CH5440- Test1

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In [1]:
```python
# import libraries

import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
from math import sqrt
```

OLS

In [2]:
```python
'''
Defining Ordinary least squares
y=ax+b
from formulae, a= Syu/Suu and b=y_mean-a(x_mean)
'''
def getOLS(data,fea1, fea2):
    #x,y
    # calculating b

    x_mean=np.mean(data[fea1])
    y_mean=np.mean(data[fea2])

    # calculating numerator and denominator for a

    Syu= np.multiply(np.subtract(data[fea2],y_mean),np.subtract(data[fea1],x_mean))
    Syu=np.sum(Syu)/len(data)
    numerator=Syu

    Suu= np.multiply(np.subtract(data[fea1],x_mean),np.subtract(data[fea1],x_mean))
    Suu=np.sum(Suu)/len(data)
    denominator=Suu
    print('Syu:',Syu,'Suu:',Suu)
    a_OLS=numerator/denominator

    b_OLS=y_mean-a_OLS*x_mean

    print('Slope Parameter of OLS:',a_OLS)
    print('Offset Parameter of OLS:',b_OLS)
#     print(Syu,Suu,x_mean,y_mean)
    return a_OLS,b_OLS
```

TLS

In [3]:
```python
'''
Defining Total least squares
from formulae, a= ((Syy-Suu)+sqrt((Syy-Suu)^2+4*(Syu)^2))/2*Syu and
b=y_mean-a(x_mean)
'''
def getTLS(data,fea1,fea2):

    # calculating b

    x_mean=np.mean(data[fea1])
    y_mean=np.mean(data[fea2])

    # calculating numerator and denominator for a

    Syu= np.multiply(np.subtract(data[fea2],y_mean),np.subtract(data[fea1],x_mean))
    Syu=np.sum(Syu)/len(data)

    Suu= np.multiply(np.subtract(data[fea1],x_mean),np.subtract(data[fea1],x_mean))
    Suu=np.sum(Suu)/len(data)

    Syy=np.multiply(np.subtract(data[fea2],y_mean),np.subtract(data[fea2],y_mean))
    Syy=np.sum(Syy)/len(data)

    numerator=(Syy-Suu)+sqrt((Syy-Suu)**2+4*((Syu)**2))
    denominator=2*Syu
    print('Syu',Syu,'Suu',Suu,'Syy',Syy)
    a_TLS=numerator/denominator

    b_TLS=y_mean-a_TLS*x_mean

    print('Slope Parameter of TLS:',a_TLS)
    print('Offset Parameter of TLS:',b_TLS)
#     print(Syu,Suu,Syy,x_mean,y_mean)
    return a_TLS,b_TLS
```

Question 1

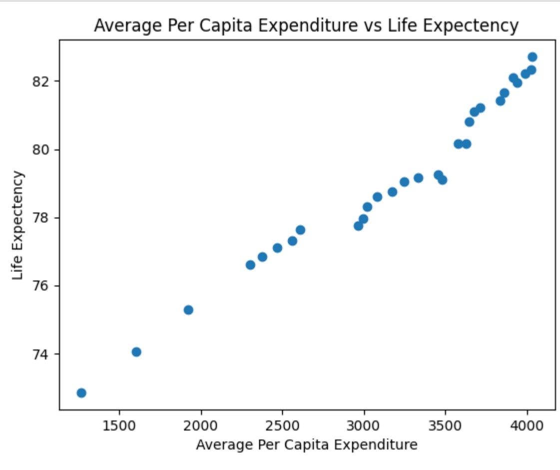
In [92]:
```python
# Data

data= pd.read_excel('MM17B113Data.xlsx')
```

In [88]:
```python
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 29 entries, 0 to 28
Data columns (total 2 columns):
 #   Column          Non-Null Count  Dtype
---  ------          --------------  -----
 0   Life Exp        29 non-null     float64
 1   Avg Per Capita  29 non-null     float64
dtypes: float64(2)
memory usage: 592.0 bytes
```

Data Visualization

In [14]:
```python
plt.scatter(data['Avg Per Capita'],data['Life Exp'])
plt.ylabel('Life Expectency')
plt.xlabel('Average Per Capita Expenditure')
plt.title('Average Per Capita Expenditure vs Life Expectency')
plt.show()
```



Linear trend is observed between Average Per Capita Expenditure and Life Expectency

Data Preprocessing

The offset parameter is assumed to be non zero, therefore we need to mean shift the data

In [84]:
```python
# mean shifting of data for desired columns

def meanshift(data,cols):
    new_data=data
    for fea in cols:
        fea_mean=np.mean(data[fea])
        new_data[fea]=np.subtract(data[fea],fea_mean)
    return new_data
```

In [90]:
```python
# data_ms=meanshift(data,data.columns)
```

part (a)

In [93]:
```python
# sample mean of life expectency

print('sample mean of life expectency is', data['Life Exp'].mean(),'years.')

# Average per capita expenditure

print('Average per capita expenditure is', data['Avg Per Capita'].mean(),'USD.')

# print('------after mean shifting------')

# print('sample mean of life expectency is', data_ms['Life Exp'].mean(),'years.')

# # Average per capita expenditure

# print('Average per capita expenditure is', data_ms['Avg Per Capita'].mean(),'USD.')
```

```
sample mean of life expectency is 79.08797308662743 years.
Average per capita expenditure is 3162.7446965517242 USD.
```

part (b)

In [25]:
```python
# standard deviation

print('Sample Standard deviation of life expectency is', data['Life Exp'].std(),'years.')

print('Average per capita expenditure is', data['Avg Per Capita'].std(),'USD.')
```

```
Sample Standard deviation of life expectency is 2.5350266528114096 years.
Average per capita expenditure is 753.1932859443003 USD.
```

part (c)

In [39]:
```python
print('------Avg Per Capita Expenditure vs Life Expectency-------')
# OLS
print('OLS')
a_OLS,b_OLS=getOLS(data,'Avg Per Capita','Life Exp')
```

```
------Avg Per Capita Expenditure vs Life Expectency-------
OLS
Syu: 1814.7775364934319 Suu: 547738.0526815183
Slope Parameter of OLS: 0.0033132215802954856
Offset Parameter of OLS: 68.60909910504716
```

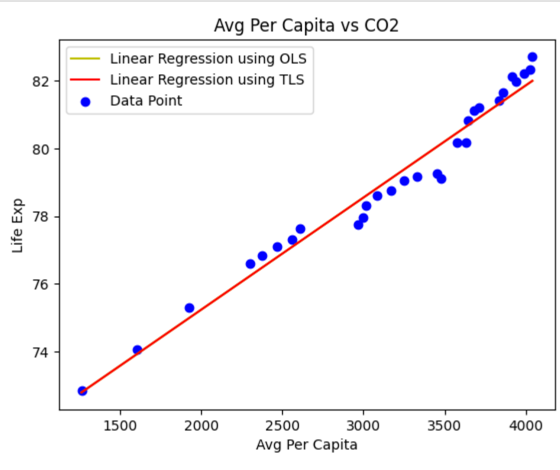
part (d)

In [40]:
```python
print('------Avg Per Capita Expenditure vs Life Expectency-------')
# TLS
print('TLS')
a_TLS,b_TLS=getTLS(data,'Avg Per Capita','Life Exp')
```

```
------Avg Per Capita Expenditure vs Life Expectency-------
TLS
Syu 1814.7775364934319 Suu 547738.0526815183 Syy 6.2047615052757985
Slope Parameter of TLS: 0.0033132227416671654
Offset Parameter of TLS: 68.60909543192504
```

part (e)

In [41]:
```python
#plotting values

x_max = np.max(data['Avg Per Capita']) + 5
x_min = np.min(data['Avg Per Capita']) - 5

#calculating line values of x and y

x = np.linspace(x_min, x_max, 1000)
y_OLS = b_OLS + a_OLS * x
y_TLS = b_TLS + a_TLS * x

#plotting line

plt.plot(x, y_OLS, color='y', label='Linear Regression using OLS')
plt.plot(x, y_TLS, color='r', label='Linear Regression using TLS')
plt.scatter(data['Avg Per Capita'], data['Life Exp'], color='b', label='Data Point')
plt.xlabel('Avg Per Capita')
plt.ylabel('Life Exp')
plt.title('Avg Per Capita vs CO2' )
plt.legend()
plt.show()
```



In [43]:
```python
# add five years to max value

max_val=np.max(data['Life Exp'])
print('Required Life expectency:',max_val+5)

print('--------IOLS-------')
a_IOLS,b_IOLS=getOLS(data,'Life Exp','Avg Per Capita')
```

```
Required Life expectency: 87.719512195122
--------IOLS-------
Syu: 1814.7775364934319 Suu: 6.2047615052757985
Slope Parameter of OLS: 292.481433001149
Offset Parameter of OLS: -19969.019004981375
```

Estimation of per capita expenditure

In [44]:
```python
max_life_Exp=max_val+5
max_per_capita_OLS= (max_life_Exp-b_OLS)/a_OLS
max_per_capita_IOLS= b_IOLS + a_IOLS * max_life_Exp
print('Incresed per capita expenditure from OLS:',max_per_capita_OLS)
print('Incresed per capita expenditure from IOLS:',max_per_capita_IOLS)
```

```
Incresed per capita expenditure from OLS: 5767.924851066101
Incresed per capita expenditure from IOLS: 5687.309624009671
```

In [50]:
```python
max(data['Avg Per Capita'])
```

Out[50]:
```
4035.9507
```

In [54]:
```python
print('additional per capita amount required from OLS:',-max(data['Avg Per Capita'])+max_per
_capita_OLS)
print('additional per capita amount required from IOLS:',-max(data['Avg Per Capita'])+max_pe
r_capita_IOLS)
print('Average is',(-2*max(data['Avg Per Capita'])+max_per_capita_OLS+max_per_capita_IOLS)/2
)
```

```
additional per capita amount required from OLS: 1731.974151066101
additional per capita amount required from IOLS: 1651.3589240096712
Average is 1691.6665375378861
```

part (f)

In [64]:
```python
df = {'Life Exp': 70,'Avg Per Capita':1000}
data_new = data.append(df, ignore_index = True)
data_new
```

Out[64]:

	Life Exp	Avg Per Capita
0	72.853659	1267.5022
1	74.051220	1604.7333
2	75.300000	1923.1369
3	76.600000	2300.8976
4	76.848780	2377.8353
5	77.100000	2469.4117
6	77.300000	2561.9933
7	77.648780	2607.9071
8	77.751220	2965.8229
9	77.953659	2997.7557
10	78.304878	3018.3427
11	78.604878	3082.7392
12	78.756098	3171.9960
13	79.056098	3248.9504
14	79.158537	3331.4384
15	79.260976	3454.0558
16	79.114634	3478.5787
17	80.163415	3578.9042
18	80.163415	3629.2090
19	80.812195	3644.7304
20	81.112195	3680.6859
21	81.214634	3713.7856
22	81.414634	3836.7055
23	81.663415	3859.5252
24	82.114634	3917.4283
25	81.968293	3942.3884
26	82.219512	3989.2368
27	82.719512	4035.9507
28	82.321951	4027.9490
29	70.000000	1000.0000

In [65]:
```python
print('------Avg Per Capita Expenditure vs Life Expectency-------')
# OLS
print('OLS')
a_OLS,b_OLS=getOLS(data_new,'Avg Per Capita','Life Exp')
```

```
------Avg Per Capita Expenditure vs Life Expectency-------
OLS
Syu: 2387.6116211322505 Suu: 680198.4220937073
Slope Parameter of OLS: 0.003510169302926319
Offset Parameter of OLS: 67.93632463837439
```

part (g)

In [95]:
```python
print('------Avg Per Capita Expenditure vs Life Expectency-------')
# OLS
print('OLS')
a_OLS,b_OLS=getOLS(data,'Avg Per Capita','Life Exp')

# print('------Avg Per Capita Expenditure vs Life Expectency-------')
# TLS
print('TLS')
a_TLS,b_TLS=getTLS(data,'Avg Per Capita','Life Exp')

print('upper and lower bounds are:',max(a_OLS,a_TLS),min(a_OLS,a_TLS))
```

```
------Avg Per Capita Expenditure vs Life Expectency-------
OLS
Syu: 1814.7775364934319 Suu: 547738.0526815183
Slope Parameter of OLS: 0.0033132215802954856
Offset Parameter of OLS: 68.60909910504716
TLS
Syu 1814.7775364934319 Suu 547738.0526815183 Syy 6.2047615052757985
Slope Parameter of TLS: 0.0033132227416671654
Offset Parameter of TLS: 68.60909543192504
upper and lower bounds are: 0.0033132227416671654 0.0033132215802954856
```

part (h)

In [72]:
```python
def estimate(x):
    return a_OLS*x+b_OLS
```

In [75]:
```python
data['estimate']=data['Avg Per Capita'].apply(estimate)
data['error']=np.subtract(data['Life Exp'],data['estimate'])
```

In [78]:
```python
print('Standard deviationof errors is',data['error'].std())
```

```
Standard deviationof errors is 0.46996092217241814
```

part (i)

In [79]:
```python
x_mean=np.mean(data['Avg Per Capita'])
Suu= np.multiply(np.subtract(data['Avg Per Capita'],x_mean),np.subtract(data['Avg Per Capit
a'],x_mean))
Suu=np.sum(Suu)/len(data)

sum_er_OLS=np.sum(np.power(np.subtract(data['Life Exp'],np.add(np.multiply(a_OLS,data['Avg P
er Capita']),b_OLS)),2))
sigma_ep_OLS=sqrt(sum_er_OLS/(len(data)-2)) # variance of errors
sigma_a_OLS=sqrt(sigma_ep_OLS/len(data)*Suu) # variance of a

print(sigma_a_OLS,a_OLS)
a_OLS_min=a_OLS-2.16*sigma_a_OLS
a_OLS_max=a_OLS+2.16*sigma_a_OLS
print('95% CI for a_OLS is [',a_OLS_min,',',a_OLS_max,']')
```

```
95.35122532242947 0.003510169302926319
95% CI for a_OLS is [ -205.95513652714473 , 205.9621568657506 ]
```

In [ ]: