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CFRM 460
Homework 3 Solutions
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The Black-Scholes price for an European Put option is defined as:

$$P(S,t,K,T,r,q,\sigma) = Ke^{-r(T-t)}\Phi(-d_-) - Se^{-q(T-t)}\Phi(-d_+)$$

Where

$$d_+ = \frac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_- = d_+ - \sigma\sqrt{T-t} = \frac{\log\left(\frac{S}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$\Phi(x) = \int_{-\infty}^{x} \phi(u)\, du$$

$$\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$$

The Put-Call parity of a European call option and European put option is defined as:

$$P = C - Se^{-q(T-t)} + Ke^{-r(T-t)}$$

Question 1

Part (a)

$$\Delta(P) = \frac{\partial P}{\partial S}$$

$$\Rightarrow \frac{\partial P}{\partial S} = \frac{\partial}{\partial S}\left[Ke^{-r(T-t)}\Phi(-d_-) - Se^{-q(T-t)}\Phi(-d_+)\right] = Ke^{-r(T-t)}\frac{\partial}{\partial S}\left[\Phi(-d_-)\right] - e^{-q(T-t)}\frac{\partial}{\partial S}\left[S\Phi(-d_+)\right]$$

By applying the product rule to the 2nd term, we have:

$$= Ke^{-r(T-t)}\frac{\partial}{\partial S}\left[\Phi(-d_-)\right] - e^{-q(T-t)}\left[\Phi(-d_+) + S\frac{\partial}{\partial S}\left[\Phi(-d_+)\right]\right]$$

Consider $\frac{\partial}{\partial S}\left[\Phi(-d_\pm)\right]$:

Applying the chain rule: $\frac{\partial}{\partial S}\left[\Phi(-d_\pm)\right] = \frac{\partial}{\partial(-d_\pm)}\left[\Phi(-d_\pm)\right] \cdot \frac{\partial}{\partial S}\left[-d_\pm\right]$

As per the fundamental theorum of calculus (FTC): $\frac{\partial}{\partial(-d_\pm)}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm)$

$$\Rightarrow \frac{\partial}{\partial S}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm) \cdot \frac{\partial}{\partial S}\left[-d_\pm\right]$$

We know $-d_\pm = -\frac{\log\left(\frac{S}{K}\right) + \left(r - q \pm \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

$$\Rightarrow \phi(-d_\pm) \cdot \frac{\partial}{\partial S}\left[-d_\pm\right] = \phi(-d_\pm) \cdot \frac{\partial}{\partial S}\left[-\frac{\log\left(\frac{S}{K}\right) + \left(r - q \pm \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right]$$

$$= \phi(-d_{\pm}) \cdot \frac{\partial}{\partial S}\left[-\frac{\log\left(\frac{S}{K}\right)}{\sigma\sqrt{T-t}} - \frac{\left(r - q \pm \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right] = \phi(-d_{\pm}) \cdot \frac{-1}{\sigma\sqrt{T-t}} \cdot \frac{K}{S} \cdot \frac{1}{K} = \phi(-d_{\pm}) \cdot \frac{-1}{S\sigma\sqrt{T-t}}$$

$$\frac{\partial}{\partial S}\left[\Phi(-d_{\pm})\right] = \frac{-\phi(-d_{\pm})}{S\sigma\sqrt{T-t}}$$

Substituting $\frac{\partial}{\partial S}\left[\Phi(-d_{\pm})\right] = \frac{-\phi(-d_{\pm})}{S\sigma\sqrt{T-t}}$ in $\Delta(P)$:

$$\Rightarrow \Delta(P) = Ke^{-r(T-t)}\frac{\partial}{\partial S}\left[\Phi(-d_-)\right] - e^{-q(T-t)}\left[\Phi(-d_+) + S\frac{\partial}{\partial S}\left[\Phi(-d_+)\right]\right]$$

$$\Delta(P) = \frac{e^{-q(T-t)}\phi(-d_+)}{\sigma\sqrt{T-t}} - \frac{Ke^{-r(T-t)}\phi(-d_-)}{S\sigma\sqrt{T-t}} - e^{-q(T-t)}\Phi(-d_+)$$

Let *Conjecture 1:* $\frac{e^{-q(T-t)}\phi(-d_+)}{\sigma\sqrt{T-t}} = \frac{Ke^{-r(T-t)}\phi(-d_-)}{S\sigma\sqrt{T-t}}$

We know $\phi(-d_-) = \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-(-d_-)^2}{2}\right)$

We also know $d_- = d_+ - \sigma\sqrt{T-t}$

$$\Rightarrow \phi(-d_-) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(d_+ - \sigma\sqrt{T-t})^2}{2}\right) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{d_+^2 - 2d_+\sigma\sqrt{T-t} + \sigma^2(T-t)}{2}\right)$$

$$= \left[\frac{1}{\sqrt{2\pi}}\exp\left(\frac{(-d_+)^2}{2}\right)\right]\exp\left(\frac{-\sigma^2(T-t)}{2}\right)\exp\left(d_+\sigma\sqrt{T-t}\right) = \phi(-d_+)\exp\left(\frac{-\sigma^2(T-t)}{2}\right)\exp\left(d_+\sigma\sqrt{T-t}\right)$$

But, we know $d_+ = \frac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

$$\Rightarrow \phi(-d_+)\exp\left(\frac{-\sigma^2(T-t)}{2}\right)\exp\left(d_+\sigma\sqrt{T-t}\right) = \phi(-d_+)\exp\left(\frac{-\sigma^2(T-t)}{2}\right)\exp\left(\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)\right)$$

$$= \phi(-d_+) \cdot \exp\left(\frac{-\sigma^2(T-t)}{2}\right) \cdot \frac{S}{K} \cdot \exp\left((r-q)(T-t)\right) \cdot \exp\left(\frac{\sigma^2(T-t)}{2}\right) = \frac{S\phi(-d_+)e^{r(T-t)}e^{-q(T-t)}}{K}$$

Substituting $\phi(-d_-) = \frac{S\phi(-d_+)e^{r(T-t)}e^{-q(T-t)}}{K}$ in *Conjecture 1*:

$$\Rightarrow \text{RHS} = \frac{Ke^{-r(T-t)}\phi(-d_-)}{S\sigma\sqrt{T-t}} = \frac{Ke^{-r(T-t)}}{S\sigma\sqrt{T-t}} \cdot \frac{S\phi(-d_+)e^{r(T-t)}e^{-q(T-t)}}{K} = \frac{e^{-q(T-t)}\phi(-d_+)}{\sigma\sqrt{T-t}} = \text{LHS}$$

$$\therefore \frac{e^{-q(T-t)}\phi(-d_+)}{\sigma\sqrt{T-t}} = \frac{Ke^{-r(T-t)}\phi(-d_-)}{S\sigma\sqrt{T-t}}$$

Applying *Conjecture 1* $\Rightarrow \Delta(P) = \frac{e^{-q(T-t)}\phi(-d_+)}{\sigma\sqrt{T-t}} - \frac{Ke^{-r(T-t)}\phi(-d_-)}{S\sigma\sqrt{T-t}} - e^{-q(T-t)}\Phi(-d_+)$

$$\therefore \Delta(P) = \frac{\partial P}{\partial S} = \underline{\underline{-e^{-q(T-t)}\Phi(-d_+)}}$$

Verifying answer using Put-Call parity and $\Delta(C)$:

We know $\Delta(C) = \frac{\partial C}{\partial S} = e^{-q(T-t)}\Phi(d_+)$

Put-Call parity states $P = C - Se^{-q(T-t)} + Ke^{-r(T-t)}$

$$\frac{\partial P}{\partial S} = \frac{\partial}{\partial S}\left[C - Se^{-q(T-t)} + Ke^{-r(T-t)}\right] = \frac{\partial C}{\partial S} + \frac{\partial}{\partial S}\left[Ke^{-r(T-t)} - Se^{-q(T-t)}\right] = \frac{\partial C}{\partial S} - e^{-q(T-t)}$$

$$\Rightarrow \text{Let } \frac{\partial P}{\partial S} + e^{-q(T-t)} = \frac{\partial C}{\partial S}$$

$$\text{LHS} = \frac{\partial P}{\partial S} + e^{-q(T-t)} = \frac{\partial P}{\partial S} = -e^{-q(T-t)}\Phi(-d_+) + e^{-q(T-t)} = e^{-q(T-t)}(1 - \Phi(-d_+))$$

$$\text{But, } 1 - \Phi(-x) = \Phi(x)$$

$$\Rightarrow e^{-q(T-t)}(1 - \Phi(-d_+)) = e^{-q(T-t)}\Phi(d_+) = \frac{\partial C}{\partial S} = \text{RHS}$$

$$\boxed{\therefore \Delta(P) = -e^{-q(T-t)}\Phi(-d_+)}$$

Part (b)

$$\Gamma(P) = \frac{\partial^2 P}{\partial S^2} = \frac{\partial \Delta(P)}{\partial S}$$

$$\Rightarrow \frac{\partial \Delta(P}{\partial S} = \frac{\partial}{\partial S}\left[-e^{-q(T-t)}\Phi(-d_+)\right] = -e^{-q(T-t)}\frac{\partial}{\partial S}\left[\Phi(-d_+)\right]$$

$$\text{From Part (a), we know } \frac{\partial}{\partial S}\left[\Phi(-d_\pm)\right] = \frac{-\phi(-d_\pm)}{S\sigma\sqrt{T-t}}$$

$$\therefore \Gamma(P) = -e^{-q(T-t)}\frac{\partial}{\partial S}\left[\Phi(-d_+)\right] = \underline{\underline{\frac{e^{-q(T-t)}\phi(-d_+)}{S\sigma\sqrt{T-t}}}}$$

Verifying answer using Put-Call parity and $\Gamma(C)$:

$$\text{We know } \Gamma(C) = \frac{\partial^2 C}{\partial S^2} = \frac{e^{-q(T-t)}\phi(d_+)}{S\sigma\sqrt{T-t}}$$

$$\text{Put-Call parity states } P = C - Se^{-q(T-t)} + Ke^{-r(T-t)}$$

$$\frac{\partial^2 P}{\partial S^2} = \frac{\partial^2}{\partial S^2}\left[C - Se^{-q(T-t)} + Ke^{-r(T-t)}\right] = \frac{\partial^2 C}{\partial S^2} + \frac{\partial^2}{\partial S^2}\left[Ke^{-r(T-t)} - Se^{-q(T-t)}\right] = \frac{\partial^2 C}{\partial S^2} - \frac{\partial}{\partial S}\left[e^{-q(T-t)}\right]$$

$$\Rightarrow \text{Let } \frac{\partial^2 P}{\partial S^2} = \frac{\partial^2 C}{\partial S^2}$$

$$\text{LHS} = \frac{\partial^2 P}{\partial S^2} = \frac{e^{-q(T-t)}\phi(-d_+)}{S\sigma\sqrt{T-t}} = \frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\frac{1}{\sqrt{2\pi}}e^{-(-d_+^2)/2} = \frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\frac{1}{\sqrt{2\pi}}e^{-d_+^2/2}$$

$$\text{But, } \phi(d_+) = \frac{1}{\sqrt{2\pi}}e^{-d_+^2/2}$$

$$\Rightarrow \frac{e^{-q(T-t)}}{S\sigma\sqrt{T-t}}\frac{1}{\sqrt{2\pi}}e^{-d_+^2/2} = \frac{e^{-q(T-t)}\phi(d_+)}{S\sigma\sqrt{T-t}} = \frac{\partial^2 C}{\partial S^2} = \text{RHS}$$

$$\boxed{\therefore \Gamma(P) = \frac{\partial^2 P}{\partial S^2} = \frac{e^{-q(T-t)}\phi(-d_+)}{S\sigma\sqrt{T-t}}}$$

Part (c)

$$\Theta(P) = \frac{\partial P}{\partial t}$$

$$\Rightarrow \frac{\partial P}{\partial t} = \frac{\partial}{\partial t}\left[Ke^{-r(T-t)}\Phi(-d_-) - Se^{-q(T-t)}\Phi(-d_+)\right] = K\frac{\partial}{\partial t}\left[e^{-r(T-t)}\Phi(-d_-)\right] - S\frac{\partial}{\partial t}\left[e^{-q(T-t)}\Phi(-d_+)\right]$$

Applying the product rule:

$$= K\Phi(-d_-)\frac{\partial}{\partial t}\left[e^{-r(T-t)}\right] + Ke^{-r(T-t)}\frac{\partial}{\partial t}\left[\Phi(-d_-)\right] - S\Phi(-d_+)\frac{\partial}{\partial t}\left[e^{-q(T-t)}\right] - Se^{-q(T-t)}\frac{\partial}{\partial t}\left[\Phi(-d_+)\right]$$

$$= rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+) + Ke^{-r(T-t)}\frac{\partial}{\partial t}\left[\Phi(-d_-)\right] - Se^{-q(T-t)}\frac{\partial}{\partial t}\left[\Phi(-d_+)\right]$$

Consider $\frac{\partial}{\partial t}\left[\Phi(-d_\pm)\right]$

Applying the chain rule: $\frac{\partial}{\partial t}\left[\Phi(-d_\pm)\right] = \frac{\partial}{\partial(-d_\pm)}\left[\Phi(-d_\pm)\right] \cdot \frac{\partial}{\partial t}\left[-d_\pm\right]$

As per the fundamental theorum of calculus (FTC): $\frac{\partial}{\partial(-d_\pm)}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm)$

$$\therefore \frac{\partial}{\partial t}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm)\frac{\partial}{\partial t}\left[-d_\pm\right]$$

Substiting $\frac{\partial}{\partial t}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm)\frac{\partial}{\partial t}\left[-d_\pm\right]$ in $\Theta(P)$:

$$= rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+) + Ke^{-r(T-t)}\phi(-d_-)\frac{\partial}{\partial t}\left[-d_-\right] - Se^{-q(T-t)}\phi(-d_+)\frac{\partial}{\partial t}\left[-d_+\right]$$

Recall *Conjecture 1:* $\frac{e^{-q(T-t)}\phi(-d_+)}{\sigma\sqrt{T-t}} = \frac{Ke^{-r(T-t)}\phi(-d_-)}{S\sigma\sqrt{T-t}}$

$$\Rightarrow \frac{Se^{-q(T-t)}\phi(-d_+)}{\sigma\sqrt{T-t}} = \frac{Ke^{-r(T-t)}\phi(-d_-)}{\sigma\sqrt{T-t}}$$

$\therefore$ Let *Conjecture 2:* $Se^{-q(T-t)}\phi(-d_+) = Ke^{-r(T-t)}\phi(-d_-)$

Applying *Conjecture 2*:

$$\Rightarrow \Theta(P) = rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+) + Ke^{-r(T-t)}\phi(-d_-)\left[\frac{\partial}{\partial t}\left[-d_-\right] - \frac{\partial}{\partial t}\left[-d_+\right]\right]$$

$$= rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+) + Ke^{-r(T-t)}\phi(-d_-)\frac{\partial}{\partial t}\left[(-d_-) - (-d_+)\right]$$

We know $d_- = d_+ - \sigma\sqrt{T-t}$

$$\Rightarrow (-d_-) - (-d_+) = d_+ - d_- = d_+ - (d_+ - \sigma\sqrt{T-t}) = \sigma\sqrt{T-t}$$

$$\Rightarrow \Theta(P) = rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+) + Ke^{-r(T-t)}\phi(-d_-)\frac{\partial}{\partial t}\left[\sigma\sqrt{T-t}\right]$$

$$\therefore \Theta(P) = \frac{\partial P}{\partial t} = \underline{\underline{-\frac{\sigma K e^{-r(T-t)}\phi(-d_-)}{2\sqrt{T-t}} + rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+)}}$$

Verifying answer using Put-Call parity and $\Theta(C)$:

$$\text{We know } \Theta(C) = \frac{\partial C}{\partial t} = -\frac{\sigma K e^{-r(T-t)}\phi(d_-)}{2\sqrt{T-t}} + qSe^{-q(T-t)}\Phi(d_+) - rKe^{-r(T-t)}\Phi(d_-)$$
$$\text{Put-Call parity states } P = C - Se^{-q(T-t)} + Ke^{-r(T-t)}$$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial t}\left[C - Se^{-q(T-t)} + Ke^{-r(T-t)}\right] = \frac{\partial C}{\partial t} + \frac{\partial}{\partial t}\left[Ke^{-r(T-t)} - Se^{-q(T-t)}\right] = \frac{\partial C}{\partial t} + rKe^{-r(T-t)} - qSe^{-q(T-t)}$$

$$\Rightarrow \text{Let } \frac{\partial P}{\partial t} + qSe^{-q(T-t)} - rKe^{-r(T-t)} = \frac{\partial C}{\partial t}$$
$$\text{LHS} = \frac{\partial P}{\partial t} + qSe^{-q(T-t)} - rKe^{-r(T-t)}$$
$$= -\frac{\sigma K e^{-r(T-t)}\phi(-d_-)}{2\sqrt{T-t}} + rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+) + qSe^{-q(T-t)} - rKe^{-r(T-t)}$$
$$= -\frac{\sigma K e^{-r(T-t)}\phi(-d_-)}{2\sqrt{T-t}} - rKe^{-r(T-t)}(1 - \Phi(-d_-)) + qSe^{-q(T-t)}(1 - \Phi(-d_+))$$
$$\text{But, } 1 - \Phi(-x) = \Phi(x)$$
$$\Rightarrow -\frac{\sigma K e^{-r(T-t)}\phi(-d_-)}{2\sqrt{T-t}} - rKe^{-r(T-t)}(1 - \Phi(-d_-)) + qSe^{-q(T-t)}(1 - \Phi(-d_+))$$
$$= -\frac{\sigma K e^{-r(T-t)}\phi(-d_-)}{2\sqrt{T-t}} + qSe^{-q(T-t)}\Phi(d_+) - rKe^{-r(T-t)}\Phi(d_-)$$
$$= -\frac{\sigma K e^{-r(T-t)}}{2\sqrt{T-t}}\frac{1}{\sqrt{2\pi}}e^{-(-d_-)^2/2} + qSe^{-q(T-t)}\Phi(d_+) - rKe^{-r(T-t)}\Phi(d_-)$$
$$= -\frac{\sigma K e^{-r(T-t)}}{2\sqrt{T-t}}\frac{1}{\sqrt{2\pi}}e^{-(d_-)^2/2} + qSe^{-q(T-t)}\Phi(d_+) - rKe^{-r(T-t)}\Phi(d_-)$$
$$= -\frac{\sigma K e^{-r(T-t)}\phi(d_-)}{2\sqrt{T-t}} + qSe^{-q(T-t)}\Phi(d_+) - rKe^{-r(T-t)}\Phi(d_-) = \frac{\partial C}{\partial t} = \text{RHS}$$

$$\boxed{\therefore \Theta(P) = -\frac{\sigma K e^{-r(T-t)}\phi(-d_-)}{2\sqrt{T-t}} + rKe^{-r(T-t)}\Phi(-d_-) - qSe^{-q(T-t)}\Phi(-d_+)}$$

Part (d)

$$\rho(P) = \frac{\partial P}{\partial r}$$
$$\Rightarrow \frac{\partial P}{\partial r} = \frac{\partial}{\partial r}\left[Ke^{-r(T-t)}\Phi(-d_-) - Se^{-q(T-t)}\Phi(-d_+)\right] = K\frac{\partial}{\partial r}\left[e^{-r(T-t)}\Phi(-d_-)\right] - Se^{-q(T-t)}\frac{\partial}{\partial r}\left[\Phi(-d_+)\right]$$
$$\text{Applying the product rule:}$$
$$\Rightarrow K\frac{\partial}{\partial r}\left[e^{-r(T-t)}\Phi(-d_-)\right] - Se^{-q(T-t)}\frac{\partial}{\partial r}\left[\Phi(-d_+)\right]$$

$$= K\Phi(-d_-)\frac{\partial}{\partial r}\left[e^{-r(T-t)}\right] + Ke^{-r(T-t)}\frac{\partial}{\partial r}\left[\Phi(-d_-)\right] - Se^{-q(T-t)}\frac{\partial}{\partial r}\left[\Phi(-d_+)\right]$$

$$= -K(T-t)e^{-r(T-t)}\Phi(-d_-) + Ke^{-r(T-t)}\frac{\partial}{\partial r}\left[\Phi(-d_-)\right] - Se^{-q(T-t)}\frac{\partial}{\partial r}\left[\Phi(-d_+)\right]$$

Consider $\frac{\partial}{\partial r}\left[\Phi(-d_\pm)\right]$. Applying the chain rule: $\frac{\partial}{\partial r}\left[\Phi(-d_\pm)\right] = \frac{\partial}{\partial(-d_\pm)}\left[\Phi(-d_\pm)\right]\cdot\frac{\partial}{\partial r}\left[-d_\pm\right]$

As per the fundamental theorum of calculus (FTC): $\frac{\partial}{\partial(-d_\pm)}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm)$

$$\therefore \frac{\partial}{\partial r}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm)\frac{\partial}{\partial r}\left[-d_\pm\right]$$

Substituting $\frac{\partial}{\partial r}\left[\Phi(-d_\pm)\right] = \phi(-d_\pm)\frac{\partial}{\partial r}\left[-d_\pm\right]$ in $\rho(P)$:

$$= -K(T-t)e^{-r(T-t)}\Phi(-d_-) + Ke^{-r(T-t)}\phi(-d_-)\frac{\partial}{\partial r}\left[-d_-\right] - Se^{-q(T-t)}\phi(-d_+)\frac{\partial}{\partial r}\left[-d_+\right]$$

Recall *Conjecture 2:* $Se^{-q(T-t)}\phi(-d_+) = Ke^{-r(T-t)}\phi(-d_-)$. Applying *Conjecture 2* to $\rho(P)$:

$$\Rightarrow \rho(P) = -K(T-t)e^{-r(T-t)}\Phi(-d_-) + Ke^{-r(T-t)}\phi(-d_-)\frac{\partial}{\partial r}\left[(-d_-)-(-d_+)\right]$$

We know $d_- = d_+ - \sigma\sqrt{T-t}$

$$\Rightarrow (-d_-)-(-d_+) = d_+ - d_- = d_+ - (d_+ - \sigma\sqrt{T-t}) = \sigma\sqrt{T-t}$$

$$\Rightarrow \rho(P) = -K(T-t)e^{-r(T-t)}\Phi(-d_-) + Ke^{-r(T-t)}\phi(-d_-)\frac{\partial}{\partial r}\left[\sigma\sqrt{T-t}\right]$$

$$\therefore \rho(P) = \frac{\partial P}{\partial r} = \underline{\underline{-K(T-t)e^{-r(T-t)}\Phi(-d_-)}}$$

Verifying answer using Put-Call parity and $\rho(C)$:

We know $\rho(C) = K(T-t)e^{-r(T-t)}\Phi(d_-)$

Put-Call parity states $P = C - Se^{-q(T-t)} + Ke^{-r(T-t)}$

$$\frac{\partial P}{\partial r} = \frac{\partial}{\partial r}\left[C - Se^{-q(T-t)} + Ke^{-r(T-t)}\right] = \frac{\partial C}{\partial r} + \frac{\partial}{\partial r}\left[Ke^{-r(T-t)} - Se^{-q(T-t)}\right] = \frac{\partial C}{\partial r} - K(T-t)e^{-r(T-t)}$$

$$\Rightarrow \text{Let } \frac{\partial P}{\partial r} + K(T-t)e^{-r(T-t)} = \frac{\partial C}{\partial r}$$

$$\text{LHS} = \frac{\partial P}{\partial r} + K(T-t)e^{-r(T-t)} = -K(T-t)e^{-r(T-t)}\Phi(-d_-) + K(T-t)e^{-r(T-t)}$$

$$= K(T-t)e^{-r(T-t)}(1-\Phi(-d_-))$$

But, $1-\Phi(-x) = \Phi(x)$

$$\Rightarrow K(T-t)e^{-r(T-t)}(1-\Phi(-d_-)) = K(T-t)e^{-r(T-t)}\Phi(d_-) = \frac{\partial C}{\partial r} = \text{RHS}$$

$$\boxed{\therefore \rho(P) = -K(T-t)e^{-r(T-t)}\Phi(-d_-)}$$