ME5430/AI4000:Introduction to Robotics/Robotics

Exam-2
Duration: 1 hour 20 minutes
Total Marks: 25

Date: 25 September, 2024

Your name: __ ________________________ Roll No: _____

Instructions:
- Answer all questions in the sheet.
- You may use loose sheet for additional or rough work
- Marks for each questions are written at the end of the question within the bracket.

Section A: Multiple-choice questions.

4½

Tick on the correct answer. [Marks. 0.5 each question]

1. Given a desired (x,y) position of the end effector, how many solutions are there to the inverse kinematics of the three-link planar arm?
   (a) 2
   (b) 3
   (c) 4
   (d) Infinite
2. If the orientation (θ) of the end effector is also specified, how many solutions are there?
   (a) 0
   (b) 1
   (c) 2
   (d) Infinite
3. The velocity of the end-effector in a robotic manipulator is calculated using:
   (a) The inverse kinematics
   (b) The Jacobian matrix
   (c) The DH parameters
   (d) Euler angles
4. In velocity kinematics, what does the term "redundancy" imply for a robotic manipulator?

(a) The manipulator has more joints than required for a specific task

(b) The manipulator has fewer joints than required

(c) The manipulator is in a singular configuration

(d) The Jacobian matrix is rank-deficient

5. In the context of velocity kinematics, which of the following is true when the manipulator is near a singularity?

   (a) The end-effector speed is maximum

   (b) Joint velocities may become very large

   (c) The Jacobian matrix is well-conditioned

   (d) The manipulator can perform infinite motions

6. In a non-redundant manipulator, if the Jacobian matrix is square and full-rank, then:

   (a) There are multiple solutions to the inverse kinematics

   (b) The manipulator is at a singularity

   (c) The Jacobian inverse can be used to solve for joint velocities

   (d) The manipulator is in a collision state

7. The manipulability ellipsoid is used to describe:

   (a) The workspace of the robot

   (b) The relationship between joint torques and joint velocities

   (c) The direction in which the end-effector can move most easily and with the highest velocity

   (d) The potential collision regions for the manipulator

8. In velocity kinematics, the relationship between the end-effector velocity and joint velocity is linear because:

   (a) The Jacobian matrix is non-linear

   (b) The joint velocity affects the translational and angular velocities directly

   (c) The end-effector's motion is governed by quadratic functions

   (d) The D-H parameters linearize the system

9. If a robotic manipulator is at a singular configuration, which of the following can occur?

   (a) The Jacobian becomes non-invertible

   (b) The manipulator can no longer move

   (c) The manipulator's joint velocities become undefined

   (d) The manipulator has infinite joint velocity solutions

10. The condition number of the Jacobian matrix is important because it:

   (a) Helps determine how far the manipulator is from a singularity

   (b) Determines the force exerted by the end-effector

   (c) Is used to compute the manipulator's workspace

   (d) Tells if the manipulator has reached a maximum velocity state

Section B: Broad questions

1. Given the DH parameters of a 2-DOF planar manipulator as given in Table 1,

$link_i$	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2

Table 1: DH Parameters for 2-DOF manipulator.

2

(a) Compute the inverse kinematics. [Marks. 2]

Answer:

(x,y)

(b) Derive the Jacobian matrix for the above 2-link planar manipulator. [Marks. 1.5]

Answer:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

(c) Derive the singularity condition (Note: derivation is required. Do not directly write the final answer). [Marks. 1.5]

Answer:

(d) Write the expression for deriving manipulability measure (μ) and the manipulability ellipsoid from the Jacobian matrix for the manipulator. [Marks. 2]

Answer:

(e) Derive the static end effector force (F) and the joint torque (τ) relationship for the manipulator. [Marks. 2]

Answer:

2.

2. What is redundant and non-redundant manipulator? Discuss at least two roles of optimization techniques in solving inverse kinematics for redundant and non-redundant manipulators. [Marks. 2]

Answer:

3. Two frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by the homogeneous transformation $H = \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. A particle has velocity $\mathbf{v}_1(t) = (3, 1, 0)$ relative to frame $o_1x_1y_1z_1$. What is the velocity of the particle in frame $o_0x_0y_0z_0$? [Marks. 1]

Answer:

4. Consider the three-link planar manipulator of Figure 1. Compute the linear velocity v and the angular velocity ω of the center of link 2 as shown. [Marks. 2]

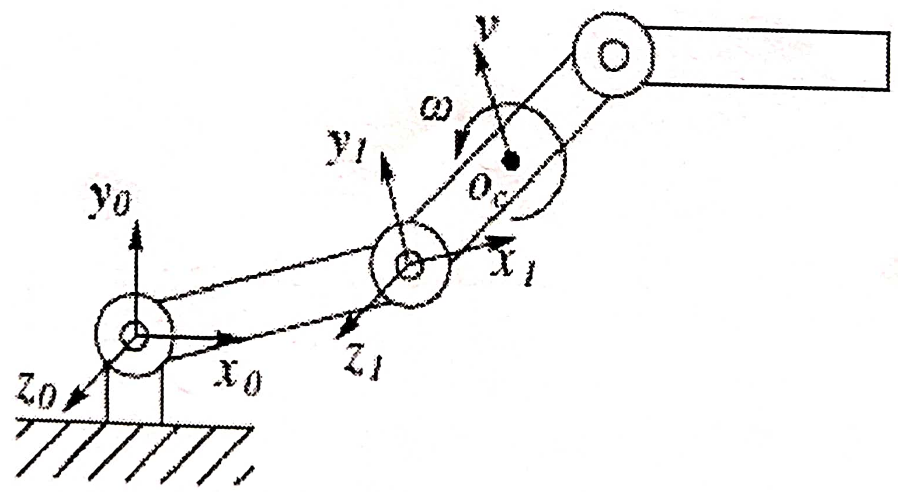

Figure 1: Three link planner manipulator.

Answer:

5. Derive the Jacobian matrix for SCARA manipulator with one prismatic and three revolute joints (R-R-P-R configuration) by computing the linear and angular velocity Jacobian step-by-step. Also, derive the singularity condition for the manipulator (No need to multiply and simplify the product of matrices) [Marks: 4+2]

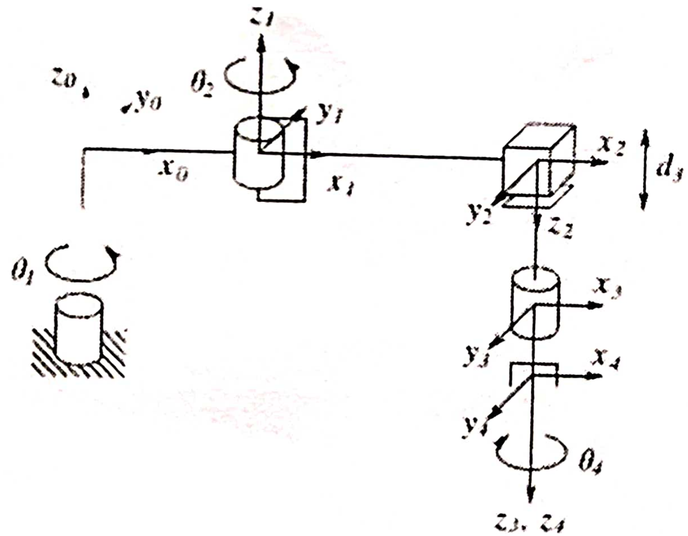

Figure 2: SCARA manipulator.

Answer:

Appendix:

A: the homogeneous transformation matrix A_i is given by:

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B: The ZYZ Euler angle transformation matrix is given by:

$$R_{ZYZ} = \begin{bmatrix} \cos\phi\cos\theta\cos\psi - \sin\phi\sin\psi & -\cos\phi\cos\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta \\ \sin\phi\cos\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\cos\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta \\ -\sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{bmatrix}$$

C: Transformation matrix for SCARA manipulators

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

USE SPACE FOR ROUGH WORK