Solve the exercises by hand and verify your answers using Mathematica.

1. Compute the following antiderivatives.

   (a) $\displaystyle\int x^2 \log(x)\, dx$

   (b) $\displaystyle\int x^2 e^x\, dx$

   (c) $\displaystyle\int \big[\log(x)\big]^2\, dx$

2. Evaluate the following definite integrals.

   (a) $\displaystyle\int_4^7 x^2 \log(x)\, dx$

   (b) $\displaystyle\int_0^\infty \frac{1}{(1+x)^2}\, dx$

3. Let K, T, σ, and r be positive constants and let

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_0^{b(x)} e^{-\frac{y^2}{2}}\, dy$$

   where $b(x) = \frac{1}{\sigma\sqrt{T}}\left[\log\left(\frac{x}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right]$. Compute $g'(x)$.

4. Let $\phi(u) = \dfrac{1}{\sqrt{2\pi}} e^{-u^2/2}$ so that $\Phi(x) = \displaystyle\int_{-\infty}^{x} \phi(u)\, du$ (i.e., the $\Phi(x)$ in Black-Scholes).

   (a) For $x > 0$, show that $\phi(-x) = \phi(x)$.

   (b) Given that $\lim_{x\to\infty} \Phi(x) = 1$, use the properties of the integral as well as a substitution to show that $\Phi(-x) = 1 - \Phi(x)$ (again, assuming $x > 0$).