## Final Exam ODE (MA1150)

Full Marks: 50

Time- 1 hour 50 Minutes

## Please answer all the questions:

f. Find the general solution of the given ODE 
$$\left( y + \frac{y^3}{3} + \frac{x^2}{2} \right) dx + \frac{1}{4} (x + xy^2) dy = 0.$$

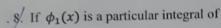
[5 marks]

- 2. If  $\phi(x)$  and  $\psi(x)$  are two solutions of the differential equation  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3f(x), \text{ then find the differential equation of which } \phi(x) + \psi(x) \text{ is a solution.}$
- 3. Whether two functions  $x^3$  and  $x^2|x|$  are linearly independent or not on the interval  $(-\infty, 0]$ . Justify your answer. [2 marks]
- Let  $y_1(x)$  and  $y_2(x)$  be twice differentiable functions on an interval I satisfying the differential equations  $\frac{dy_1}{dx} y_1 y_2 = e^x \sin x$  and  $\frac{dy_2}{dx} \frac{dy_1}{dx} + 2y_1 = 0$ . Then find the  $y_1(x)$ . [5 marks]
- Let  $e^{-x}\cos\left(\sqrt{3}x\right)$ ,  $e^{-x}\sin\left(\sqrt{3}x\right)$ , and  $e^{2x}$  be three linearly independent solutions of the linear ODE L[y(x)] = 0. Then find the general solution of the  $L[y(x)] = e^{2x}x^2$ . [5 marks]
- 6. Find the general solution of linear ODE  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$

[5 marks]

A. Find the solution of the ODE

 $y'' - 3y' + 2y = \cos(e^{-x})$  by using the method of variation of parameter. [5 marks]



$$L[y(x)] = \frac{d^2y}{dx^2} - a\frac{dy}{dx} + b y = e^{ax} + f(x),$$

and  $\phi_2(x)$  is a particular integral of  $L[y(x)] = e^{ax} - f(x)$ , a, b being constants, then find the particular integral of  $L[y(x)] = 2be^{ax}$ . [5 marks]

9. Let v(t) be a solution of the differential equation x'(t) + t x(t) = 0 and let  $\phi(t) = v(t)\xi(t)$  be a solution of the differential equation

$$x''(t) + 2 t x'(t) + (t^2 + 2)x(t) = 0$$

satisfying  $\phi(0) = 0$  and  $\phi'(0) = 1$ . Then find the solution  $\phi(t)$ .

[6 marks]

- 10. If  $y_1(x)$  and  $y_2(x)$  are two solutions of the differential equation  $y'' + \tan x \ y' \sec x \ (1 + e^{-x^2})y = 0 \text{ for } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  with  $y_1(0) = 2$ ,  $y_1'(0) = -1$ ,  $y_2(0) = 2$  and  $y_2'(0) = 2$ , then find the Wronskian of  $y_1(x)$  and  $y_2(x)$  at  $x = \pi/4$ . [5 marks]
- 1. Find the solution of the boundary value problem

$$4x^2 \frac{d^2y}{dx^2} + 8x \frac{dy}{dx} + y = 4/\sqrt{x}$$
,  $y(1) = 0$ ,  $y(e) = 0$ . [5 marks]