-A122 B4Ct1 11009

Roll number:

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MA1150 - Differential Equations

Date: May 27, 2023

Time: 120 minutes, 09:30 AM - 11:30 AM

Maximum Marks 40

Note. Begin writing by making an index on the first page (Q.No and Page numbers in two coloums).

Q.1 Answer the following:

(a) Solve the initial value problem: (3 marks)

$$6y'' - y' - y = 0, \quad y(0) = 10, \quad y'(0) = 0.$$

(b) Find the general solution of ODE: $x^2y'' - 7xy' + 7y = 0$ on $(0, \infty)$. (2 marks)

(c) Suppose that $\mu(x,y)$ is an integrating factor of ODE $-y + x\dfrac{dy}{dx} = 0$. For every statement below, decide whether they are True or False (No justification required). For each correct answer (+1 **mark**), wrong answer (**−.5 mark**), and in case of no attempt **(0 mark)**.

(i) $\mu(x,y)$ must be a function of x only;

(ii) $\mu(x,y)$ must be a function of y only,

(iii) $\mu(x,y)$ must be of the product form $P(x)Q(y)$ only, where P and Q are functions of x and y respectively,

(iv) None of the above are True. (4 marks)

Q.2 Answer the following:

(a) Consider the following ODE for $y(x)$: (4 marks)

$$y' + \sin^2(x+y) = 0.$$

(i) Is it linear or non-linear?

(ii) Is it Separable or non-separable?

(iii) Convert the ODE into IVP by setting the initial condition $y(x_0) = y_0$ for some $x_0, y_0 \in \mathbb{R}$. What can you say about the existence and uniqueness of solutions to the given initial value problem?

(b) Solve the initial value problem and find the interval of validity: (3 marks)

$$y' = y^2 \cos x, \quad y(0) = \sqrt{2}$$

(c) Assuming that $p(x) \not\equiv 0$, state conditions under which the linear equation (2 marks)

$$y' + p(x)y = f(x)$$

is separable. If the equation satisfies these conditions, solve it by the separation of variables method.

Q.3 Answer the following:

(a) Let $A, B, C, D > 0$ be positive constants with $B^2 > 4AC$. For the following ODE

$$Ay'' + By' + Cy = D.$$

(i) Find a particular solution; (3 marks)

(ii) Find the general solution $y(x)$,

(iii) How does $y(x)$ behave when $x \to +\infty$.

(b) Suppose $f_1(x)$, $f_2(x)$, $g_1(x)$, and $g_2(x)$ are continuous on open interval (a, b) and the equations

$$y'' + f_1(x)y' + g_1(x)y = 0 \quad \text{and} \quad y'' + f_2(x)y' + g_2(x)y = 0$$

have the same solutions on (a, b). Show that $f_1(x) = f_2(x)$ and $g_1(x) = g_2(x)$ on (a, b).

(4 marks)

(c) Compute the Wronskian of the given functions $y_1 = e^x \cos x$ and $y_2 = e^x \sin x$. Check whether y_1 and y_2 are linearly independent on $\mathbb{R}$. (2 marks)

Q.4 Answer the following:

(a) Solve the initial value problem

$$y'' - 4y' - 5y = 9e^{2x}(1 + x), \quad y(0) = 2, \quad y'(0) = -10.$$

(4 marks)

(b) Find ϕ_1, ϕ_2, ϕ_3 and ϕ_4 the first 4 Picard's iterations for the following ODE

$$y' = x + y, \quad y(0) = 0.$$

(3 marks)

Q.5 Answer the following

(a) Find conditions on the constants A, B, C, D, E, and F such that the ODE

$$(Ax^2 + Bxy + Cy^2)\,dx + (Dx^2 + Exy + Fy^2)\,dy = 0$$

is exact. (2 marks)

(b) Find an integrating factor for $(3xy + 6y^2) + (2x^2 + 9xy)\dfrac{dy}{dx} = 0$. (4 marks)