

Assignment - 1

AI1110: Probability and Random Variables

K Anuraga Chandan, AI22BTECH11011

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Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are (i) mutually exclusive (ii) independent.

Solution:

Given,

$$P(A) = \frac{1}{2}$$

$$P(A + B) = \frac{3}{5}$$

$$P(B) = p = ?$$

(i) A and B are *mutually exclusive* events

$$AB = 0$$

$$(A + B) = (A) + (B) - (AB)$$

$$(A + B) = (A) + (B)$$

$$P(A + B) = P(A) + P(B)$$

$$P(B) = P(A + B) - P(A)$$

$$P(B) = \frac{3}{5} - \frac{1}{2} = \frac{6-5}{10}$$

$$P(B) = \frac{1}{10}$$

(ii) A and B are *independent* events

$$P(AB) = P(A)P(B)$$

$$P(A+B) = P(A) + P(B) - P(AB)$$

$$P(A+B) = P(A) + P(B) - P(A)P(B)$$

$$P(A+B) - P(A) = P(B)[1 - P(A)]$$

$$P(B) = \frac{P(A+B)-P(A)}{1-P(A)}$$

$$P(B) = \frac{\frac{3}{5}-\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{6-5}{10}}{\frac{1}{2}} = \frac{1}{10}$$

$$P(B) = \frac{1}{10}$$