CFRM 460: Mathematical Methods for Quantitative Finance Homework 1

Instructions: Solve each of the exercises both by hand (must show all work for credit) and using Mathematica to verify that you get the same answers. Be sure to include the Mathematica code used to verify the solution. For example,

$$\lim_{x\to 0}\frac{\sin(x)}{x} \qquad \text{would be} \qquad \texttt{Limit[Sin[x] / x, x -> 0]}.$$

1. Compute the following limits.

   (a) $\displaystyle\lim_{h\to 0}\frac{4(x+h-3)^2-4(x-3)^2}{h}$

   (b) $\displaystyle\lim_{x\to\infty}\frac{1}{\sqrt{4x^2-2x-10}+2x}$

2. Compute the derivatives of the following functions.

   (a) $f(x)=\dfrac{1}{1-x}$

   (b) $f(x)=\displaystyle\sum_{k=1}^{7} ke^{-a_k x^3}$ (the $\{a_k\}$ are constants)

   (c) $f(x)=\dfrac{\log\left(\frac{x}{K}\right)+\left(r-q+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ ($K>0$, r, q, $\sigma>0$, and $T>t$ constant)

   (d) $f(x)=\dfrac{\log\left(\frac{S}{K}\right)+\left(r-q+\frac{x^2}{2}\right)(T-t)}{x\sqrt{T-t}}$ ($S>0$, $K>0$, r, q, and $T>t$ constant)

   (e) $f(x)=\dfrac{\log\left(\frac{S}{K}\right)+\left(x-q+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ ($S>0$, $K>0$, q, $\sigma>0$, and $T>t$ constant)

3. Use l'Hôpital's rule to compute the following limits.

   (a) $\displaystyle\lim_{x\to 0}\frac{\sin(x)}{x}$

   (b) $\displaystyle\lim_{x\to\infty}\frac{\log(x^3)}{x}$