

1. Let  $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 1 \\ -2 & 1 & 1 \end{bmatrix}$

- (a) Use elimination to turn  $A$  into an upper triangular matrix. How many pivots does  $A$  have?
- (b) Let  $b = (1, 6, 3)$ . Does  $Ax = b$  have a solution?
- (c) Let  $b = (1, 6, 5)$ . Does  $Ax = b$  have a solution?
- (d) Can you find multiple solutions in either part (b) or part (c)? If so, find 2.
- (e) Does  $A$  have an inverse? Justify your answer using results from this exercise.

2. Suppose  $AB = I$  and  $CA = I$  where  $I$  is the  $n \times n$  identity matrix.

- (a) What are the dimensions of the matrices  $A$ ,  $B$  and  $C$ ?
- (b) Show that  $B = C$ .
- (c) Is  $A$  invertible?

[Hint: you can write  $B = IB$ ]

3. Let  $A$  be a square matrix with the property that  $A^2 = A$ . Simplify  $(I - A)^2$  and  $(I - A)^7$ .

4. (a) Write the vector  $(9, 2, -5)$  as a linear combination of the vectors  $(1, 2, 3)$  and  $(6, 4, 2)$  or explain why it can't be done.
- (b) How many pivots does a system of equations with coefficient matrix

$$A = \begin{bmatrix} 1 & 6 & 9 \\ 2 & 4 & 2 \\ 3 & 2 & -5 \end{bmatrix}$$

have?