

AI1110 Assignment 2 in L^AT_EX

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12.13.6.16: Bag I contains 3 red balls and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Solution:

Table: Random variables and their values

| Random variable | Description | Values |
|-----------------|----------------------------|------------------------|
| X | Colour of transferred ball | 0 for red, 1 for black |
| Y | Colour of drawn ball | 0 for red, 1 for black |

By using Bayes' theorem, the probability that the transferred ball is black given that the drawn ball is red

$$\Pr(X = 1|Y = 0) = \frac{\Pr(Y = 0|X = 1) \times \Pr(X = 1)}{\Pr(Y = 0)} \quad (1)$$

where $P(Y = 0|X = 1)$ is the probability of drawing a red ball from Bag II given that the transferred ball is black, $P(X = 1)$ is the probability of transferring a black ball from Bag I to Bag II, and $P(Y = 0)$ is the probability of drawing a red ball from Bag II.

We can calculate each of these probabilities as follows:

$$\Pr(Y = 0|X = 1) = \frac{4}{10} \quad (2)$$

$$\Rightarrow \Pr(Y = 0|X = 1) = \frac{2}{5} \quad (3)$$

$$\Pr(X = 1) = \frac{4}{7} \quad (4)$$

$$\Pr(Y = 0) = \Pr(Y = 0|X = 0) \times \Pr(X = 0) + \Pr(Y = 0|X = 1) \times \Pr(X = 1) \quad (5)$$

$$\Rightarrow \Pr(Y = 0) = \frac{5}{10} \times \frac{3}{7} + \frac{4}{10} \times \frac{4}{7} \quad (6)$$

$$\Rightarrow \Pr(Y = 0) = \frac{15}{70} + \frac{16}{70} \quad (7)$$

$$\Rightarrow \Pr(Y = 0) = \frac{31}{70} \quad (8)$$

$$\text{Substituting the values in equation number (1),} \quad (9)$$

$$\Pr(X = 1|Y = 0) = \frac{\frac{4}{10} \times \frac{4}{7}}{\frac{31}{70}} \quad (10)$$

$$\Rightarrow \Pr(X = 1|Y = 0) = \frac{\frac{16}{70}}{\frac{31}{70}} \quad (11)$$

$$\Rightarrow \Pr(X = 1|Y = 0) = \frac{16}{31} \quad (12)$$

\therefore The probability that the transferred ball is black given that the drawn ball is red is $\frac{16}{31}$.

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