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CFRM 460
Homework 5 Solutions
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Question 1

$$\text{Let } A =$$

Part (a)

$$A : n \quad n$$
$$B : n \quad n$$
$$C : n \quad n$$

Part (b)

$$) \;\; AB = CA$$
$$\text{Multiplying both sides by } B\text{:}$$
$$ABB = CAB \;\;) \;\; (AB)B = C(AB)$$
$$\text{We know } AB = I \;\;) \;\; IB = CI$$
$$\text{But, } IX = X \text{ and } XI = X$$
$$\therefore B = C$$

Part (c)

$$\text{We know } AB = I,\ CA = I \text{ and } B = C$$
$$\text{Using the property that } A^{\ 1}A = I \text{ and } AA^{\ 1} = I,$$
$$\text{we can conclude } B = C = A^{\ 1}$$
$$\therefore A \text{ is invertible.}$$

Question 3

Part (a)

$$(I \quad A)^2 = I^2 \quad 2IA + A^2$$
$$\text{But, we know } A^2 = A \text{ and } I^n = I$$
$$) \;\; I^2 \quad 2IA + A^2 = I \quad 2A + A = \underline{\underline{I \quad A}}$$

Part (b)

$$(I \quad A)^7 = (I \quad A)\left[(I \quad A)^2\right]^3$$
$$\text{Recall, } (I \quad A)^2 = I \quad A$$
$$) \;\; (I \quad A)[(I \quad A)^2]^3 = (I \quad A)(I \quad A)^3 = (I \quad A)^4 = [(I \quad A)^2]^2$$
$$) \;\; [(I \quad A)^2]^2 = (I \quad A)^2 = \underline{\underline{I \quad A}}$$

Question 4

Part (a)

$$) \; a \begin{bmatrix}1\\2\\3\end{bmatrix} + b \begin{bmatrix}6\\4\\2\end{bmatrix} = \begin{bmatrix}9\\2\\5\end{bmatrix}$$

Multiplying both sides by a constant c, we have:

$$ac \begin{bmatrix}1\\2\\3\end{bmatrix} + bc \begin{bmatrix}6\\4\\2\end{bmatrix} = c \begin{bmatrix}9\\2\\5\end{bmatrix} \;) \; ac \begin{bmatrix}1\\2\\3\end{bmatrix} + bc \begin{bmatrix}6\\4\\2\end{bmatrix} \quad c \begin{bmatrix}9\\2\\5\end{bmatrix} = 0$$

Reversing the dot product, we have: $\begin{bmatrix}1 & 6 & 9\\2 & 4 & 2\\3 & 2 & 5\end{bmatrix} \begin{bmatrix}ac\\bc\\c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

Using Gaussian elimination to determine solutions:

$$\left[\begin{array}{ccc|c}1 & 6 & 9 & 0\\2 & 4 & 2 & 0\\3 & 2 & 5 & 0\end{array}\right]$$

$$R_2 \;!\; \frac{r_2}{2} \left[\begin{array}{ccc|c}1 & 6 & 9 & 0\\1 & 2 & 1 & 0\\3 & 2 & 5 & 0\end{array}\right]$$

$$R_3 \;!\; r_3 \quad r_2 \left[\begin{array}{ccc|c}1 & 6 & 9 & 0\\1 & 2 & 1 & 0\\2 & 0 & 6 & 0\end{array}\right]$$

$$R_1 \;!\; r_1 \quad 3r_2 \left[\begin{array}{ccc|c}2 & 0 & 6 & 0\\1 & 2 & 1 & 0\\2 & 0 & 6 & 0\end{array}\right]$$

$$R_3 \;!\; r_3 + r_2 \left[\begin{array}{ccc|c}2 & 0 & 6 & 0\\1 & 2 & 1 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

$$R_2 \;!\; r_2 + \frac{r_1}{2} \left[\begin{array}{ccc|c}2 & 0 & 6 & 0\\0 & 2 & 4 & 0\\0 & 0 & 0 & 0\end{array}\right]$$

Thus, as all of the coefficients are 0 as per the solution to the equation, the linear expression of the vector cannot be done.

Part (b)

The Gaussian elimination performed above shows that $\begin{bmatrix}1 & 6 & 9\\2 & 4 & 2\\3 & 2 & 5\end{bmatrix}$ has 2 pivots.