

Agenda:

1. Intro to Graphs
2. Types of Graphs
3. Traversal: DFS
4. Detect cycle in directed graph

Additional  
Mocks ?

1. Mandatory Skill Evaluation DSA  
2.5 hrs

2. Mock Interview (2 hrs)

Intro (5 min)

Ques and Ans (25 min)

Feedback (5 min)

Vertices

Graphs : Collection of nodes and edges

Real life examples of Graph :

1. Computer Networks
2. Google Maps
3. Social Media

Types of Graph

1. Cyclic Graph - Graph which contains atleast one cycle. If we start from a node and return to same node without repeating an edge  $\rightarrow$  cycle

$\downarrow$   
closed path

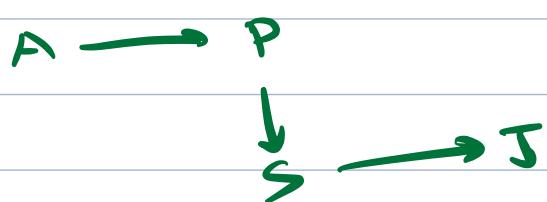


2. Acyclic Graph : graph with no cycles

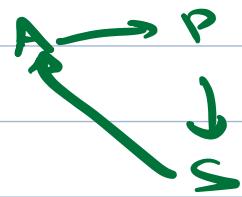
$\downarrow$   
No closed paths



3. Directed Graph: Edges have a direction, indicating one way relationship b/w nodes

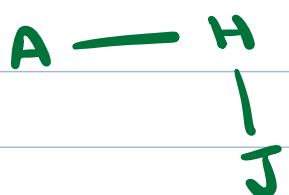


Directed Acyclic

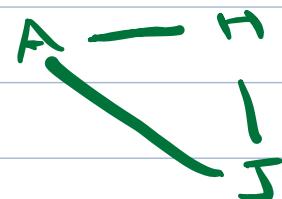


Directed Cyclic

4. Undirected Graph: Edges have no direction, representing symmetric relationship b/w nodes

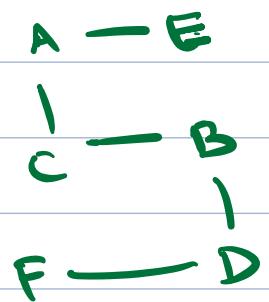


Undirected Acyclic

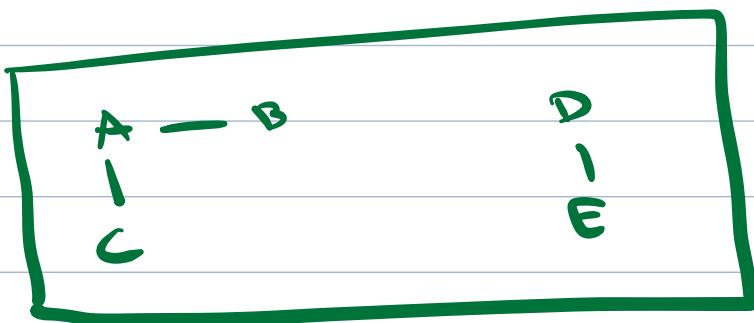


Undirected Cyclic

5. Connected Graph: A connected graph has a path b/w every pair of nodes. There are no isolated nodes.



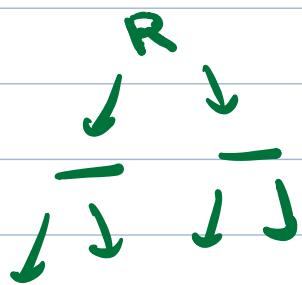
6. Disconnected Graph : Graph with at least 2 disconnected components (meaning there is no path b/w them)



Q. Is tree a graph? Yes

① Directed Acyclic Graph

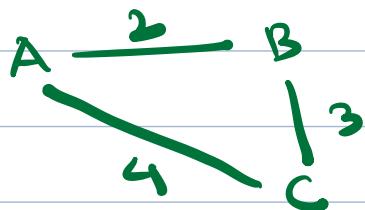
Tree  $\rightarrow$  DAG



② Connected

③ N nodes, N-1 edges

7. Weighted Graph : Edges have associated weights → used to represent distances / time / costs or other metrics.



8. Unweighted Graph : No associated weight on edges



①  
Directed  
undirected

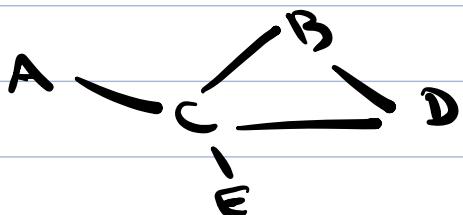
②  
Cyclic  
Acyclic

③  
Weighted  
unweighted

---

Degree of a vertex : No. of edges connected to vertex

Degree → Undirected graph



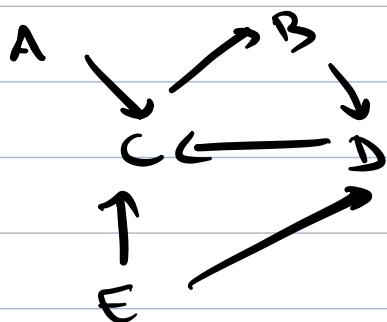
|   |   |
|---|---|
| A | 1 |
| B | 2 |
| C | 4 |
| D | 2 |
| E | 1 |

# Indegree and outdegree of a vertex

Directed graph

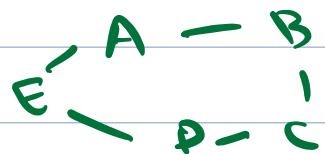
No. of incoming edges for a vertex

No. of outgoing edges for a vertex



|   | Indegree | Outdegree |
|---|----------|-----------|
| A | 0        | 1         |
| B | 1        | 1         |
| C | 3        | 1         |
| D | 2        | 1         |
| E | 0        | 2         |

Simple graph: No self loops or multiple edges b/w same pair of nodes

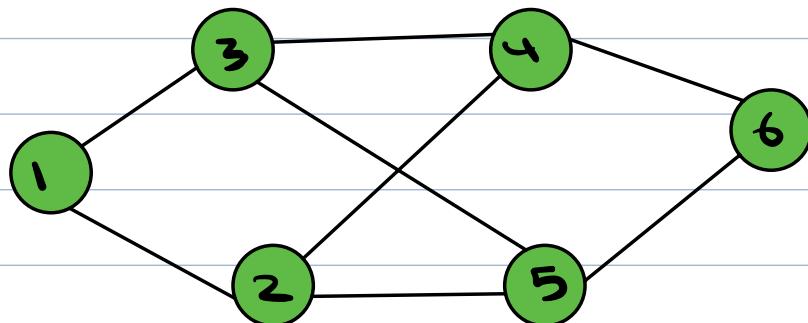


How to store graph?

Adjacency Matrix

Adjacency List

## ① Adjacency Matrix



$N \in$

edges

|   |   |   |
|---|---|---|
| 6 | 8 | ✓ |
| 1 | 3 | ✓ |
| 2 | 5 | ✓ |
| 3 | 3 | ✓ |
| 4 | 2 | ✓ |
| 5 | 5 | ✓ |
| 6 | 6 | ✓ |
| 7 | 5 | ✓ |
| 8 | 2 | ✓ |

①  $N \rightarrow 1$  to 6

②  $\text{adj}[7][7] = \text{adj}[N-1][N-1]$

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 1 | 1 |
| 3 | 0 | 1 | 0 | 0 | 1 | 1 |
| 4 | 0 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 0 | 1 | 1 | 0 | 1 |
| 6 | 0 | 0 | 0 | 0 | 1 | 0 |

①  $N$  nodes, 1 to  $N \rightarrow \text{adj}[N+1][N+1]$

②  $N$  nodes, 0 to  $N-1 \rightarrow \text{adj}[N][N]$

Q. Given array of edges, construct adjacency matrix

① // I/P

input << N << edges

int adj[N+1][N+1] = {0}

for (i=0 ; i < edges ; i++) {

    | input (u and v)

    | adj[u][v] = 1

    | adj[v][u] = 1

② // N, edges [ ] [ ]

[ [1,3]  
[2,5]  
[4,3] ]

int adj[N+1][N+1] = {0}

for (i=0 ; i < edges.size() ; i++) {

    | int u = edges[i][0]

    | int v = edges[i][1]

    | adj[u][v] = 1

    | adj[v][u] = 1

Directed ( $u \rightarrow v$ )

$u \sim v$

$$\text{adj}[u][v] = 1$$

Undirected ( $u - v$ )

$u \sim v$

$$\begin{aligned}\text{adj}[u][v] &= 1 \\ \text{adj}[v][u] &= 1\end{aligned}$$

Directed weighted

$u \sim v \omega$

$$\text{adj}[u][v] = \omega$$

Undirected weighted

$u \sim v \omega$

$$\begin{aligned}\text{adj}[u][v] &= \omega \\ \text{adj}[v][u] &= \omega\end{aligned}$$

Advantage: Easy to add | delete | update edges.

Disadvantage: Space wastage bcoz of non-existent edges

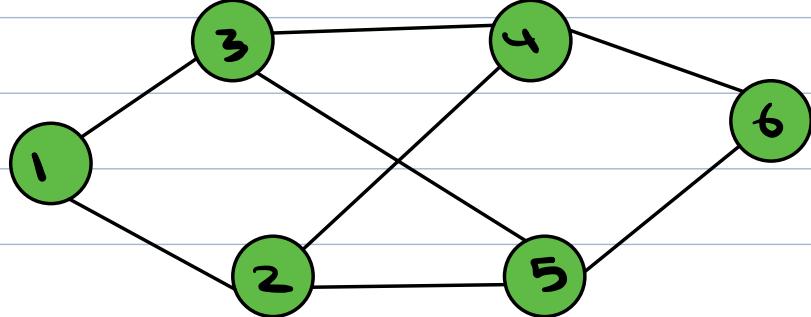
$$N = 10^5 \quad \text{adj} \rightarrow 10^5 \times 10^5 = 10^{10}$$

Only  $N = 10^3$ , we can use adj matrix  
 $10^3 \times 10^3 = 10^6$

SC:  $O(N^2) / O(V^2)$

## 2. Adjacency List

Store neighbours of every node



list<int> adj[7]

0 →

1 → 3 2

2 → 5 4 1

|   |   |
|---|---|
| 6 | 8 |
| 1 | 3 |
| 2 | 5 |
| 4 | 3 |
| 2 | 4 |
| 3 | 5 |
| 4 | 6 |
| 6 | 5 |
| 1 | 2 |

$3 \rightarrow 1 \quad 4 \quad 5$        $N/V = 6$  (1 to 6)  
 $4 \rightarrow 3 \quad 2 \quad 6$   
 $5 \rightarrow 2 \quad 3 \quad 6$   
 $6 \rightarrow 4 \quad 5$        $E = 8$

$$\text{SC: } O(V + 2E)$$

$$= O(V + E)$$

① N nodes (1 to N)

list<int> adj[N+1]

10:30

② N nodes (0 to N-1)

list<int> adj[N]

Q. Given array of edges, construct adjacency list

① II I/P

input << N << edges

list<int> adj[N+1]

for (i=0 ; i < edges ; i++) {

    input (u and v)

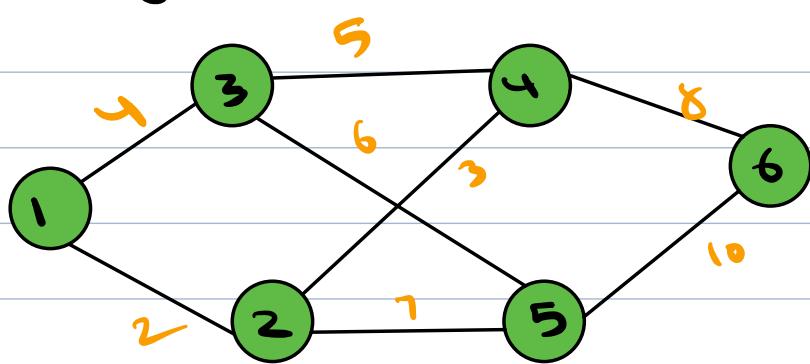
    adj[u].add(v)  $\nearrow u-v$

    adj[v].add(u)  $\nearrow v-u$

    adj[u].add(v)  $\nearrow u \rightarrow v$

}

# Weighted graph



$G \rightarrow (V, \omega)$

$0 \rightarrow$

$1 \rightarrow (3, 4) \quad (2, 2)$

$2 \rightarrow (5, 7) \quad (4, 3) \quad (1, 2)$

$3 \rightarrow 1 \quad 4 \quad 5$

$4 \rightarrow 3 \quad 2 \quad 6$

$5 \rightarrow 2 \quad 3 \quad 6$

$6 \rightarrow 4 \quad 5$

list <pair<int,int>> adj[N+1]

for (i=0 ; i < edges ; i++) {

} input (u, v and w)

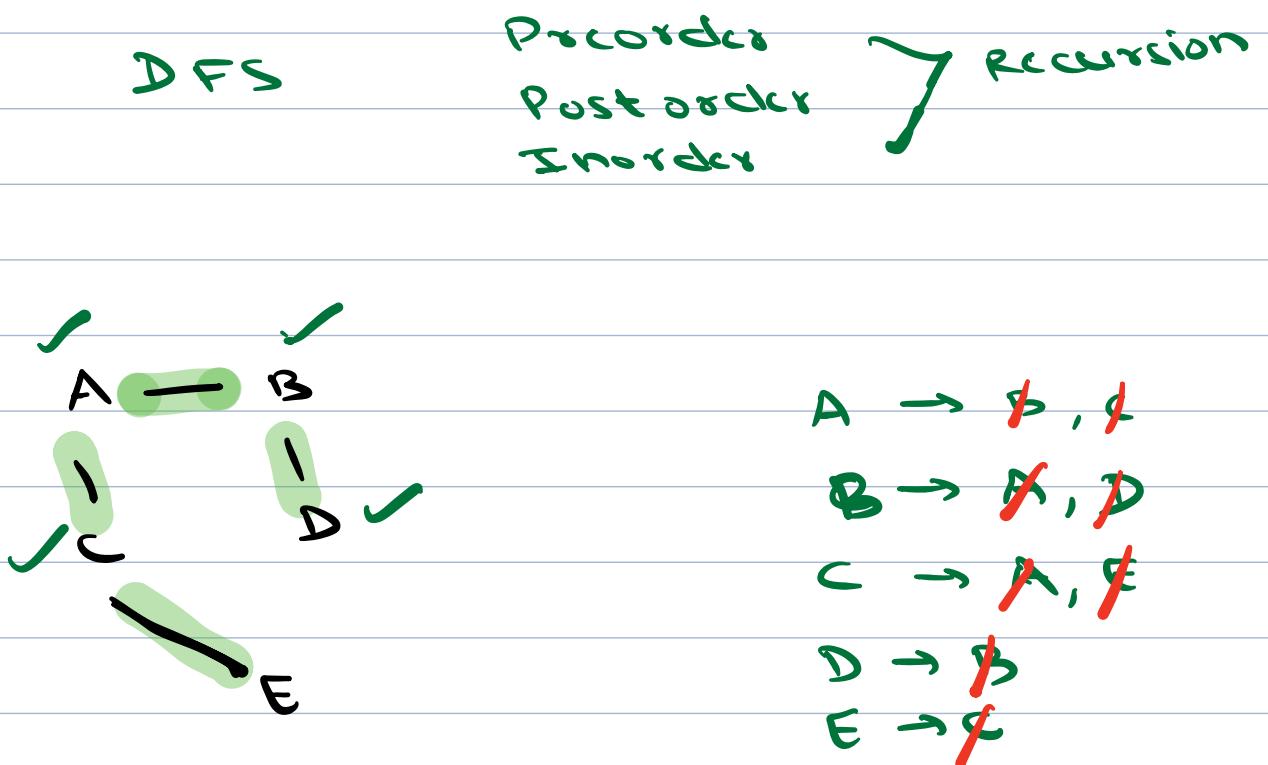
adj[u].add(<v,w>) } u →

adj[v].add(<u,w>) } v →

adj[u].add(<v,w>) } u → v

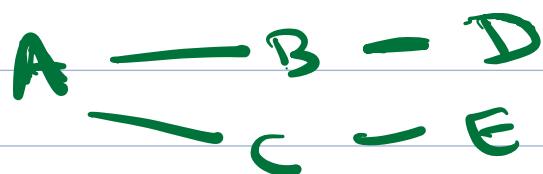
# Graph Traversals

- ① DFS - Depth First Search
- ② BFS - Breadth First Search



| Visited | A              | B              | C              | D              | E              |
|---------|----------------|----------------|----------------|----------------|----------------|
|         | ✓ <sub>T</sub> |

OP: A B D C E



// N nodes (1 to N)

bool visited[N+1] = {false}

list<int> adj[N+1]

void dfs(int curNode) <

print (curNode)

visited[curNode] = true

for (i=0 ; i < adj[curNode].size(); i++) <

int nbr = adj[curNode][i]

if (visited[nbr] == false)

dfs(nbr)

void main() <

// 1 to N

for (i=1; i <= N; i++) <

if (visited[i] == false)

dfs(i)

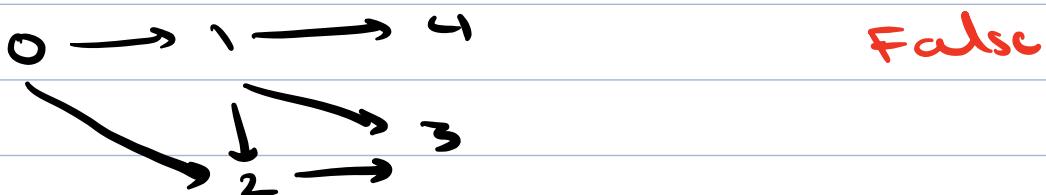
TC: O(V + 2E)

$\approx O(V+E)$

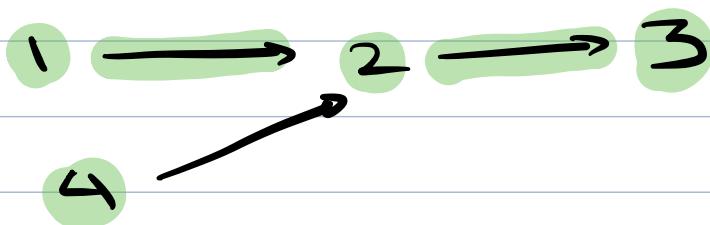
SC: O(V )

visit[] and stack

1. Given a directed graph, check if it contains a cycle or not?



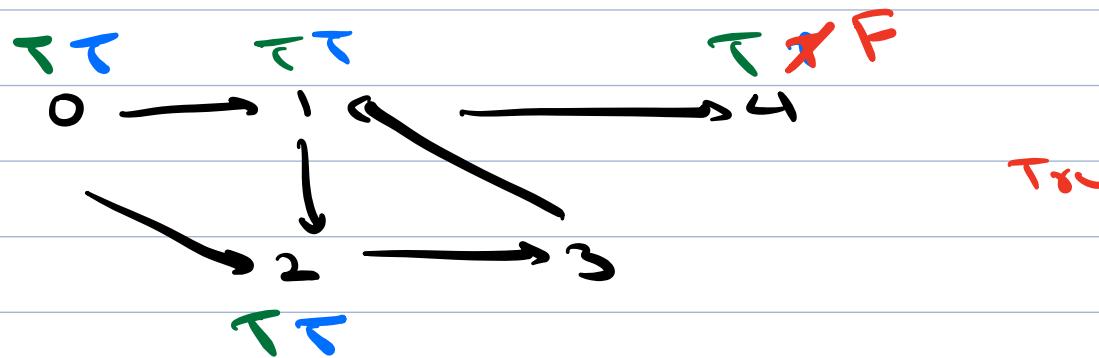
Approach : Apply DFS and if we come across a visited node, there's a cycle.



$1 \rightarrow 2$   
 $2 \rightarrow 3$   
 $3 \rightarrow$   
 $4 \rightarrow 2$

vis    1    2    3    4  
F T    F F    F T    F T

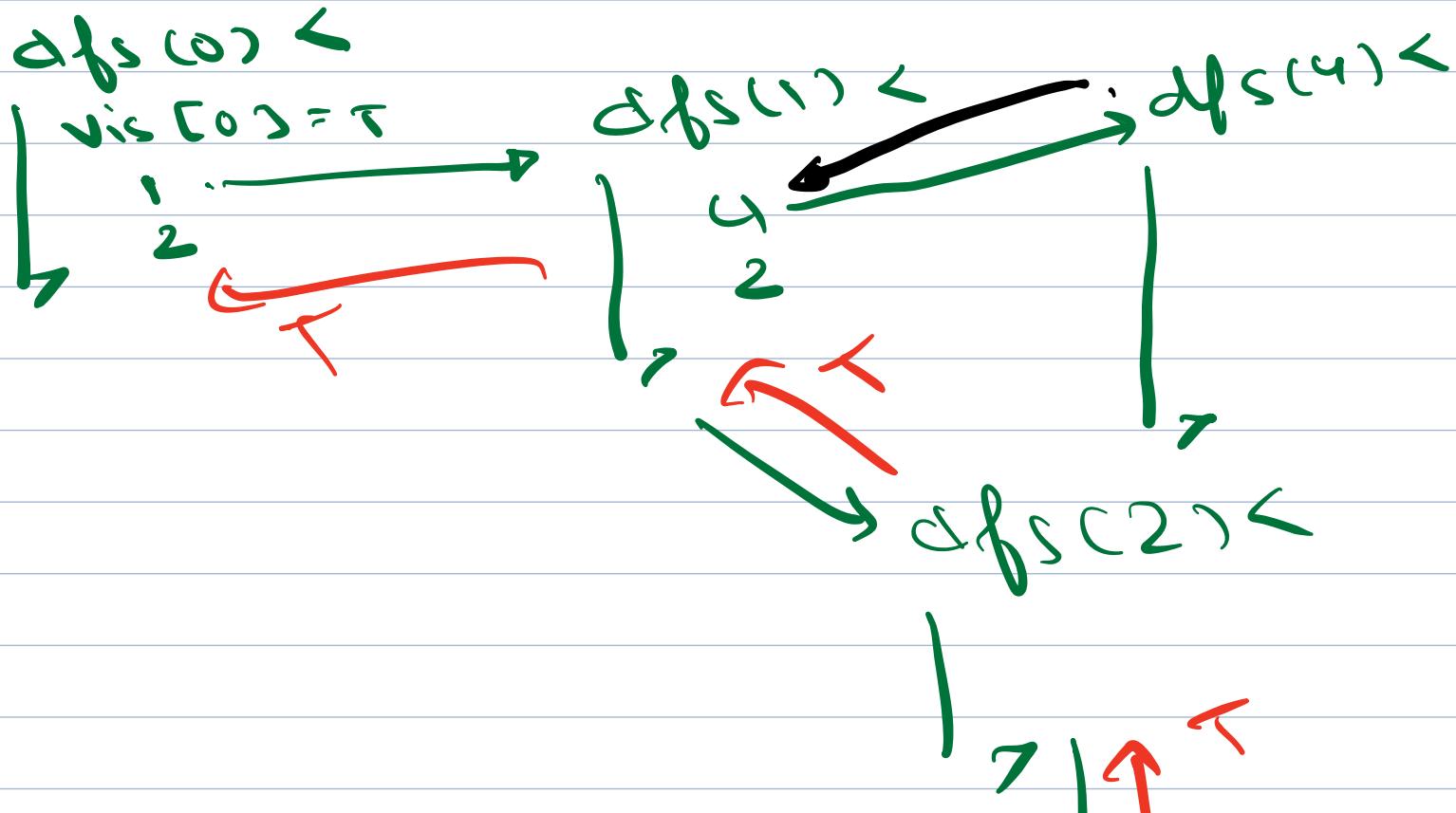
$1 \rightarrow 2 \rightarrow 3$   
 $4 \rightarrow 2$



vis      0    1    2    3    4  
~~F~~ ~~F~~ ~~F~~ ~~F~~ ~~T~~

Green  $\rightarrow$  vis  
 Blue  $\rightarrow$  Path

path      0    1    2    3    4  
~~F~~ ~~F~~ ~~F~~ ~~F~~ ~~F~~  
~~T~~

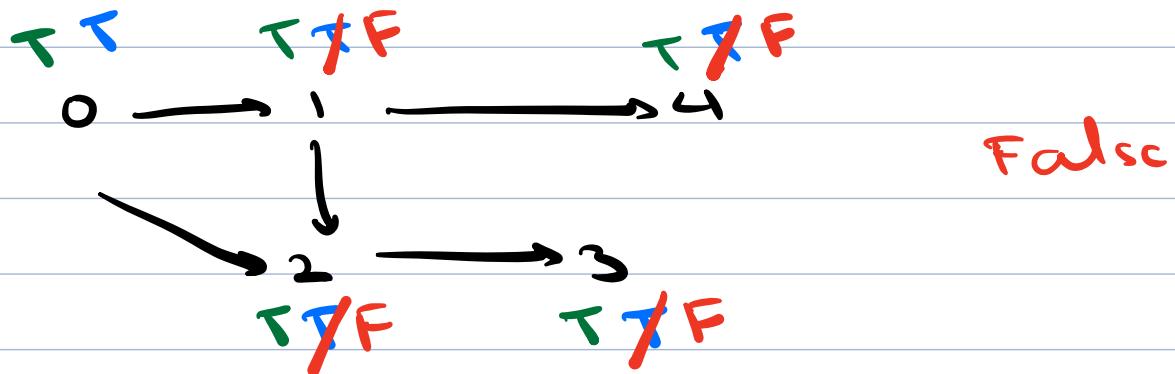


↓

$\alpha \beta \gamma (3) <$

|   |   X  
↓   ,

Correct Approach: Look for an already visited node in same path  $\rightarrow$  cycle



$0 \rightarrow 1 \rightarrow 4$

$0 \rightarrow 1$   
↓  
 $2 \rightarrow 3$

$0 \rightarrow 2 \dots$

// return true if curNode has a cycle  
or any of the future neighbours

bool dfs (int curNode) {

vis [curNode] = true

path [curNode] = true

for (i=0 ; i < adj [curNode].size(); i++) {

int nbr = adj [curNode][i];

if (visited [nbr] == false) {

if (dfs (nbr) == true)

return true

else {

if (path [nbr] == true)

return true

path [curNode] = false

return false

}

TC: O(V+E)

SC: O(V)

3 ✓

vis[], path[], stack