

Agenda

Modular Arithmetic Intro

Count pairs whose sum mod m is 0

GCD Intro

Properties of GCD

Delete one

$$x \cdot y \cdot 5 =$$

Modulo (\div)

$A \div M = \text{Remainder}$ when A is divided by M

Range of $A \div M \rightarrow [0, m-1]$

Why do we need Mod?



Limit the range of the data

$-\infty$



∞

$\div M =$

0



$m-1$

Int $\rightarrow 2 \times 10^9$

Long $\rightarrow 4 \times 10^{18}$

ans $\div 10^9 + 7$

$[0 \rightarrow 10^9 + 6]$

(Int)

Properties of Mod (on arithmetic operators)

$$\textcircled{1} (a + b) \div m = (a \div m + b \div m) \div m$$

Eg. $a = 9, b = 8, m = 5$

$$(9+8) \cdot 1.5 \quad | \quad (9 \cdot 1.5 + 8 \cdot 1.5) \cdot 1.5$$

$$= 17 \cdot 1.5 = 2 \quad | \quad (4 + 3) \cdot 1.5 = 7 \cdot 1.5$$

$$= 2$$

$$(2) \quad (a \times b) \cdot m = (a \cdot m \times b \cdot m) \cdot m$$

Eg. $a=9, b=8 \quad m=5$

$$(9 \times 8) \cdot 1.5 \quad | \quad (9 \cdot 1.5 \times 8 \cdot 1.5) \cdot 1.5$$

$$\Rightarrow 12 \cdot 1.5 \quad | \quad \Rightarrow (4 + 3) \cdot 1.5$$

$$\Rightarrow 2 \quad | \quad \Rightarrow 12 \cdot 1.5 = 2$$

$$(3) \quad (a \cdot m) \cdot m = a \cdot m$$

$$12 \cdot 1.5 = 2 \cdot 1.5 = 2 \cdot 1.5 = 2 \dots$$

$$(4) \quad (a + m) \cdot m = (a \cdot m + m \cdot m) \cdot m$$

$$= (a \cdot m) \cdot m$$

$$(a + m) \cdot m = a \cdot m$$

$$(18 + 5) \cdot 1.5$$

$$= 23 \cdot 1.5 = 3$$

$$18 \cdot 1.5 = 3$$

$$\underline{11111} \quad \underline{11111} \quad \underline{11111} \quad 1111$$

eg. $(-4) \cdot 1.6 = (-4 + 6) \cdot 1.6 = 2 \cdot 1.6 = 2$

$$(-8) \cdot 1.6 = (-8 + 6) \cdot 1.6 = -2 \cdot 1.6 = (-2 + 6) \cdot 1.6$$

$$= 4 \cdot 6 = 4$$

$$(5) \quad (a - b) \% m = (a \% m - b \% m + m) \% m$$

For eg. $a = 17$ $b = 8$ $m = 5$

$$\Rightarrow (17 - 8) \% 5 \quad | \quad (17 \% 5 - 8 \% 5) \% 5$$

$$\Rightarrow 9 \% 5 = 4 \quad | \quad (2 - 3) \% 5 = (-1) \% 5$$

$$= 4$$

$$(-1) \% 5 = (-1 + 5) \% 5 = 4 \% 5 = 4$$

$$(6) \quad (a^b) \% m = (a \times a \times a \dots \text{b times}) \% m$$

$$= ((a \% m) \times (a \% m) \times (a \% m) \dots$$

$$\text{b times}) \% m$$

$$= ((a \% m)^b) \% m$$

$$\text{Quiz : } (37^{103} - 1) \% 12$$

$$\Rightarrow ((37^{103}) \% 12 - 1 \% 12 + 12) \% 12$$

$$\Rightarrow (1 - 1 + 12) \% 12 = 0$$

$$\textcircled{1} (a - b) \% m = (a \% m - b \% m + m) \% m$$

$$\textcircled{2} a^b \% m = ((a \% m)^b) \% m$$

$$\begin{aligned} 37^{103} \% 12 &= ((37 \% 12)^{103}) \% 12 \\ &= (1^{103}) \% 12 = 1 \end{aligned}$$

$$\text{If } A \% m = 0$$

A is completely divisible by m
m is a factor of A

Prob 1: Given N array elements, find count of pairs (i, j) such that $(a[i] + a[j]) \% m = 0$
 $i \neq j$ and pair (i, j) is same as pair (j, i)

2 idx $(i, j) \rightarrow$ pair sum divisible by m

$A = \langle 4, 3, 6, 3, 8, 12 \rangle$ $ans = 3$
 $m = 6$

$$(6 + 12) \% 6 = 0$$

$$(3 + 3) \% 6 = 0$$

$$(4 + 8) \% 6 = 0$$

BF: Go to all unique pairs, calculate their sum. If their sum $\% m == 0$
cnt++

int ans = 0

```
for (i = 0 ; i < n ; i++) <
|   for (j = i+1 ; j < n ; j++) <
|   |   if (a[i] + a[j] \% m == 0)
|   |       ans++
|   >
>
return ans
```

TC: $O(n^2)$

SC: $O(1)$

Optimised Approach

$$(a+b) \% m = (a \% m + b \% m) \% m$$

\downarrow \downarrow
 0 0

Look for a pair whose sum is divisible by m

sum of remainders should be divisible by m

$$(x_1 + x_2) \% m = 0$$

$$x_1, x_2 \rightarrow [0 \quad m-1]$$

$$1 \rightarrow m-1$$

$$2 \rightarrow m-2$$

$$3 \rightarrow m-3$$

...

$$x \rightarrow m-x$$

$$(x_1 + x_2) \% 6 = 0$$

$$x_1, x_2 \rightarrow [0 \quad 5]$$

$$1 \rightarrow 5$$

$$2 \rightarrow 4$$

$$3 \rightarrow 3$$

$$0 \rightarrow 0$$

Observation : $x_1 + x_2 = m$

$A =$

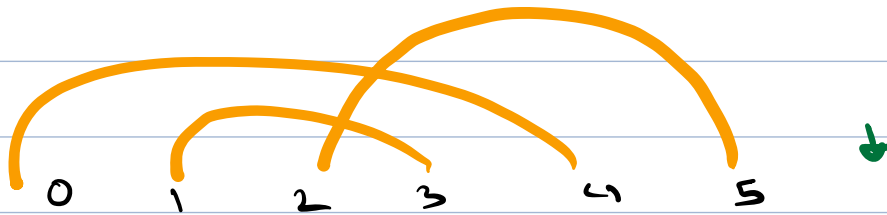
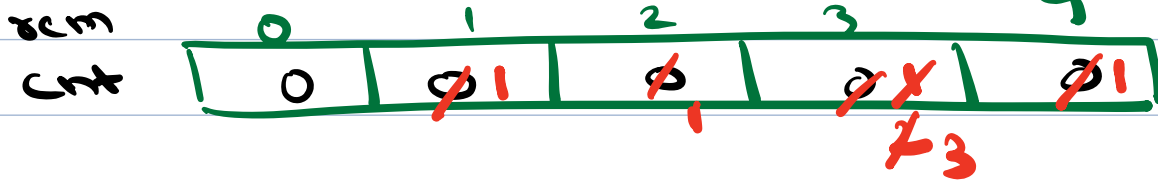
0	1	2	3	4	5
4	3	6	3	8	12
\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
4	3	1	3	3	2
	1	2	4	2	3

$x_1 = A \% M$

x_2

$M = 5$

ans = 4

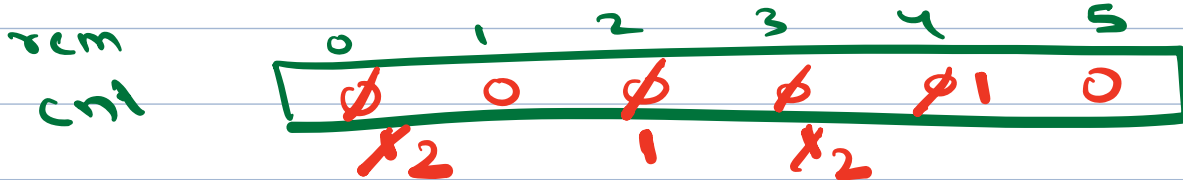


A = 1, 3, 6, 9, 8, 12, 9 M = 6

A % M =

4	3	0	3	2	0	3
↓	↓	↓	↓	↓	↓	
2	3	0	3	4	0	3

ans = 0
1
2
3



ans = 3
↓ + 2
5

idx → rem freq[rem]

int countPairs (int A[], int N, int M) {

int freq[M] = {0} // idx → 0 to M-1
int ans = 0

for (int i = 0 ; i < N ; i++) {

int x1 = A[i] % M

int x2 = M - x1

if (x1 == 0)

 x2 = 0

ans += freq[x2]

freq[x1] ++

1
return ans

TC: $O(N)$
SC: $O(1)$

10:42

17

GCD \rightarrow Greatest Common Divisor

HCF \rightarrow Highest Common Factor

If x is a factor of A

$$\Rightarrow A \div x = 0$$

$\text{GCD}(A, B) =$ Greatest factor that divides both A and B

$$\text{GCD}(A, B) = x$$

① $A \div x = 0$

② $B \div x = 0$

③ x is largest no. which divides both

$$\text{GCD}(15, 25) =$$

$$\text{GCD}(12, 30) =$$

$$\text{GCD}(0, 4) =$$

$$\text{GCD}(0, a) =$$

$$\text{GCD}(0, 0) =$$

$$\text{GCD}(15, 25) =$$

Properties of GCD

$$\textcircled{1} \quad \text{GCD}(A, B) =$$

$$\textcircled{2} \quad \text{GCD}(0, A) =$$

$$\textcircled{3} \quad \text{GCD}(A, B, C) =$$

$$\textcircled{4} \quad \text{GCD}(1, A) =$$

$$\textcircled{5} \quad \text{Given} \quad A \geq B > 0$$

$$\text{GCD}(A, B) =$$

$$\text{GCD}(A, B) =$$

$$\text{eg.} \quad \text{GCD}(17, 5)$$

$$\textcircled{6} \quad \text{gcd}(A, B) =$$

$$\text{e.g.} \quad \text{gcd}(24, 16) =$$

Prob: Calculate GCD of entire array

$$ar[3] = \langle 6, 12, \frac{15}{7} \rangle$$