1. (a) Under what condition does the following hold?

$$\iint_D f(x,y)\,dA = \iint_D f(x,y)\,dy\,dx = \iint_D f(x,y)\,dx\,dy$$

   (b) Evaluate the double integral

$$\iint_D e^{y^2}\,dA$$

   where $D = \{(x,y) : 0 \le y \le 1,\ \ 0 \le x \le y\}$.

2. (a) Transform the double integral

$$\iint_D e^{\frac{x+y}{x-y}}\,dA$$

   into an integral of u and v using the change of variables

$$u = x + y \qquad\qquad v = x - y$$

   and call the domain in the uv plane S.

   (b) Let D be the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,-2)$ and $(0,-1)$. Find the corresponding region S in the uv plane by evaluating the transformation at the vertices of D and connecting the dots. Sketch both regions.

   (c) Compute the integral found in part (a) over the domain S from part (b).

3. Consider the Black-Scholes formula for the price of a European call option as a function of a single variable S (i.e., treat all the other inputs as constant). Write down a second order Taylor polynomial around S_0 for $C(S)$ in terms of Δ and Γ.

4. (a) Let $D = \{(x,y) : 1 \le x^2 + y^2 \le 9,\ y \ge 0\}$. Compute the integral

$$\iint_D \sqrt{x^2+y^2}\, dx\, dy$$

by changing to polar coordinates. Sketch the domains of integration in both the xy and $r\theta$ (that means r on one axis and θ on the other) planes.

(b) Compute the integral

$$\iint_D \sin(\sqrt{x^2+y^2})\, dx\, dy$$

where $D = \{(x,y) : \pi^2 \le x^2 + y^2 \le 4\pi^2\}$.

5. Compute the Taylor series expansion $T(x)$ of the function

$$f(x) = \frac{1}{1+x}$$

around the point $a = 0$ and find its radius of convergence. Does $T(x) = f(x)$ on the domain of convergence?

6. Let $P_0, P_1, \ldots$ be a sequence of market prices for the same asset. The *arithmetic* return on the asset during period t is defined to be

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

and the *log* return is defined to be

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right).$$

Show that $r_t \approx R_t$ for small values of R_t (say on the order of 1%).