Agenda

- What is Binary search ?
- Find an element in array
- Find first occurrence of element in array
- Unique Element
- Local Minima

Introduction to Searching

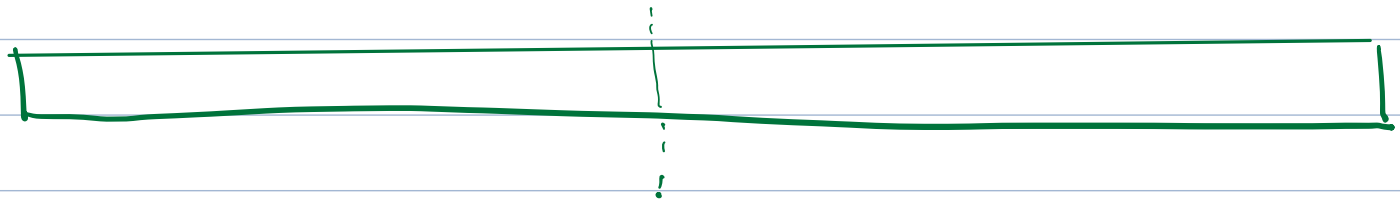
Target - What to search for

Search space - Where to search

Searching in organised search space is easier:

1. Dictionary
2. Phone book

Binary Search



Divide search space into 2 parts and based on some condition, we repeatedly neglect one-half of search space

↓

1. Till we get ans
2. No search space is left

At each step, search space is halved

1. Given a sorted array with distinct elements search index of an element k. If k is not present, return -1.

0	1	2	3	4	5	6	7	8	9
3	6	9	12	14	19	20	23	25	27

K = 12 ans = 3

K = 13 ans = -1

Brute Force: Linear search

Iterate entire array to check if k is present

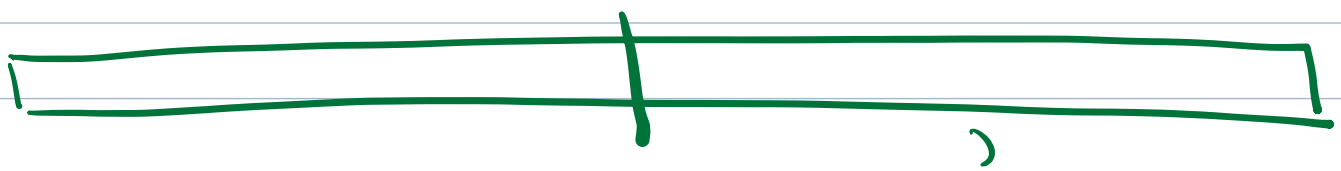
TC: O(N) SC: O(1)

Binary Search

Search space: Array (0 → N-1 idx)

Target: k

Condition:



A[mid] == k

return mid

else if (A[mid] < k)

go to right

else

go to left

0	1	2	3	4	5	6	7	8	9
3	6	9	12	14	19	20	23	25	27

K = 12

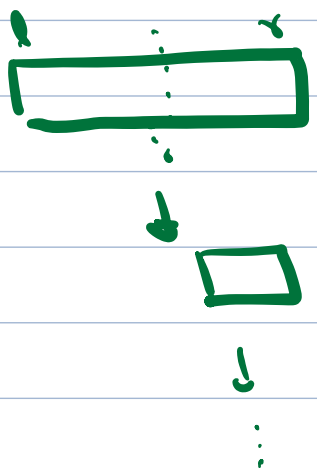
l	r	mid $\frac{l+r}{2}$	A[mid]	where to go
0	9	4	14	14 > 12 Go to left r = mid - 1
0	3	1	6	6 < 12 Go to right l = mid + 1
2	3	2	9	9 < 12 Go to right l = mid + 1
3	3	3	12	12 == 12 return 3

---

l	r	mid		
3	3	3	12	K = 13 12 < 13 Go to right l = mid + 1
4	3			l > r STOP

```
int search(int a[], int N, int K){
    l = 0, r = N-1
    while(l <= r){
        mid = (l+r)/2
        if(a[mid] == K)
            return mid
        else if(a[mid] < K)      // right
            l = mid+1
        else                     // K < a[mid]   // left
            r = mid-1
    }
    return -1
}
```

TC: $O(\log_2 N)$
SC: $O(1)$



N → N/2 → N/4 → ... → 1

Best practice to compute Mid

Let's assume that we have a datatype → dtype which has a range -100 to 100.

Array of length 100 ⇒ 0 to 99 indices

① dtype $l = 0$, $r = 99$

dtype $mid = \frac{l+r}{2}$

⇒ go right $l = mid + 1$

② $l = 50$ $r = 99$

$mid =$

$$Mid = \frac{l+r}{2} = \boxed{l + \frac{(r-l)}{2}}$$ ✓ Mid

$l = 50$ $r = 99$

2. All emails in your mailbox are sorted chronologically. Can you find first email that you received in 2020?

0	1	2	3	4	5	6	7	8
2005	2009	2013	2018	2019	2019	2020	2020	2020

9	10	11	12	13	14
2020	2021	2021	2022	2023	2024

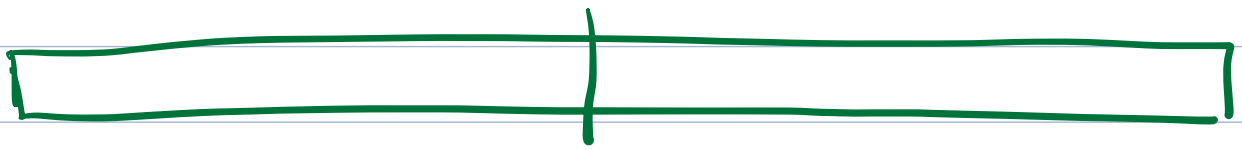
ans = 6 Sorted array of duplicates

Brute Force: Linear search from left to right and return idx / first time you find 2020

TC: O(N) SC: O(1)

Binary Search:

Search space: Array (0 → N-1 idx)
Target: First occurrence of 2020
Condition:



A[mid] == 2020
ans = mid
Search on left
r = mid-1

A[mid] < 2020 Right

l = mid + 1

A[mid] > 2020 Left

r = mid - 1

0	1	2	3	4	5	6	7	8
2005	2009	2013	2018	2019	2019	2020	2020	2020

9	10	11	12	13	14
2020	2021	2021	2022	2023	2024

K = 2020

l	r	mid $\frac{l+r}{2}$	A[mid]	Where to go	ans -1
0	14	7	2020	2020 == 2020 Update ans Left, r = mid - 1	7
0	6	3	2018	2018 < 2020 Right, l = mid + 1	7
4	6	5	2019	2019 < 2020 Right, l = mid + 1	7
6	6	6	2020	2020 == 2020 Update ans Left, r = mid - 1	6

6 5 l > r End of search

ans = 6

```
int search( int a[], int N, int K) {
    l = 0, r = N-1 , ans = -1
    while ( l <= r) {
        mid = (l+r) / 2
        if ( a[mid] == K) {
                ans = mid
                r = mid - 1          // left
        }
        else if ( a[mid] < K)        // right
                l = mid + 1
        else                         // a[mid] > K  -> left
                r = mid - 1
    }
    return ans
}
```

N → N/2 → ... → 1

TC : O(log₂N)

SC : O(1)

Q: Find last occurrence of an element k

Q: Find freq of an element in sorted arr

last occurrence idx - first occurrence idx + 1

10:25

3. Every element occurs twice except for 1, find the unique element

NOTE: Duplicate elements are adjacent to each other but the array is not sorted

	0	1	2	3	4	5	6
A →	8	8	5	5	6	2	2

BF : check every elem A[i] with A[i+1]

. . . .

Binary Search

Search space :

Target :

Condition :

```
int search (int a[], int N) {
```

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	3	1	1	8	8	10	10	19	6	6	2	2	4	4

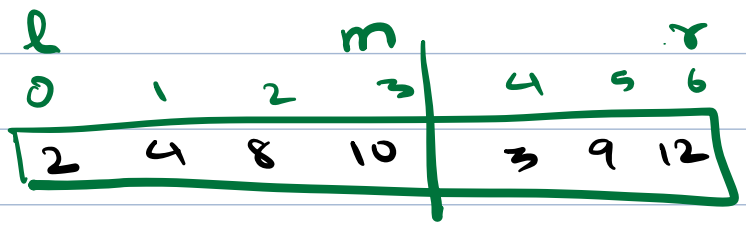
l r mid isAns firstOcc where to go

```
l = 0        r = N-1
while (l <= r) {
    mid = l + (r-l)/2

Case 1  if((mid==0 || A[mid] < A[mid-1])
        && (mid==N-1 || A[mid] < A[mid+1]))
            return A[mid]

Case 2  if(mid!=0 && A[mid] > A[mid-1]){
and 4       // left
            r = mid-1
        }
Case 3  else
            l = mid+1        // right
}
```

TC : $O(\log_2 N)$ SC : $O(1)$

Hashing, Sorting, Searching

↓ 2 ↓ 2 ↓ 3



$M = \frac{0+6}{2}$