Agenda

Modular Arithmetic Intro

Count pairs whose sum mod m is 0

GCD Intro

Properties of GCD

Delete one

$x \% 5 =$

Modulo (%)

A % M = Remainder when A is divided by M

Range of A % M → [0, M-1]

Why do we need Mod?

↓

Limit the range of the data

$$[-\infty, \infty] \ \% M = [0, M-1]$$

Int → 2×10^9

Long → 4×10^{18}

ans % $10^9 + 7$

$[0 \rightarrow 10^9 + 6]$

(Int)

Properties of Mod (on arithmetic operators)

① $(a+b) \% m = (a \% m + b \% m) \% m$

Eg. $a = 9$, $b = 8$ $m = 5$

$$(9+8)\%5 = 17\%5 = 2 \quad\Big|\quad (9\%5 + 8\%5)\%5 = (4 + 3)\%5 = 7\%5 = 2$$

② $(a \times b)\%m =$ $(a\%m \times b\%m)\%m$

Eg. $a=9,\ b=8 \quad m=5$

$$(9\times 8)\%5 \Rightarrow 72\%5 \Rightarrow 2 \quad\Big|\quad (9\%5 \times 8\%5)\%5 \Rightarrow (4 \times 3)\%5 \Rightarrow 12\%5 = 2$$

③ $(a\%m)\%m = a\%m$

$$72\%5 = 2\%5 = 2\%5 = 2 \ldots$$

④ $(a+m)\%m = (a\%m + \cancelto{0}{m\%m})\%m$

$$= (a\%m)\%m$$

$$\boxed{(a+m)\%m = a\%m}$$

$(18+5)\%5$
$= 23\%5 = 3$

$18\%5 = 3$

||||| ||||| ||||| |||

eg. $(-4)\%6 = (-4+6)\%6 = 2\%6 = 2$

$(-8)\%6 = (-8+6)\%6 = -2\%6 = (-2+6)\%6$

$= 4 \% 6 = 4$

(5) $(a-b)\% m = (a\% m - b\% m + m)\% m$

For eg. $a = 17 \quad b = 8 \quad m = 5$

$\Rightarrow (17-8)\% 5$

$\Rightarrow 9\% 5 = 4$

$(17\% 5 - 8\% 5)\% 5$

$(2 - 3)\% 5 = (-1)\% 5$

$= 4$

$(-1)\% 5 = (-1+5)\% 5 = 4\% 5 = 4$

(6) $(a^b)\% m = (a \times a \times a \ldots \text{ b times })\% m$

$= ((a\% m) \times (a\% m) \times (a\% m) \ldots$

$\text{b times})\% m$

$= ((a\% m)^b)\% m$

Quiz : $(37^{103} - 1) \% 12$

$$\Rightarrow ((37^{103}) \% 12 - 1 \% 12 + 12) \% 12$$

$$\Rightarrow (1 - 1 + 12) \% 12 = 0$$

① $(a-b) \% m = (a \% m - b \% m + m) \% m$

② $a^b \% m = ((a \% m)^b) \% m$

$$37^{103} \% 12 = ((37 \% 12)^{103}) \% 12$$
$$= (1^{103}) \% 12 = 1$$

If $A \% M = 0$

A is completely divisible by M

M is a factor of A

Prob 1 : Given N array elements, find count of pairs (i, j) such that (ar[i] + ar[j]) % m = 0
i != j and pair (i, j) is same as pair (j, i)

2 idx (i, j) → pair sum divisible by m

A = < 4, 3, 6, 3, 8, 12 > ans = 3
m = 6

(6 + 12) % 6 = 0
(3 + 3) % 6 = 0
(4 + 8) % 6 = 0

BF : Go to all unique pairs, calculate their sum. If their sum % m == 0
cnt ++

```
int ans = 0
for (i = 0 ; i < n ; i++) {
    for (j = i+1 ; j < n ; j++) {
        if (a[i] + a[j] % m == 0)
            ans++
    }
}
return ans
```

TC : O(N²)
SC : O(1)

Optimised Approach

$$(a+b)\,\%\,m = \left(a\%m + b\%m\right)\%\,m$$

$(a+b)\%m \to 0$: Look for a pair whose sum is divisible by m

$(r_1 + r_2)\%m \to 0$, where $r_1 = a\%m$ and $r_2 = b\%m$: sum of remainders should be divisible by m

$(r_1 + r_2)\%m = 0$

$r_1, r_2 \to [0 \quad m-1]$

$1 \to M-1$

$2 \to M-2$

$3 \to M-3$

...

$\boxed{x \to M-x}$

$(r_1 + r_2)\%6 = 0$

$r_1, r_2 \to [0 \quad 5]$

$1 \to 5$

$2 \to 4$

$3 \to 3$

$0 \to 0$

Observation : $r_1 + r_2 = M$

	0	1	2	3	4	5
A =	4,	3,	6,	3,	8,	12
$r_1 = A\%M$	[4	3	1	3	3	2
r_2	1	2	4	2	2	3

M = 5

ans = ~~0~~ ~~1~~ 4

rem	0	1	2	3	4
cnt	0	~~0~~ 1	~~0~~ 1	~~0~~ ~~1~~ ~~2~~ 3	~~0~~ 1

	0	1	2	3	4	5	
A =	10,	3,	6,	9,	8,	12	9
A % M =	4	3	0	3	2	0	3
	↓ 2	↓ 3	↓ 0	↓ 3	↓ 4	↓ 0	3

M = 6

ans = ~~0~~ ~~1~~ ~~2~~ 3

rem	0	1	2	3	4	5
cnt	~~0~~ ~~1~~ 2	0	~~0~~ 1	~~0~~ ~~1~~ 2	~~0~~ 1	0

ans = 3 $\downarrow +2$ 5

idx → rem   freq[rem]

```
int  countPairs (int A[], int N, int M) {

    int freq[M] = {0}      // idx → 0 to M-1
    int ans = 0

    for (int i = 0 ; i < N ; i++) {

        int r1 = A[i] % m
        int r2 = m - r1
        if (r1 == 0)
                 r2 = 0

        ans += freq[r2]
        freq[r1] ++
```

}
return ans
}

TC : O(N)

SC : O(M)

10 :42

}

GCD → Greatest Common Divisor

HCF → Highest Common Factor

If x is a factor of A

$\Rightarrow A \,\%\, x = 0$

GCD(A, B) = Greatest factor that divides both A and B

GCD (A, B) = x

1. $A \,\%\, x = 0$
2. $B \,\%\, x = 0$
3. x is largest no. which divides both

GCD (15, 25) =

GCD (12, 30) =

GCD (0, 4) =

GCD $(0, a) =$

GCD $(0, 0) =$

GCD $(15, 25) =$

Properties of GCD

① GCD $(A, B) =$

② GCD $(0, A) =$

③ GCD $(A, B, C) =$

④ GCD (1, A) =

⑤ Given $A \geq B > 0$

GCD (A, B) =

GCD (A, B) =

eg. GCD (17, 5)

⑥ $GCD(A, B) =$

e.g. $GCD(24, 16) =$

Prob: Calculate GCD of entire array

$$arr[3] = \langle 6, 12, \underset{7}{15} \rangle$$