Quiz 1

MA4240/MA2540: Applied Statistics

Total time: 50 minutes **Total marks: 40**

Q 1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform distribution on the interval $[\theta, 2\theta]$, $\theta > 0$.

(a) Find the method of moments estimator of θ.

(b) Find the maximum likelihood estimator of θ.

(c) Find the bias of the maximum likelihood estimator. $\boxed{\text{3+4+8 MARKS}}$

Q 2.

(a) The observed value of the mean of a random sample from $N(\theta, 1)$ distribution is 2.3. Given the parameter space is $\Theta = \{0, 1, 2, 3\}$, find the maximum likelihood estimator of θ.

(b) Let X_1, X_2, X_3 be iid $N(0, \theta^2)$ random variables, $\theta > 0$. Find the value of k for which the estimator $\left(k \sum_{i=1}^{3} |X_i|\right)$ is an unbiased estimator of θ. $\boxed{\text{6+6 MARKS}}$

Q 3.

(a) Let $X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_n$ be i.i.d. $N(0, 1)$ random variables. Then find the distribution of

$$W = \frac{n(\sum_{i=1}^{m} X_i)^2}{m(\sum_{j=1}^{n} Y_j^2)}.$$

$\boxed{\text{5 MARKS}}$

(b) Suppose

$$\frac{2X_1^2 + 2X_2^2 + X_3^2 + 4X_1X_2 + 2\sqrt{2}X_1X_3 + 2\sqrt{2}X_2X_3}{Y_1^2 + Y_2^2 + Y_3^2 + 2Y_1Y_2} \sim F_{1,\,2}.$$

$\frac{|x_1|+|x_2|+|x_3|}{3} = E(|x_i|)$

$N(0,\theta^2)$

Then, which of the following is/are true? Explain all the steps. $\boxed{\text{8 MARKS}}$

$N(0,1)$ i.i.d

$\frac{n-1}{n}\theta$

$E(k\sum_{i=1}^{3}|x_i|) = \frac{2}{3}\theta^2$

$3k\,E(|x_i|) = \frac{2}{3}\theta^2$

$E(k\sum_{i=1}^{3}|x_i|)$

(i) $(\sqrt{2}X_1 + \sqrt{2}X_2 + X_3) \sim N(0, 1)$, $\dfrac{(Y_1 + Y_2)}{\sqrt{2}} \sim N(0, 1)$, and $Y_3^2 \sim \chi_1^2$.

(ii) $\left(X_1 + X_2 + \dfrac{X_3}{2}\right) \sim N(0, \sqrt{2})$, $(Y_1 + Y_2) \sim N(0, 1)$, and $\sqrt{2}Y_3 \sim N(0, 1)$.

(iii) $\left(X_1 + X_2 + \dfrac{X_3}{2}\right)^2 \sim \chi_1^2$, $(Y_1 + Y_2) \sim N(0, 1)$, and $Y_3^2 \sim \chi_1^2$.

(iv) $\left(X_1 + X_2 + \dfrac{X_3}{\sqrt{2}}\right)^2 \sim \chi_1^2$, $(Y_1 + Y_2) \sim N(0, 1)$, and $Y_3^2 \sim \chi_1^2$.

(v) $\left(X_1 + X_2 + \dfrac{X_3}{\sqrt{2}}\right) \sim N(0, 1)$, $(Y_1 + Y_2)^2 \sim \chi_1^2$, and $Y_3 \sim N(0, 1)$.

$\left(\frac{1}{\theta}\right)^n = L(\theta)$

θ^{-n}

$\frac{1}{\theta^n}$ $\quad -n \frac{1}{\theta} \frac{1}{\theta^{n+1}}$

$\left(\frac{1}{\theta^n}\right) = L(\theta)$

$-\frac{n}{\theta^{n+1}} = 0$