$$\boldsymbol{D(x \pm a) = |0.5771(x \pm a)|, \qquad -7.79 \mp a < x < 7.79 \mp a} \tag{2}$$

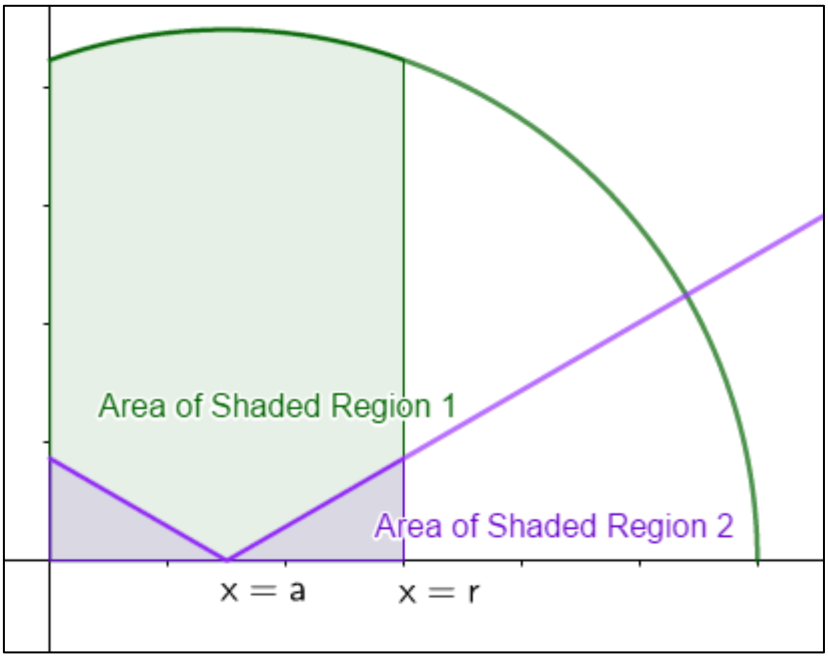

Figure 18: Area of Shaded Region 1 and Area of Shaded Region 2

Now, the function $C(x - a)$ is integrated with the boundary value of $x = 0$ to the distance between the reference point at the centre of a large room and the maximum half-width of the hall or large area, $x = r$. The graphical representation of the area is presented as shown in Figure 18. The integral $A1$ is defined as the area under the curve of Equation (1), the $C(x-a)$ function. The mathematical definition of the forementioned statement is as follows,

$$\text{Area of Shaded Region 1, A1} = \int_0^r C(x-a)\,dx = \int_0^r \sqrt{81-(x-a)^2}\,dx \tag{3}$$

The substitution method is used to solve the problem. The function $C(x-a)$ can also be represented by a triangle and by using basic trigonometry, the function can be substituted. Figure 19 shows the trigonometric representation of $C(x-a)$.

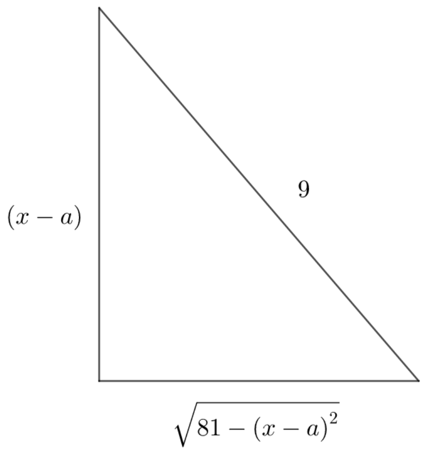

Figure 19: Trigonometric representation of $\sqrt{81-(x-a)^2}$

$$\sin\theta = \frac{x-\alpha}{9}$$

$$9\sin\theta = x-\alpha$$

$$x = 9\sin\theta + \alpha$$

By substituting $x = 9\sin\theta + \alpha$ into equation (3),

$$\text{A1} = \int_0^r \sqrt{81-(9\sin\theta)^2}\,dx = \int_0^r \sqrt{81-81\sin^2\theta}\,dx$$

$$\text{A1} = \int_0^r 9\sqrt{1-\sin^2\theta}\,dx \tag{3.1}$$

Math HL

By applying the Pythagorean identity of $1-\sin^2\theta = \cos^2\theta$ into equation (3.1),

$$\text{A1} = \int_0^r 9\sqrt{\cos^2\theta}\,dx = \int_0^r 9\cos\theta\,dx \tag{3.2}$$

From here, the variable of the integral must be the same. Therefore, we can have the same variable of integration by differentiating the equation of $x = 9\sin\theta + \alpha$ in respect to θ.

$$x = 9\sin\theta + \alpha$$
$$\frac{dx}{d\theta} = 9\cos\theta$$
$$dx = 9\cos\theta\, d\theta$$

The integral boundaries of $x = r$ and $x = 0$ will also have to be adjusted,

$$x = 0, \qquad 0 = 9\sin\theta + \alpha$$
$$\sin\theta = \frac{-\alpha}{9}$$
$$\theta = \sin^{-1}\left(-\frac{\alpha}{9}\right)$$

$$x = r, \qquad r = 9\sin\theta + \alpha$$
$$\sin\theta = \frac{r-a}{9}$$
$$\theta = \sin^{-1}\left(\frac{r-\alpha}{9}\right)$$

By substituting the integral boundary of $\theta = \sin^{-1}\left(-\frac{\alpha}{9}\right)$ and $\theta = \sin^{-1}\left(\frac{r-\alpha}{9}\right)$ and $dx = 9\cos\theta\, d\theta$ into equation (3.3),

$$A1 = \int_{\sin^{-1}\left(-\frac{\alpha}{9}\right)}^{\sin^{-1}\left(\frac{r-\alpha}{9}\right)} 81\cos^2\theta\, d\theta \tag{3.3}$$

By substituting the double angle identity of $\cos^2\theta = \frac{\cos 2\theta + 1}{2}$ into equation (3.3),

$$A1 = \int_{\sin^{-1}\left(-\frac{\alpha}{9}\right)}^{\sin^{-1}\left(\frac{r-\alpha}{9}\right)} 81\left(\frac{\cos 2\theta + 1}{2}\right) d\theta = 40.5\left(\int_{\sin^{-1}\left(-\frac{\alpha}{9}\right)}^{\sin^{-1}\left(\frac{r-\alpha}{9}\right)} (\cos 2\theta + 1)\, d\theta\right) \tag{3.4}$$

Finishing on the integration of the area of shaded region 1 will result in the general formula of the definite integral with $x = r$, as the half of the width of a large room and $x = \alpha$, as the measure of movement of the speakers to the right.

$$A1 = 40.5\left[\frac{\sin 2\theta}{2} + \theta\right]_{\sin^{-1}\left(-\frac{\alpha}{9}\right)}^{\sin^{-1}\left(\frac{r-\alpha}{9}\right)}$$

$$A1 = 20.25\sin\left(2\sin^{-1}\left(\frac{r-\alpha}{9}\right)\right) + 40.5\sin^{-1}\left(\frac{r-\alpha}{9}\right) - 20.25\sin\left(2\sin^{-1}\left(-\frac{\alpha}{9}\right)\right) - 40.5\sin^{-1}\left(-\frac{\alpha}{9}\right) \tag{3.5}$$

correct calculation

Next, equation (2) which is the function $D(x-a)$ will also undergo definite integration for the determination of area of nominal speaker coverage, between the same boundary value of $x = r$ and $x = 0$.

$$\text{Area of Shaded Region 2, A2} = \int_0^r D(x-\alpha)dx = \int_0^r |0.5771(x-\alpha)|dx$$

$$A2 = \int_a^r 0.5771(x-\alpha)dx + \int_0^a -0.5771(x-\alpha)dx = 0.5771\left[\frac{x^2}{2} - \alpha x\right]_a^r - 0.5771\left[\frac{x^2}{2} - \alpha x\right]_0^a$$

$$A2 = 0.28855r^2 - 0.5771ar - 0.28855\alpha^2 + 0.5771\alpha^2 - 0.28855\alpha^2 + 0.5771\alpha^2$$

$$A2 = 0.28855r^2 - 0.5771ar + 0.5771\alpha^2 \tag{4}$$

Therefore, we can find the general formula for the area of nominal coverage, $A_{NC}(a,r)$. In this case, integration will occur at one quadrant rather than both quadrants. This is because the shape of functions at the first quadrant is reflective to the shape of functions at the second quadrant. Hence, the final calculations of the area of nominal speaker coverage will result in the double value of the area calculated at when equation (3.5) subtracted to equation (4).

$$A_{NC}(a,r) = 2(Area\ of\ Shaded\ Region\ 1 - Area\ of\ Shaded\ Region\ 2)$$

$$A_{NC}(a,r) = 40.5\sin\left(2\sin^{-1}\left(\frac{r-\alpha}{9}\right)\right) + 81\sin^{-1}\left(\frac{r-\alpha}{9}\right) - 40.5\sin\left(2\sin^{-1}\left(-\frac{\alpha}{9}\right)\right) - 81\sin^{-1}\left(-\frac{\alpha}{9}\right)$$
$$- 0.5771r^2 + 1.1542ar - 1.1542\alpha^2, \quad 0 \le a \le 9, \qquad a, r \in \mathbb{R}$$

Now, consider the scenario of Dewan Orang Ramai Bukit Changgang with the width of the hall of 11 metres. Therefore, the half width of the hall, $r = 5.5$. The formula for the area at Dewan Orang Ramai Bukit Changgang, defined as formula $N(\alpha)$ is as follows,

$$N(\alpha) = 40.5\sin\left(2\sin^{-1}\left(\frac{5.5-\alpha}{9}\right)\right) + 81\sin^{-1}\left(\frac{5.5-\alpha}{9}\right) - 40.5\sin\left(2\sin^{-1}\left(-\frac{\alpha}{9}\right)\right) - 81\sin^{-1}\left(-\frac{\alpha}{9}\right)$$
$$- 1.1542\alpha^2 + 6.3481a - 17.457275, \qquad 0 < a \le 5.5, \qquad \alpha \in \mathbb{R}$$

From the formula, the maximum point of the function is determined to identify the maximum area of nominal speaker coverage for when the width of the room or hall is at 11 metres or the half-width of the room is at 5.5 metres. By using GeoGebra application, the aforementioned maximum point of $N(\alpha)$ when $r = 5.5$ metres can be determined.

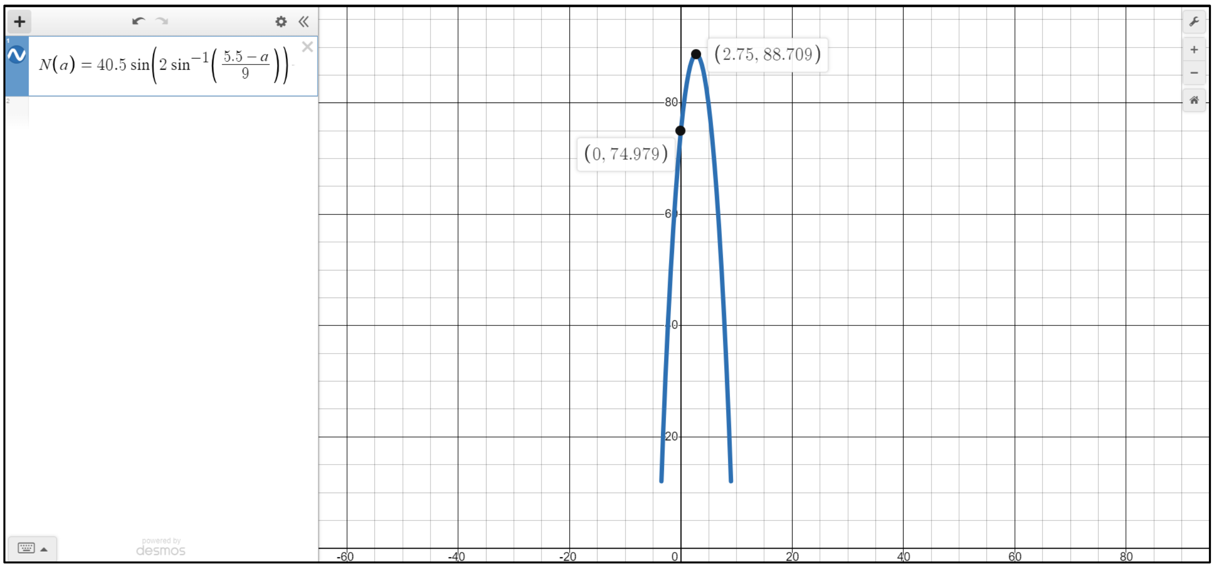

Figure 20: The graph of $N(\alpha)$

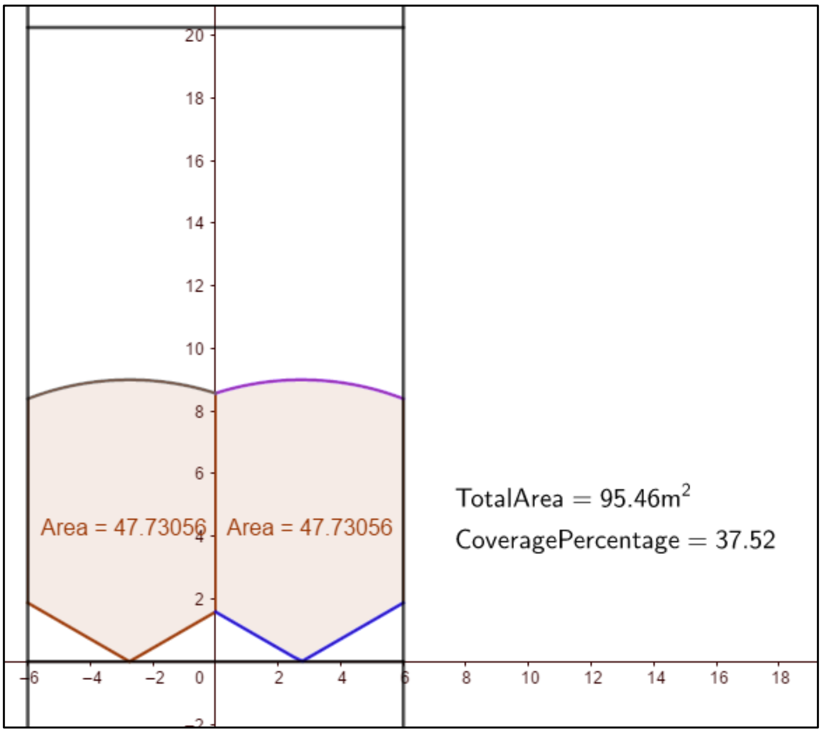

Figure 21: The Maximum Speaker Coverage Area in Dewan Orang Ramai Bukit Changgang

From Figure 20, it is shown that the maximum point of $N(\alpha)$ is $(2.75, 88.709)$. Theoretically, as shown in Figure 21, the maximum area that can be achieved by the use of the personal wireless speakers (see Figure 1) at Dewan Orang Ramai Bukit Changgang is 88.709 m^2, when the distance between the centre of the hall to the speaker is 2.75 m. The percentage of speaker nominal coverage for in this case is 37.52%. This implies that the best position of putting a pair of portable speakers is the best distance between the wireless speakers that can be put at Dewan Orang Ramai Bukit Changgang is at 5.5 metres, which coincidentally becomes the centre of the half-width of the hall.

From the coverage percentage and the area covered shown in Figure 21, it can be observed that less than half of the audience can hear the wireless sound system. In this case, the outcome of the theoretical maximum area speaker coverage is logical, only if it is the interpretation of the outcome is described for an optimum sound system area only. At a non-optimum context, the outcome described for the best position to put the speakers, even if the theoretical speaker coverage area is at a very low percentage. At this point, providing with the best position to put the speakers would indirectly presume that the non-optimum coverage percentage is at the maximum value. Graphically, it is logical to observe that the centre of the half-width of the hall is the best position to put the wireless speakers because the sound system would spread to equally to the left and right side of the hall. Therefore, the outcome of this theoretical analysis can be accepted.

5. EVALUATION AND CONCLUSION

In this mathematical exploration, it can be observed that the aim of the exploration, which is to investigate the best distance to put a pair of speakers in a large room or hall has been achieved. It is shown when the position of the speakers can be identified by determining the maximum point of the function, $A_{NC}(a,r) = 40.5\sin\left(2\sin^{-1}\left(\frac{r-\alpha}{9}\right)\right) + 81\sin^{-1}\left(\frac{r-\alpha}{9}\right) - 40.5\sin\left(2\sin^{-1}\left(-\frac{\alpha}{9}\right)\right) - 81\sin^{-1}\left(-\frac{\alpha}{9}\right) - 0.5771r^2 + 1.1542ar - 1.1542\alpha^2$, provided that $0 \leq a \leq 9, a, r \in \mathbb{R}$. Based on this general formula of the area of nominal speaker coverage, it is certain that there would be a trend of the approximate area of nominal speaker coverage best and the distance between speakers according to the variation of the half width of a certain large hall or room.

This mathematical exploration has been pursued with two approaches, which is the experimental analysis and the theoretical analysis. By comparing the two analysis, we can predict whether the best theoretical distance to put the pair of speakers is similar to the best experimental position. Unexpectedly, there is a significant difference between the experimental and the theoretical analysis. By considering Dewan Orang Ramai Bukit Changgang with the half width, $r = 5.5$ metres as a large hall, the best theoretical distance to put the wireless speakers in the hall is 5.5 metres whilst experimentally, it is 6.6 metres. There is this a difference between the two outcomes because of the experimental outcome of the analysis considered the reality of the sound behaviours together with the real behaviour of humans when engaging with the audio performance. The theoretical model or the general formula is made based on assumptions of simplification that made the model with no external factors which disturbs the system of the simulation.[9]

During the pursuant of this investigation, there were many limitations that have been identified. During the measurement takings in the aforementioned methodology, there were many errors such as the uncertainties

[9] Antonio J. Ibáñez-Molina and Sergio Iglesias-Parro, ‘A Comparison between Theoretical and Experimental Measures of Consciousness as Integrated Information in an Anatomically Based Network of Coupled Oscillators’, *Complexity*, 2018 (2018) <https://doi.org/10.1155/2018/6101586>.

of the exact measurement of the distance. Also, there is a subjectivity issue in terms of hearing an audio clearly. Experimenter was not the only person who did the measurements as there were a few of the experimenter's assistants who were also involved. In this case, the hearing ability of each person is varied. For example, the audio can be heard clearly at one point however others do not. This has influenced the uncertainties of measurements during the experimental analysis process. In the theoretical analysis, the general formula of the area of speaker coverage that have been founded has sufficiently increase the understanding towards the role of integral calculus in approximating the area of the region under the curve. This self-approach of using integral calculus to find the general formula of the approximate area with unknown variables has allowed to reflect broadly about the necessary skills of attempting algebraic methods in unfamiliar areas of mathematics.

In both methodologies, the results that has been taken from this mathematical exploration is satisfied. It can be observed that both methodologies that there is in fact the best position to put a pair of wireless speakers, which has clearly shown that the aim for this exploration has been achieved. There is high hope that the result of this mathematical exploration can be used by all people who wants the best audio quality that they can give to the audience, even these people were only to brought in their personal pair of wireless speakers. With the help of the application of differentiation and integral calculus, the best position to put a pair of wireless speakers can be determined mathematically. However, in this mathematical exploration, it does not consider the theories of Physics as there are many connections between the audio waves and the distance of speakers. There is a possibility that there is a connection between this mathematical exploration with the use of differential equations in this topic. The theoretical analysis focuses more on the nominal speaker coverage in 2-dimension plane, instead of in a 3-dimensional plane. From there, the use of 3-dimensional vectors would be the main focus on this study. Therefore, it is with a very big hope that there would be an extension to this mathematical exploration which seeks the volume that can be filled by the speaker which can maximize in a large room or hall, which will improve the current research topic.

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APPENDICES

Appendix 1: Table of measurement of maximum position that can be heard by a person relative to its *x*-coordinates, p_{max} when the distance between its speakers, $d_s = 1, 2, 3, \ldots, 11$ metres. (Measurements are in metres, m)

x-coordinate, x_0	*Distance between two speakers, $d_s = 0$*	$d_s = 1$	$d_s = 2$	$d_s = 3$	$d_s = 4$	$d_s = 5$	$d_s = 6$	$d_s = 7$	$d_s = 8$	$d_s = 9$	$d_s = 10$	$d_s = 11$
-5.5	3.428	6.002	8.939	8.661	8.782	8.5	10.02	12.141	13.306	16.416	18.291	20.186
-5	4.386	9.752	10.132	9.492	9.5	11.88	12.366	12.912	16.04	17.99	19.99	19.462
-4.5	6.948	11.858	10.47	10.73	10.912	13.216	14.66	16.234	17.454	19.165	18.49	18.411
-4	9.382	9.422	10.252	10.731	11.642	14.57	16.632	17.921	19.714	17.984	16.854	17.2
-3.5	12.34	11.346	11.885	11.693	13.1	17.02	18.44	18.462	18.442	17.158	15.272	15.7
-3	13.296	12.63	12.501	13.171	14.212	18.097	19.66	17.026	18.076	15.93	13.458	14.471
-2.5	14.859	14.034	13.832	15.273	14.252	18.731	19.13	14.846	17.743	15.025	12.172	12.79
-2	13.11	14.18	15.514	14.741	15.78	18.55	18.38	13.362	16.702	14.24	9.606	10.792
-1.5	12.948	13.61	16.61	14.322	14.352	18.706	18.801	12.18	15.832	12.8	8.042	8.846
-1	14.328	18.144	17.994	14.032	13.9	18.252	18.384	11.545	13.71	12.013	6.46	5.721
-0.5	16.492	16.1	16.9	13.254	11.79	17..012	17.76	10.944	12.028	10.171	5.471	3.277
0	17.228	14.033	16.326	14.377	11.154	16.792	16.712	9.792	9.85	8.432	4.51	2.567
0.5	16.998	17.164	16.94	17.826	17.4	17.33	17.7	11.65	12.09	9.21	5.552	3.11
1	15.392	17.422	18.778	16.342	18.008	16.372	18.1	13.1	14.312	11.662	6.96	4.252

1.5	12.022	16.224	16.186	16.866	18.612	18.462	18.77	14.55	15.704	12.97	8.652	7.076
2	10.858	13.952	14.014	15.574	17.712	19.62	20.17	15.712	17.436	14.322	10.502	10.292
2.5	7.066	12.632	12.124	15.422	16.526	19.486	20.31	17.552	18.386	16.136	12.25	12.332
3	5.042	9.572	10.622	13.972	15.2	15.972	21.3	18.99	19.062	17.732	13.694	14.352
3.5	9.332	8.236	9.026	13.576	14.752	15.32	22.3	19.962	19.424	18.452	15.746	15.886
4	5.604	6.138	8.164	13.132	13.858	13.442	19.192	19.288	20.332	20.364	17.684	17.252
4.5	6.888	5.018	10.944	12.172	12.91	10.516	17.102	17.73	19.492	21.033	18.45	18.372
5	3.928	4.058	8.288	11.6	11.822	8.622	14.486	15.72	17.422	19.174	18.966	19.14
5.5	3.554	2.912	7.25	10.262	10.92	7.23	11.35	11.99	15.055	17.43	18.32	19.236

Appendix 2: Table of Comparison of Best Fit Functions for the Determination of the Approximate Area of Speaker Coverage by Variation of Distance between Speakers, d_s

Distance between Speakers, d_s	Three Possible Best Fit Functions, $q_n(x), n = 1, 2, 3$ of respective d_s	R^2	Best Fit Function
0	$q_1(x) = -0.0004\,x^7 - 0.00084\,x^6 + 0.02075\,x^5 + 0.04514\,x^4 - 0.25725\,x^3 - 1.02601\,x^2 - 0.06015\,x + 16.0125$	0.9186	$q_3(x)$
	$q_2(x) = 0.00016\,x^8 - 0.0004\,x^7 - 0.01017\,x^6 + 0.02075\,x^5 + 0.21619\,x^4 - 0.25725\,x^3 - 2.01122\,x^2 - 0.06015\,x + 16.87694$	0.9449	
	$q_3(x) = 0.00005\,x^9 + 0.00016\,x^8 - 0.0038\,x^7 - 0.01017\,x^6 + 0.09549\,x^5 + 0.21619\,x^4 - 0.85769\,x^3 - 2.01122\,x^2 + 1.21859\,x + 16.87694$	0.9643	
2	$q_1(x) = -0.00016\,x^7 - 0.00079\,x^6 + 0.00794\,x^5 + 0.04861\,x^4 - 0.10603\,x^3 - 1.07308\,x^2 + 0.06362\,x + 18.03054$	0.9435	$q_3(x)$
	$q_2(x) = -0.00007\,x^8 - 0.00016\,x^7 + 0.00351\,x^6 + 0.00794\,x^5 - 0.03018\,x^4 - 0.10603\,x^3 - 0.61921\,x^2 + 0.06362\,x + 17.63231$	0.9531	
	$q_3(x) = 0.00002\,x^9 - 0.00007\,x^8 - 0.00118\,x^7 + 0.00351\,x^6 + 0.0304\,x^5 - 0.03018\,x^4 - 0.28642\,x^3 - 0.61921\,x^2 + 0.4478\,x + 17.63231$	0.9562	
3	$q_1(x) = -0.00019\,x^7 + 0.0003\,x^6 + 0.01066\,x^5 - 0.01333\,x^4 - 0.17946\,x^3 - 0.06564\,x^2 + 1.07456\,x + 15.42576$	0.8872	$q_3(x)$
	$q_2(x) = -0.00007\,x^8 - 0.00019\,x^7 + 0.00423\,x^6 + 0.01066\,x^5 - 0.08531\,x^4 - 0.17946\,x^3 + 0.34893\,x^2 + 1.07456\,x + 15.06201$	0.9047	
	$q_3(x) = 0.00003\,x^9 - 0.00007\,x^8 - 0.00233\,x^7 + 0.00423\,x^6 + 0.0578\,x^5 - 0.08531\,x^4 - 0.55816\,x^3 + 0.34893\,x^2 + 1.88107\,x + 15.06201$	0.9339	

4	$q_1(x) = -0.00031\,x^7 + 0.00101\,x^6 + 0.01832\,x^5 - 0.05034\,x^4 - 0.32986\,x^3 + 0.44876\,x^2 + 1.98914\,x + 14.63221$	0.8065	$q_3(x)$
	$q_2(x) = -0.00011\,x^8 - 0.00031\,x^7 + 0.00779\,x^6 + 0.01832\,x^5 - 0.17463\,x^4 - 0.32986\,x^3 + 1.16466\,x^2 + 1.98914\,x + 14.00406$	0.8467	
	$q_3(x) = 0.00004\,x^9 - 0.00011\,x^8 - 0.00268\,x^7 + 0.00779\,x^6 + 0.07055\,x^5 - 0.17463\,x^4 - 0.74944\,x^3 + 1.16466\,x^2 + 2.8827\,x + 14.00406$	0.8741	
5	$q_1(x) = 0.0001\,x^7 + 0.00162\,x^6 - 0.00087\,x^5 - 0.07151\,x^4 - 0.0603\,x^3 + 0.44301\,x^2 + 0.17483\,x + 17.56753$	0.9627	$q_3(x)$
	$q_2(x) = -0.0001433447\,x^8 - 0.0001887072\,x^7 + 0.0091443589\,x^6 + 0.010670167\,x^5 - 0.1941952674\,x^4 - 0.1878133393\,x^3 + 1.0766250662\,x^2 + 0.507697041\,x + 17.0646785634$	0.97819	
	$q_3(x) = 0.0000022089\,x^9 - 0.0001383748\,x^8 - 0.0003181201\,x^7 + 0.008912088\,x^6 + 0.0131871174\,x^5 - 0.1909031852\,x^4 - 0.205833939\,x^3 + 1.0622792856\,x^2 + 0.5421979299\,x + 17.073908472$	0.97821	
6	$q_1(x) = 0.0001040115\,x^7 + 0.0015140142\,x^6 - 0.0038383721\,x^5 - 0.084371466\,x^4 + 0.0074799959\,x^3 + 0.9382656632\,x^2 + 0.5008965405\,x + 17.4703546754$	0.9806	$q_3(x)$
	$q_2(x) = 0.0000018052\,x^8 + 0.0001076218\,x^7 + 0.0014192432\,x^6 - 0.0039836875\,x^5 - 0.0828265179\,x^4 + 0.0090857553\,x^3 + 0.9302864172\,x^2 + 0.4967046745\,x + 17.4766870953$	0.9814	
	$q_3(x) - 0.0000027252\,x^9 - 0.0000043265\,x^8 + 0.0002672863\,x^7 + 0.0017058097\,x^6 - 0.007088999\,x^5 - 0.0868881554\,x^4 + 0.0313188411\,x^3 + 0.9479856601\,x^2 + 0.4541388757\,x + 17.465299608$	0.9815	

7	$q_1(x) = -0.0001643118\,x^7 + 0.0008100915\,x^6 + 0.0090159996\,x^5 - 0.0686773667\,x^4 - 0.1590147888\,x^3 + 1.3789061724\,x^2 + 1.1144965859\,x + 10.5642448315$	0.9784	$q_3(x)$
	$q_2(x) = 0.0000538389\,x^8 - 0.0001643118\,x^7 - 0.002393322\,x^6 + 0.0090159996\,x^5 - 0.0099523466\,x^4 - 0.1590147888\,x^3 + 1.0406568267\,x^2 + 1.1144965859\,x + 10.8610316723$	0.9857	
	$q_3(x) = -0.0000135744\,x^9 + 0.0000538389\,x^8 + 0.0007423801\,x^7 - 0.002393322\,x^6 - 0.0109352637\,x^5 - 0.0099523466\,x^4 + 0.0012729372\,x^3 + 1.0406568267\,x^2 + 0.7731401417\,x + 10.8610316723$	0.9890	
8	$q_1(x) = -0.0000092794\,x^7 + 0.0009166319\,x^6 + 0.0004241787\,x^5 - 0.069825512\,x^4 - 0.0037559115\,x^3 + 1.3484625228\,x^2 + 0.1377197457\,x + 12.0841211957$	0.9269	$q_3(x)$
	$q_2(x) = -0.0000789865\,x^8 - 0.0000092794\,x^7 + 0.0056163296\,x^6 + 0.0004241787\,x^5 - 0.1559804351\,x^4 - 0.0037559115\,x^3 + 1.8447049471\,x^2 + 0.1377197457\,x + 11.6487080212$	0.945392	
	$q_3(x) = -0.0000004107\,x^9 - 0.0000789865\,x^8 + 0.0000181559\,x^7 + 0.0056163296\,x^6 - 0.0001795199\,x^5 - 0.1559804351\,x^4 + 0.0010941807\,x^3 + 1.8447049471\,x^2 + 0.1273907562\,x + 11.6487080212$	0.945394	
9	$q_1(x) = 0.000087015\,x^7 + 0.0002866573\,x^6 - 0.0054743016\,x^5 - 0.0379035107\,x^4 + 0.0970771713\,x^3 + 1.1173654435\,x^2 - 0.2433999137\,x + 9.9529374526$	0.9706	$q_3(x)$
	$q_2(x) = -0.0000605702\,x^8 + 0.000087015\,x^7 + 0.0038905868\,x^6 - 0.0054743016\,x^5 - 0.1039707957\,x^4 + 0.0970771713\,x^3 + 1.4979053884\,x^2 - 0.2433999137\,x + 9.6190439817$	0.9774	

	$q_3(x) = -0.0000025296\,x^9 - 0.0000605702\,x^8 + 0.0002559748\,x^7 + 0.0038905868\,x^6 - 0.0091921708\,x^5 - 0.1039707957\,x^4 + 0.1269463978\,x^3 + 1.4979053884\,x^2 - 0.3070108539\,x + 9.6190439817$	0.9775	
10	$q_1(x) = 0.000035803\,x^7 + 0.0002350764\,x^6 - 0.0015392599\,x^5 - 0.0332741368\,x^4 + 0.0092739622\,x^3 + 1.2266752219\,x^2 + 0.1231416147\,x + 5.3807805088$	0.9934	$q_3(x)$
	$q_2(x) = -0.0000653289\,x^8 + 0.000035803\,x^7 + 0.0041221431\,x^6 - 0.0015392599\,x^5 - 0.1045318977\,x^4 + 0.0092739622\,x^3 + 1.6371117093\,x^2 + 0.1231416147\,x + 5.0206552047$	0.9975	
	$q_3(x) = 0.0000120035\,x^9 - 0.0000653289\,x^8 - 0.000765959\,x^7 + 0.0041221431\,x^6 + 0.0161030789\,x^5 - 0.1045318977\,x^4 - 0.1324639481\,x^3 + 1.6371117093\,x^2 + 0.4249934797\,x + 5.0206552047$	0.9984	
11	$q_1(x) = 0.0000849433\,x^7 + 0.001377732\,x^6 - 0.0055524547\,x^5 - 0.0877320016\,x^4 + 0.1078724334\,x^3 + 1.9482697081\,x^2 - 0.6117104831\,x + 3.2493100982$	0.9912	$q_3(x)$
	$q_2(x) = -0.0001009459\,x^8 + 0.0000849433\,x^7 + 0.0073840141\,x^6 - 0.0055524547\,x^5 - 0.1978392458\,x^4 + 0.1078724334\,x^3 + 2.5824747407\,x^2 - 0.6117104831\,x + 2.6928457311$	0.9979	
	$q_3(x) = -0.00000702\,x^9 - 0.0001009459\,x^8 + 0.0005538382\,x^7 + 0.0073840141\,x^6 - 0.0158702338\,x^5 - 0.1978392458\,x^4 + 0.1907650966\,x^3 + 2.5824747407\,x^2 - 0.788242682\,x + 2.6928457311$	0.9981	