

Discrete Structure for Computer Science (CS F222)

Assignment 2

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Question 1:

Rick and Morty are off on their adventures, going from one dimension to another like always. Rick is adamant on covering all the dimensions on their list for today but doesn't want to visit a dimension twice (he gets bored very quickly). Moreover, since his portal gun is almost running out of battery, he can only hop to the "nearby" dimensions from a particular dimension. Help him find an inter-dimensional path which helps him cover the entire list and leads him back home (the famed C137) without repeating any.

Input format:

First line contains N, the number of dimensions (implicitly labeled from 0 to N-1). The 0th dimension is the home. The next line contains M, the number of "links" between nearby dimensions (Read on if this is not clear). The next M lines contain two integers each. If a line has "A B", then it means that A is near to B, that is, Rick can travel from A to B or B to A.

Output format:

One line containing a list of integers, representing the order that Rick and Morty must follow, starting from dimension 0, which covers all the dimensions and leads back to 0, without repeating a single one. Assume that such a cycle is possible. Print any one if multiple are possible.

Example:

Input:

```
5
7
0 1
0 2
0 4
4 2
2 3
1 2
1 3
```

Output:

0 1 3 2 4 0

Explanation:

They can follow the path 0->1->3->2->4->0 and cover all the dimensions and come back home.

Question 2:

The demons from "Upside-Down" have dug a maze of tunnels under the city of Hawkins. Thankfully, the Hawkins National Laboratory has designed a new virus that once released, spreads through all the connected tunnels quickly and kills all the dema-dogs in them. However, these "virus bombs" are expensive and they need to find the exact number of these bombs required. Help them find that.

Input format:

The tunnel maze is represented as an undirected graph, with junctions as nodes and the tunnels as edges. The first line contains N, the number of junctions (implicitly labeled from 0 to N-1). The next line contains M, the number of tunnels. The next M lines contain two integers each. If a line has "A B", then it means that junction A is connected to B and B is connected to A.

Output format:

The minimum number of virus bombs needed to kill all the dema-dogs in the tunnel maze, assuming that a virus bombed dropped anywhere kills all the dema-dogs in all the connected tunnels.

Example:

Input:

5
4
0 4
1 2
2 3
3 1

Output:

2

Explanation:

The first virus bomb can take care of the $\langle 0, 4 \rangle$ section of the maze (since it is connected). The second can take care of the $\langle 1, 2, 3 \rangle$ section of the maze.

Question 3:

NOTE: The "graph" of all the dimensions is directed in this question, unlike Question 1 where it is undirected.

Our friend Rick has just returned from an inter-dimensional party and is "mildly inebriated" in his own words. Therefore, he's traversing the multi-verse in a random manner. He can go from one dimension to another using his portal gun, but since the battery is low, he can only go to the nearby dimensions. He traverses in the following manner:

1) He starts from dimension 0.

2) When in a particular dimension:

If there are any "nearby" dimensions (remember that "nearby-ness" is directed):

Pick one of the reachable dimensions at random (with equal probability) and go there.
(Example: If there are 3 reachable dimensions, each can be picked with a probability of $1/3$)

If there are no "nearby" dimensions:

Select a dimension from ALL the dimensions at random (with equal probability of $1/N$) and go there. ALL includes all dimensions from 0 to $N-1$. Note that he can stay in the same dimension as well in this case with a probability of $1/N$.

Your job is to find out the probability of Rick being in the different dimensions after a loooooonnnngggg time.

Input format:

First line contains N , the number of dimensions (implicitly labeled from 0 to $N-1$). The 0th dimension is the home. The next line contains M , the number of "links" between nearby dimensions. The next M lines contain two integers each. If a line has "A B", then it means that A is near to B, that is, Rick can travel from A to B. This DOES NOT imply that Rick can travel from B to A.

Output format:

N lines. Each line has one decimal value. The value at the i th line represents the probability of Rick being in dimension i after a very long time. Sanity check: The sum of the values should be one.

Example:

Input:

2

1
0 1

Output:

0.3333333
0.6666667

Explanation:

Time 0: Rick at 0 with probability 1.0, Rick at 1 with probability 0.0 (initial condition)
Time 1: Rick at 0 with probability 0.0, Rick at 1 with probability 1.0 (can only move to 1)
Time 2: Rick at 0 with probability 0.5, Rick at 1 with probability 0.5
Time 3: Rick at 0 with probability $0.5 \cdot 0 + 0.5 \cdot 0.5 = 0.25$, Rick at 1 with probability $0.5 \cdot 1 + 0.5 \cdot 0.5 = 0.75$

.
.
.

Question 4:

NOTE: The graph is directed in this question.

The Survey Scouts are planning an expedition outside Wall Rose and Erwin Smith has identified certain towns to check for Titan activity, with roads between them. He wants to cover all the towns and return back to Trost without covering any town more than once. He has asked you to find the number of ways he can do that.

Input Format:

The first line contains N , the number of towns (implicitly labeled from 0 to $N-1$). Trost is town 0. The next line contains M , the number of roads. The next M lines contain two integers each. If a line has "A B", then it means that town B can be reached from A. This does not mean that town A can be reached by B.

Output Format:

One line with an integer representing the number of paths that can be taken, which cover each town exactly once and return to Trost.

Example:

Input:

5

7
0 1
0 2
2 1
1 3
2 3
3 4
4 0

Output:

1

Explanation:

The only possible path is 0->2->1->3->4->0

Question 5:

NOTE: undirected graph

Hawkins National Laboratory is laying down telephone lines in the entirety of Hawkins, connecting all the houses there. There are only a few feasible connections of laying down the lines, and each of these comes at a specific cost. Your job is to design a network which connects all the houses with the minimum total cost. However, it turns out that HNL has other shady motives of doing this. They want to be able to spy on a select few individuals. They ask you to necessarily include a few connections in your final network, minimizing the cost with this constraint in place. (Hint: Think a variation of MSTs). You need to print this minimum cost so that they can allocate the budget.

Input format:

The first line contains N, the number of houses (implicitly labeled from 0 to N-1). The second line contains M, the number of connections that are feasible (You must choose from these). The next M lines contain 3 integers each. If a line has "A B C", then house A can be connected to house B (and vice versa) with a cost of C (C is always positive). The next line contains an integer R, the number of necessary edges. The next R lines contain one integer each, representing the edge number that must necessarily be included. The "edge number" is the position (0 to M-1) where an edge appears in the preceding edge list (See example).

Output format:

One number representing the minimum cost of the network which connects all the houses but necessarily includes the given R edges.

Example:**Input:**

```
5
6
0 1 1
1 2 2
2 3 3
3 0 4
3 4 5
4 0 6
1
5
```

Output:

```
12
```

Explanation:

There is only 1 required edge ($R = 1$), that is edge number 5. The 5th position edge (starting from 0) in the edge list is "4 0 6". If this edge must be included, then the minimum tree that spans all 5 vertices is [$\langle 4, 0 \rangle$, $\langle 0, 1 \rangle$, $\langle 1, 2 \rangle$, $\langle 2, 3 \rangle$]. The sum of the weights of these edges is $6 + 1 + 2 + 3 = 12$.

Question 6:

Our local ghoul Ken Kaneki has joined the evil organization Aogiri Tree and wants to break his ghoul friends out of the Cochlea prison. However, he is not sure if there is a path from his current position to the prison. Help him figure this out.

The entire area is drawn as a graph, with nodes as buildings and edges as (bidirectional) roads.

Input format:

The first line contains N , the number of buildings (implicitly labeled from 0 to $N-1$). The next line contains M , the number of roads. The next M lines contain two integers each. If a line has "A B", then it means that town A and B are connected (undirected). The next 2 lines contain one integer each, representing Kaneki's current building and the Cochlea.

Output format:

"Yes" if there is a path from the current building to Cochlea. "No" otherwise.

Example:**Input:**

5
3
0 1
1 2
3 4
0
4

Output:

No

Question 7:

Right outside Wall Rose, the Titan activity has increased immensely. Captain Levi and his Survey Corps are heading out to check it out. Captain Levi has multiple locations of Titan activity marked on a map, but unfortunately, his team has a limited number of people. So he wants to cluster the activity locations into groups so that each member of his force can go check one such group. He needs your help for this. You are given the 2D coordinates of the various locations of Titan activity and the number of members in Levi's squad. Your job is to find the best way to group the 2D coordinates into the given number of groups. The best such grouping is defined as follows:

Definitions:

1) Let ALL be the set of all coordinates given. A GROUP is a subset of ALL. A GROUPING is a set of GROUPs such that there is no overlap between the GROUPs of a GROUPING, and the union of all the GROUPs of a GROUPING yields ALL. Basically grouping is partitioning the ALL set.

2) The number of GROUPs in your grouping must be equal to the number of members in Levi's squad.

3) We define the distance between two GROUPs A and B as follows:

$\text{dist}(A, B) = \min(\{\text{Squared_Euclid}(p, q) \text{ for } p \text{ in } A \text{ and } q \text{ in } B\})$. That is, $\text{dist}(A, B)$ is the minimum of all the $|A|*|B|$ squared Euclidean distances possible between the GROUPs.

4) Define goodness of a GROUPING G as follows:

$\text{goodness}(G) = \min(\{\text{dis}(A,B) \text{ for } A, B \text{ in } G\})$. That is, of all the combination($|G|, 2$) distances between GROUPs of the G , the minimum is the goodness of G .

Your job is to find the GROUPING with the maximum goodness. Hint: Although it looks complicated, it isn't. Think of coordinates as nodes of a complete graph with edge weights as the squared Euclidean distance. Then think about the case when $K = N-1$. Note that the number of coordinates in each GROUP of a GROUPING need not be the same. (See example).

Input format:

First line has N , the number of coordinates. The next N lines have 2 integers each, representing the N locations. Note that the locations are implicitly labeled 0 to $N-1$. The next line has K , the number of members in Levi's squad.

Output format:

K lines, each with a list of integers. The list in the i th line represents the labels of the locations in the i th group. The labels are as described in the Input Format (0 to $N-1$ based on input ordering). The order of the groups or the order of the elements in them does not matter. That is the K groups can be arranged in any order.

Example:

Input:

```
5
0 0
1 1
2 7
2 8
100 100
3
```

Output:

```
0 1
2 3
4
```

Explanation:

The first group has $\langle 0, 0 \rangle$ and $\langle 1, 1 \rangle$, so the 0th and the 1st location.
The second group has $\langle 2, 7 \rangle$ and $\langle 2, 8 \rangle$, so the 2nd and the 3rd location.
The third group has only $\langle 100, 100 \rangle$, so the 4th location.

Question 8:

Morty is studying about graphs and trees, and has come across the concept of Minimum Spanning Trees. Rick, being his usual self, tells him that MSTs taught in schools are a lie. A true MST is one where the weights of the edges are multiplied, not added. That is, a true MST is a spanning tree such that the product of its weight is the minimum of all the spanning trees. Help Morty figure out a way to calculate a true MST. Given a graph, print the "multiplicative cost" of its true MST.

Input format:

The first line contains N, the number of nodes in the graph (implicitly labeled from 0 to N-1). The next line contains M, the number of edges. The next M lines contain three integers each. If a line has "A B C", then it means that there is an undirected edge between A and B with weight C. C is always positive.

Output format:

One integer representing the "multiplicative cost" of the true MST of the graph.

Example:**Input:**

```
5
6
0 1 1
1 2 2
2 3 3
3 4 5
3 0 4
4 0 6
```

Output:

```
30
```

Explanation:

The true MST is [$\langle 0, 1 \rangle$, $\langle 1, 2 \rangle$, $\langle 2, 3 \rangle$, $\langle 3, 4 \rangle$]. The multiplication of the costs of the edges is 30.

Question 9:

Mikasa is swinging from street to street killing off all the Titans in her way. She wants to cover all the streets but doesn't want to go through a street more than once. Given the map as a graph of junctions and streets, tell her if such a path exists.

Input format:

The first line contains N , the number of nodes (junctions) in the graph (implicitly labeled from 0 to $N-1$). The next line contains M , the number of edges (streets). The next M lines contain two integers each. If a line has "A B", then it means that there is an undirected edge between A and B.

Output format:

"Yes" if such a path is possible. "No" otherwise.

Example:

Input:

```
5
8
0 1
0 2
0 3
1 2
1 3
2 3
2 4
3 4
```

Output:

```
Yes
```