Discrete Structure for Computer Science (CS F222)
Assignment 3

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Question 1:

Yukihira Soma has designed a new recipe for the upcoming cooking competition. A recipe is made up of a list of ingredients, with spices connecting them (it is weird, but that is how it is taught in the Totsuki Culinary Academy). It turns out that his opponent Subaru Mimasaka is known for copying recipes. Given both the recipes, you need to tell, as a judge, if Mimasaka has copied the recipe or not. That is, given the graphs of the two recipes, you need to tell if they are isomorphic or not.

Input format:

First line contains N, the number of nodes in the first graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the first graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) "link of spice" between node A and B. Next line contains M, the number of nodes in the second graph (implicitly labeled from 0 to M-1). Next line contains F, the number of edges in the second graph. The next F lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) edge between node A and B.

Output format:

"Yes" if the two graphs are isomorphic, "No" otherwise.

Example:

Input:

```
4
5
0 1
1 2
2 3
3 0
0 2
4
5
0 2
2 1
1 3
3 0
3 2
```

Output:

Yes

Explanation:

The following bijection from nodes(graph1) to nodes(graph2) exists: 0->3 1->0 2->2 3->1. This bijection fulfils the isomorphic condition.

Question 2:

Morty is doing his Discrete Structures in Computer Science homework, and the first question asks him to find whether two graphs are isomorphic or not (which is very poorly wrapped in a story about some cooking Anime). Rick reads the question and says that Morty should aim for more! Just finding if the two graphs are isomorphic or not is not enough. Morty should find all the possible isomorphism bijections between the nodes as well! Help Morty do exactly that.

Input format:

First line contains N, the number of nodes in the first graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the first graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) link between node A and B. Next line contains M, the number of nodes in the second graph (implicitly labeled from 0 to M-1). Next line contains F, the number of edges in the second graph. The next F lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) edge between node A and B.

Output format:

If the graphs are not isomorphic, print "No bijection possible". Otherwise, print each isomorphic bijection from the nodes of the first graph to the nodes of the second. Check the output example for the format of the bijections.

Example:

Input:

```
4
4
0 1
1 2
2 3
3 0
4
4
0 2
2 1
1 3
3 0
```

Output:

```
0->0 1->2 2->1 3->3
0->0 1->3 2->1 3->2
0->1 1->2 2->0 3->3
0->1 1->3 2->0 3->2
0->2 1->0 2->3 3->1
```

0->2 1->1 2->3 3->0
0->3 1->0 2->2 3->1
0->3 1->1 2->2 3->0

Explanation:

There are 8 isomorphic bijections possible. For example, the first one maps graph1[node0] to graph2[node0], graph1[node1] to graph2[node2], graph1[node2] to graph2[node1] and graph1[node3] to graph2[node3].

Question 3:

The Fire Nation has a very aggressive annexation policy. Each year, of all the cities not in their kingdom, they select the one which has the shortest distance to any part of their kingdom and annex it. Your job, as an Earth Kingdom data analyst, is to figure out what their territory will look like after some years. The entire continent is described to you as an undirected graph, with nodes as cities and the weights of the edges representing the distance between the two cities (need not be complete). You are also given the initial city of the Fire Nation. Print the cities in the fire nation after K years.

Input format:

First line contains N, the number of nodes in the graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the graph. The next E lines contain three integers each. If a line contains "A B C", it means that there is a (bidirectional) link between node A and B with distance C (>0). Next line contains the starting node for the Fire Nation. The next line contains the number of years (K). Note: The number of cities in the final empire will be 1 more than the number of years.

Output format:

One line containing a list of K + 1 integers, representing the cities that will be in the Fire Nation after K years. Any order is fine.

Example:

Input:

4
6
0 1 1
1 2 2
2 3 3
3 0 4
0 2 6
3 1 5
0
2

Output:

0 1 2

Explanation:

In the first year, the city 1 is conquered (since it has a distance of only 1). In the next year, since city 2 is the closest to the empire (dist(1, 2) = 2), it is annexed.

Question 4:

Yukihira has once again come up with a new recipe. His senior Gin thinks that his cooking is influenced by the his father's. Given a graph of this new recipe and a graph of the cooking style of Yukihira's father, check if the latter is a subgraph of the former. Note: The subgraph need not be a vertex-induced subgraph.

Input format:

First line contains N, the number of nodes in the first graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the first graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) link between node A and B. Next line contains M, the number of nodes in the second graph (implicitly labeled from 0 to M-1). Next line contains F, the number of edges in the second graph. The next F lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) edge between node A and B.

Output format:

"Yes" if the second graph is a subgraph of the first. "No" otherwise.

Example:

Input:

```
6
7
0 2
0 1
1 2
2 3
2 5
3 4
4 5
5
3
0 1
0 3
0 2
```

Output:

Yes

Explanation:

If you map graph2[node0] to graph1[node2], graph2[node1] to graph1[node0], graph2[node2] to graph1[node5] and graph2[node3] to graph1[node3] and graph2[node4] to graph1[node1] you see that graph2 is a subgraph of graph1. Of course, multiple such mappings may be possible.

Question 5:

Given two graphs, check if the latter is a VERTEX INDUCED SUBGRAPH of the latter.

Input format:

First line contains N, the number of nodes in the first graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the first graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) link between node A and B. Next line contains M, the number of nodes in the second graph (implicitly labeled from 0 to M-1). Next line contains F, the number of edges in the second graph. The next F lines contain two integers each. If a line contains "A B", it means that there is a (bidirectional) edge between node A and B.

Output format:

"Yes" if the second graph is a VERTEX INDUCED SUBGRAPH of the first. "No" otherwise.

Input:

```
6
7
0 2
0 1
1 2
2 3
2 5
3 4
4 5
5
3
0 1
0 3
0 2
```

Output:

No

Explanation:

Unlike the previous problem, here the subgraph must be vertex induced. That is, all the edges possible must be included.

Question 6:

Thanos is going from planet to planet in search of the Soul stone. Each planet has a positive integer label. Due to some limitations of Thanos's spaceship, he can only move to a different planet in a particular order:

1) He has K functions of the form f_i(n) = (a * n + b) % M
2) When on a planet "N", he can choose one of the K functions and apply it to N. He can then move to the resultant planet.
3) M is the same for all functions

You are given his starting planet and the planet on which the Soul stone exists. Find a path that can help Thanos reach the Soul stone. If no such path is possible, say so.

Input format:

First line contains M (see the function format). Second line contains K, the number of functions (implicitly labeled from 0 to K-1). The next K lines contain two integers each. If the ith line contains, "A B" (both >= 0), it means that the ith function is (A * n + B) % M. The next line contains one integer, representing the planet that Thanos starts from. The next line contains one integer representing the planet with the Soul stone.

Output format:

If no path is possible from the starting planet to the goal planet, print "No path possible". Else, print "function_ID New_planet_ID" for the entire path (Check example and it's explanation). If multiple paths are possible, print any one of them.

Example:

Input:

5
3
2 1
1 3
1 1
2
4

Output:

0 0
0 1
0 3
2 4

Explanation:

Initially, we start from 2. Now, the first line of output says that apply function0 (2 * 2 + 1) % 5 and you get 0. The second line says, apply funtion0 again and get (2 * 0 + 1) % 5 = 1. The third line says apply function0 again and get (2 * 1 + 1) % 5 = 3. The final line says apply function2 and get (1 * 3 + 1) % 5 = 4, finally reaching the destination.

Question 7:

Goku and his friends are participating in the Tournament of Power. The Grand Priest has decided to divide all the participants into two groups for the group stages. However, he does not want close friends to be in the same group. Help him figure out a way to divide them into such groups. All the participants are given to you as a graph, with edges between them representing "close friendship". You may assume that the given graph (which is undirected) will always have exactly one connected component.

Input Format:

First line contains N, the number of nodes (participants) in the graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (undirected) friendship between node A and B.

Output Format:

If a division in two is not possible, print "Not possible".
Else, in two lines, print the members of each group as space separated integers.

Example:

Input:

```
8
7
0 4
0 5
0 6
1 6
1 7
2 5
3 5
```

Output:

```
0 1 2 3
4 5 6 7
```

Question 8:

Given a connected, planar graph, calculate the number of faces including the unbounded face.

Input format:

First line contains N, the number of nodes in the graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (undirected) edge between node A and B.

Output format:

One integer representing the number of faces in the graph.

Example:

Input:

5
7
0 1
0 2
1 2
1 3
3 2
2 4
3 4

Output:

4

Question 9:

The local conspiracy theorist Rorschach has a new obsession with the Illuminati. He is convinced that the New York City was created by them. He thinks that on a map of the NYC, a gazillion triangles (which obviously symbolize the Illuminati) are evident. Help him find the number of triangles given the map of NYC. The map is given to you as an undirected graph. Note: A triangle in a graph is a triplet of nodes <A, B, C> such that [A B], [B C] and [C A] are all in the edge list of the graph.

Input format:

First line contains N, the number of nodes in the graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (undirected) edge between node A and B.

Output format:

One integer, representing the number of triangles in the graph.

Example:

5
7
0 1
1 2
0 2
1 3
2 3
3 4
4 2

Output:

3

Explanation:

The triangles are <0, 1, 2>, <1, 2, 3>, <2, 3, 4>.

Question 10:

Given a graph and two vertices, print a path from the first vertex to the second.

Input format:

First line contains N, the number of nodes in the graph (implicitly labeled from 0 to N-1). Next line contains E, the number of edges in the graph. The next E lines contain two integers each. If a line contains "A B", it means that there is a (undirected) edge between node A and B. The next two lines contain one integer each, representing the start and the end vertex.

Output format:

If a path from start to end does not exist, print "No path exists". Else, in one line, print a list of integers representing the nodes encountered in the path from start to end.

Example:

Input:

```
5
7
0 1
1 2
0 2
1 3
3 2
3 4
4 2
0
4
```

Output:

```
0 1 2 3 4
```