Final Exam **Full Marks: 50**

ODE (MA1150) **Time- 1 hour 50 Minutes**

Please answer all the questions:

1. Find the general solution of the given ODE
$\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\frac{1}{4}(x+xy^2)dy=0.$ [5 marks]

2. If $\phi(x)$ and $\psi(x)$ are two solutions of the differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=3f(x)$, then find the differential equation of which $\phi(x)+\psi(x)$ is a solution. [2 marks]

3. Whether two functions x^3 and $x^2|x|$ are linearly independent or not on the interval $(-\infty,0]$. Justify your answer. [2 marks]

4. Let $y_1(x)$ and $y_2(x)$ be twice differentiable functions on an interval I satisfying the differential equations $\frac{dy_1}{dx}-y_1-y_2=e^x\sin x$ and $\frac{dy_2}{dx}-\frac{dy_1}{dx}+2y_1=0$. Then find the $y_1(x)$. [5 marks]

5. Let $e^{-x}\cos(\sqrt{3}\,x)$, $e^{-x}\sin(\sqrt{3}\,x)$, and e^{2x} be three linearly independent solutions of the linear ODE $L[y(x)]=0$. Then find the general solution of the $L[y(x)]=e^{2x}x^2$. [5 marks]

6. Find the general solution of linear ODE
$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{e^x}$ [5 marks]

7. Find the solution of the ODE
$y''-3y'+2y=\cos(e^{-x})$ by using the method of variation of parameter. [5 marks]

8. If $\phi_1(x)$ is a particular integral of

$$L[y(x)] = \frac{d^2y}{dx^2} - a\frac{dy}{dx} + b\,y = e^{ax} + f(x),$$

and $\phi_2(x)$ is a particular integral of $L[y(x)] = e^{ax} - f(x)$, a, b being constants, then find the particular integral of $L[y(x)] = 2be^{ax}$. [5 marks]

9. Let $v(t)$ be a solution of the differential equation $x'(t) + t\,x(t) = 0$ and let $\phi(t) = v(t)\xi(t)$ be a solution of the differential equation

$$x''(t) + 2\,t\,x'(t) + (t^2 + 2)x(t) = 0$$

satisfying $\phi(0) = 0$ and $\phi'(0) = 1$. Then find the solution $\phi(t)$.

[6 marks]

10. If $y_1(x)$ and $y_2(x)$ are two solutions of the differential equation

$$y'' + \tan x\, y' - \sec x\,(1 + e^{-x^2})y = 0 \text{ for } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

with $y_1(0) = 2, y_1'(0) = -1, y_2(0) = 2$ and $y_2'(0) = 2$, then find the Wronskian of $y_1(x)$ and $y_2(x)$ at $x = \pi/4$. [5 marks]

11. Find the solution of the boundary value problem

$$4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + y = 4/\sqrt{x}\,,\ y(1) = 0,\ y(e) = 0.$$ [5 marks]

**********Best of Luck**********