AerE 161 Project 1: Standard Atmosphere Table

Due date: Friday, Feb 23rd, 2018 by 6 pm.

Problem Statement

Write a MATLAB program to generate Standard Atmosphere values for Troposphere and Stratosphere ($h = 0$ km to $h = 47$ km) in steps 1 km and plot each of the quantities that listed below.

The output of your MATLAB program:

I. Standard Atmosphere Table should include calculated values:

- h - geopotential altitude in *km*
- h_G - geometric altitude in *km*
- T - temperature in *Kelvin*
- ρ - density in kg/m^3
- p - pressure in N/m^2

II. Plots of Geopotential Altitude h (km) vs.:

- temperature T (K)
- density $\rho\,(kg/m^3)$
- pressure p (N/m^2)

Submission requirements:

Submit Electronically in Canvas (assignment - Project 1):

- Your Project Report (pdf document)
- Your MATLAB program file (m-file)

Name your files as follows: *LastName_FirstName_Project1.pdf* and *LastName_FirstName_Project1.m*

- **Projects turned in after the due date will not be graded and will receive a 0 grade**

Presentation requirements:

- Create a Project Report (PDF document) as a complete solution.
- Include a complete write-up (Cover Sheet, Table of Contents, Problem Statement, Theory, Solution, Discussion).
- In cover sheet, include your name (with affiliation), title of your report, the course number and the due date.
- The table of contents should provide a brief overview of the organizational structure of your report. All major parts of the reports should be listed here along with their corresponding page numbers.
- In the Theory part: include and explain all equations (use MathType or Equation Editor) used in your program.
- In the Solution part include: a copy of the <u>well-commented</u> computer code, Standard Atmosphere table and all plots. Note: do not use **subplot** MATLAB function.
- In the Discussion part: include the analysis and interpretation of your results. Also, briefly discuss the challenges faced by you and the solutions that helped you complete this project.

Theory

1. Gradient Layer

The gradient layers exist from $h = 0\,km$ to $h = 11\,km$ (*troposphere*) and from $h = 25\,km$ to $h = 47\,km$ (*stratosphere*).

Start at sea level ($h = 0$) , where standard sea level values of temperature, density, and pressure, respectively, are

$T_{SL} = 288.16\ K$, $\rho_{SL} = 1.225\ kg/m^3$, $p_{SL} = 1.01325 \times 10^5\,N/m^2$,
$g_0 = 9.80065\ m/s^2$, $R = 287\ \frac{J}{kg \cdot K}$.

Use the following equations, derived from the hydrostatic equation and the equation of state:

$$T = T_1 + a(h - h_1)$$

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}} \qquad (1)$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR} - 1}$$

where $a = \dfrac{dT}{dh}$ - the *lapse rate* for the gradient layer:

$$a = -6.5 \times 10^{-3}\,K/m \quad (\textit{troposphere})$$

$$a = 3 \times 10^{-3}\,K/m \quad (\textit{stratosphere})$$

2. Isothermal Layer

The isothermal (constant temperature) layer exists from $h = 11\ km$ to $h = 25\ km$.

Use the following equations, derived from the hydrostatic equation and the equation of state:

$$\frac{p}{p_1} = e^{\left(-\frac{g_0}{RT}\right)(h - h_1)}$$

$$(2)$$

$$\frac{\rho}{\rho_1} = e^{\left(-\frac{g_0}{RT}\right)(h - h_1)}$$

3. Geometric vs. Geopotential Altitude

Geopotential altitude is a "fictitious" altitude, which is physically compatible with the assumption of a constant gravitational acceleration ($g = const = g_0$).

Geometric altitude h_G can be related to geopotential altitude h by:

$$h_G = \frac{h \cdot r_E}{r_E - h} \tag{3}$$

where r_E the radius of the Earth $r_E = 6371.0008\ km$