Quiz 2
MA4240/MA2540: Applied Statistics

Total time: 50 minutes   **Total marks: 30**

Q 1. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, 16)$ population. Find the value of n such that the 95% confidence interval for μ is $\left[\overline{X} - 0.98, \overline{X} + 0.98\right]$.

$\boxed{7 \text{ MARKS}}$

Q 2. Consider a population with density

$$f(x, \theta) = \begin{cases} e^{\theta - x}, & \text{if } x > \theta \\ 0, & \text{otherwise} \end{cases}.$$

(a) Let X_1, X_2 be iid random variables from the given density function $f(x, \theta)$ and let $Y = \min\{X_1, X_2\}$. Find a confidence interval, for θ, of the type $[Y - b, Y]$, $0 \leq b < \infty$, having confidence coefficient 0.95.

(b) Suppose now we have $X_1, X_2, \ldots, X_n$ iid random variables from $f(x, \theta)$ and let $Z = \min\{X_1, X_2, \ldots, X_n\}$. Find the confidence level at which $\left(Z - \frac{2}{n}\log_e 5, Z\right)$ is a confidence interval for θ. $\boxed{7+6 = 13 \text{ MARKS}}$

Q 3. A coin is tossed four times and p is the probability of getting a head in a single trial. Let S be the number of heads obtained. It is decided to test

$$H_0 : p = \frac{1}{2} \quad vs \quad H_a : p \neq \frac{1}{2}$$

using the decision rule: Reject H_0 if S is 0 or 4. Then, find the

(a) probability of Type I error.

(b) probability of Type II error, when $p = 3/4$. $\boxed{5+5 = 10 \text{ MARKS}}$