AI1110 Assignment 2 in LaTeX

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12.13.6.16: Bag I contains 3 red balls and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transfered from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transfered ball is black.

Solution: Let X be the color of the ball transferred from Bag I to Bag II, where $X = 0$ if the ball is red, and $X = 1$ if the ball is black. Let Y be the color of the ball drawn from Bag II, where $Y = 0$ if the ball is red, and $Y = 1$ if the ball is black.

By using Bayes' theorem,the probability that the transferred ball is black given that the drawn ball is red

$$\Pr(X = 1|Y = 0) = \frac{\Pr(Y = 0|X = 1) \times \Pr(X = 1)}{\Pr(Y = 0)} \tag{1}$$

where $P(Y = 0|X = 1)$ is the probability of drawing a red ball from Bag II given that the transferred ball is black, $P(X = 1)$ is the probability of transferring a black ball from Bag I to Bag II, and $P(Y = 0)$ is the probability of drawing a red ball from Bag II.

We can calculate each of these probabilities as follows:

$$\Pr(Y = 0|X = 1) = \frac{4}{10} \tag{2}$$

$$\implies \Pr(Y = 0|X = 1) = \frac{2}{5} \tag{3}$$

$$\Pr(X = 1) = \frac{4}{7} \tag{4}$$

$$\Pr(Y = 0) = \Pr(Y = 0|X = 0) \times \Pr(X = 0) + \Pr(Y = 0|X = 1) \times \Pr(X = 1) \tag{5}$$

$$\implies \Pr(Y = 0) = \frac{5}{10} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7} \tag{6}$$

$$\implies \Pr(Y = 0) = \frac{15}{70} + \frac{16}{70} \tag{7}$$

$$\implies \Pr(Y = 0) = \frac{31}{70} \tag{8}$$

$$Substituting the values in equation number(1), \tag{9}$$

$$\Pr(X = 1|Y = 0) = \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{31}{70}} \tag{10}$$

$$\implies \Pr(X = 1|Y = 0) = \frac{\frac{16}{70}}{\frac{31}{70}} \tag{11}$$

$$\implies \Pr(X = 1|Y = 0) = \frac{16}{31} \tag{12}$$

$\therefore$ The probability that the transferred ball is black given that the drawn ball is red is $\frac{16}{31}$.

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