AerE 161
Project #1: Standard Atmosphere Table

Hammad Imam
Undergraduate, Aerospace Engineering
Iowa State University

February 23rd, 2018

Contents

1 Problem Statement **1**

2 Theory **1**
2.1 Introduction 1
2.2 Geometric Altitude, h_g, m 2
2.3 Temperature, T , K 2
2.4 Density, ρ, kg/m^3 3
2.5 Pressure, p, N/m 4

3 Solution **5**
3.1 Overview 5
3.2 Code 5
3.3 Standard Atmopshere Table 7
3.4 Plots 8
3.4.1 Temperature vs Geopotential Altitude 8
3.4.2 Density vs Geopotential Altitude 9
3.4.3 Pressure vs Geopotential Altitude 9

4 Discussion **10**
4.1 Analysis 10
4.2 Challenges 10

1 Problem Statement

The purpose of this project was to generate a Standard Atmosphere Table, with values for the standard atmosphere from the troposphere to the stratosphere. The table outputs the geopotential altitude from 0 to 47km in steps of 1km, and uses those values to find the geometric altitude, and standard values for temperature, density, and pressure. Additionally, three graphs were generated, plotting temperature, density, and pressure all against the geopotential altitude.

2 Theory

2.1 Introduction

In this section, the methods of calculations for each calculated value will be explained. There are three parts of the atmosphere that all behave differently, so each subsection will explain what was done differently for each value. Any constants used in calculations will be expressed in the subsection that uses those constants. Every subsection will contain a short overview of the value, and then calculations for the value derived from the hydrostatic equation and equation of state. The calculations will all be either in terms of the geopotential altitude

h, or of a previously calculated value. All work and explanations will be shown, and final results for a formula will have a number displayed next to them.

2.2 Geometric Altitude, h_g, m

Geometric altitude h_g is the "true" distance off the surface of the Earth, as opposed to geopotential altitude h which is a "fictitious" altitude that represents the height with an assumption of constant gravitational acceleration g_0, as opposed to one that varies with the distance from the surface.
Geometric altitude h_g is related to geopotential altitude h by the formula

$$h_g = \frac{h * r_E}{r_E - h} \tag{1}$$

where r_E represents the radius of Earth, $6371.0008km$. In the Standard Atmosphere Table, the geometric altitude is calculated directly from the stepped geopotential altitude.

2.3 Temperature, T , K

Temperature T is displayed in the table in units Kelvin. The temperature first decreases in the troposphere, remains constant in the isothermal layer, and then rises again in the stratosphere. Layers where the temperature changes are referred to as "gradient layers."
The equation given from the hydrostatic equation is as follows

$$T = T_1 + a(h - h_1)$$

where T_1 is the temperature 1 lower than the value being calculated, a is the lapse rate $\frac{dT}{dh}$, h is the current height, and h_1 is the height 1 lower than the value being calculated.
Since the equation depends on lower values of the temperature, an initial temperature is required to calculate higher values. T_0, the temperature at sea level, is given as

$$T_0 = 288.16K$$

The values for a, the lapse rate of the temperature, depend on the layer of atmosphere. The values are given below

$$a = \begin{cases} -6.5 \times 10^{-3}\ K/m & troposphere,\ h = [0, 11] \\ 0\ K/m & isothermal,\ h = [12, 24] \\ 3 \times 10^{-3}\ K/m & stratosphere,\ h = [25, 47] \end{cases}$$

The geopotential altitude in the table is calculated in steps of 1km, so the value for $(h - h_1)$ will remain a constant 1km.
Inserting these values into the original equation for temperature gives us the combined equation

$$T_h = \begin{cases} T_{h-1}\ K - (6.5 \times 10^{-3}\ K/m)(1km) & troposphere,\ h = [0, 11] \\ T_{h-1}\ K & isothermal,\ h = [12, 24] \\ T_{h-1}\ K + (3 \times 10^{-3}\ K/m)(1km) & stratosphere,\ h = [25, 47] \end{cases} \tag{2}$$

2.4 Density, ρ, kg/m^3

Density ρ is displayed in the table in units kg/m^3. The rate of change is reliant on the ratio between the temperature at the current height, so a different formula must be used for the isothermal layer than from the gradient layers. The original equations are

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\frac{g}{aR}-1}$$
$$\frac{\rho}{\rho_1} = e^{\left(-\frac{g}{RT}\right)(h-h_1)}$$

where gravitational acceleration $g = 9.81m/s^2$, lapse rate $a = -6.5 \times 10^{-3} K/m$ in the troposphere and $3 \times 10^{-3} K/m$ in the stratosphere, and $R = 287\frac{J}{kg\ K}$. The first equation is used in the gradient layers, while the second equation is used in the isothermal layers. With some rearranging, the equations become

$$\rho_h = \rho_{h-1}\left(\frac{T_h}{T_{h-1}}\right)^{-\frac{g}{aR}-1}$$
$$\rho_h = \rho_{h-1}\left(e^{\left(-\frac{g}{RT}\right)(h-h_1)}\right)$$

Since each value will be calculated, temperature values will be independently calculated in each row and used in the formula for density. Gravitational acceleration g, lapse rate a, and specific gas constant R are all known values. The geopotential altitude in the table is calculated in steps of 1km, so the value for $(h - h_1)$ will remain a constant 1km. Subbing in all known values gives the equation

$$\rho_h = \begin{cases} \rho_{h-1}\left(\frac{T_h}{T_{h-1}}\right)^{\frac{9.81m/s^2}{6.5\times10^{-3}K/m\ \ 287\frac{J}{kg\ K}}-1} & troposphere,\ h = [0, 11] \\ \rho_{h-1}\left(e^{\left(-\frac{9.81m/s^2}{T_h * 287\frac{J}{kg\ K}}\right)(1km)}\right) & isothermal,\ h = [12, 24] \\ \rho_{h-1}\left(\frac{T_h}{T_{h-1}}\right)^{-\frac{9.81m/s^2}{3\times10^{-3}K/m\ \ 287\frac{J}{kg\ K}}-1} & stratosphere,\ h = [25, 47] \end{cases} \tag{3}$$

2.5 Pressure, p, N/m

Pressure p is displayed in the table in units N/m. Similar to density, he rate of change is reliant on the ratio between the temperature at the current height, so a different formula must be used for the isothermal layer than from the gradient layers. The original equations are

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g}{aR}}$$
$$\frac{p}{p_1} = e^{\left(-\frac{g}{RT}\right)(h-h_1)}$$

where gravitational acceleration $g = 9.81m/s^2$, lapse rate $a = -6.5 \times 10^{-3}K/m$ in the troposphere and $3 \times 10^{-3}K/m$ in the stratosphere, and $R = 287\frac{J}{kg\ K}$. The first equation is used in the gradient layers, while the second equation is used in the isothermal layers. Rearranged, the equations become

$$p = p_{h-1}\left(\frac{T_h}{T_{h-1}}\right)^{-\frac{g}{aR}}$$
$$p_h = p_{h-1}\left(e^{\left(-\frac{g}{RT}\right)(h-h_1)}\right)$$

Since each value will be calculated, temperature values will be independently calculated in each row and used in the formula for pressure. Gravitational acceleration g, lapse rate a, and specific gas constant R are all known values as listed above. The geopotential altitude in the table is calculated in steps of 1km, so the value for $(h - h_1)$ will remain a constant 1km. Replacing variables for all known values gives the equation

$$p_h = \begin{cases} p_{h-1}\left(\frac{T_h}{T_{h-1}}\right)^{\frac{9.81m/s^2}{6.5\times10^{-3}K/m\ 287\frac{J}{kg\ K}}} & troposphere,\ h = [0, 11] \\ p_{h-1}\left(e^{\left(-\frac{9.81m/s^2}{T_h*287\frac{J}{kg\ K}}\right)(1km)}\right) & isothermal,\ h = [12, 24] \\ p_{h-1}\left(\frac{T_h}{T_{h-1}}\right)^{-\frac{9.81m/s^2}{3\times10^{-3}K/m\ 287\frac{J}{kg\ K}}} & stratosphere,\ h = [25, 47] \end{cases} \tag{4}$$

3 Solution

3.1 Overview

This section will contain the code used to generate the standard atmosphere table, the table itself, and the 3 plots of temperature T, density ρ, and pressure p all against geopotential altitude h. The code section will contain screenshots of the code as it appears in the MATLAB editor. The Standard Atmosphere Table section will contain the table as it appears in the MATLAB output, with line breaks added for readability. The plots section will contain the graphs exactly as they are generated in the MATLAB output.

3.2 Code

```
%Hammad Imam // himam@iastate.edu
%AER_E 161 Project 1
%February 23rd, 2018
clear, clc;

%initial values for each variable
h_g = 0;
T = 288.16;
d = 1.225;
p = 101325;

%constants/etc that will be used in calculations
a_t = -6.5*10^-3;
a_s = 3*10^-3;
r_e = 6371.0003;
g = 9.80065;
R = 287;

%string names to associate with each variable (g - graph, t - table)
G_h = 'Geopotential Altitude (km)'; T_h = 'Geopotential';
G_g = 'Geometric Altitude (km)'; T_g = 'Geometric';
G_T = 'Temperature (K)'; T_T = 'Temperature';
G_d = 'Density (kg/m^3)'; T_d = 'Density';
G_p = 'Pressure (N/m^2)'; T_p = 'Pressure';

%initialize resulting matrix to first row, initial values
res = [0,h_g,T,d,p;];

```

```matlab
for h = 1:47

    if(h<=11)           %if under height of troposphere
        a = a_t;        %use the troposphere a value
    elseif(h<25)        %elseif above troposphere, but under stratosphere
        a = 0;          %set a to 0 (to be used for checking later)
    else                %else, not under stratosphere, must be in it
        a = a_s;        %use the stratosphere a value
    end

    T_0 = res(h,3);  %set the initial temp to the one above current row
    d_0 = res(h,4);  %same for density
    p_0 = res(h,5);  %same for pressure

    h_g = (r_e*h)/(r_e - h);      %calculate geometric from geopotential
    T = T_0 + (a*1000);           %calculate temp from previous temp, a

    if(a == 0)                                  %if in isothermal zone
        d = d_0 * exp((-g/(R*T))*1000);        %use isothermal density calc
        p = p_0 * exp((-g/(R*T))*1000);        %same for pressure
    else                                        %else, is in gradient layer
        d = d_0 * ((T/T_0)^((-g/(a*R))-1));    %use gradient density calc
        p = p_0 * ((T/T_0)^(-g/(a*R)));        %same for pressure
    end

    res(h+1,1:end) = [h,h_g,T,d,p];  %add a matrix row with those values
end

%Standard Atmosphere Table
disp('Standard Atmosphere Table');
disp(array2table(res,'VariableNames',{T_h,T_g,T_T,T_d,T_p}));

%Geopotential Height vs Temperature
figure
plot(res(1:end, 1), res(1:end, 3),'.b-')
xlabel(G_h);
ylabel(G_T);
title([G_T, ' vs. ', G_h])

%Geopotential Height vs Density
figure
plot(res(1:end, 1), res(1:end, 4),'.r-')
xlabel(G_h);
ylabel(G_d);
title([G_d, ' vs. ', G_h])

%Geopotential Height vs Pressure
figure
plot(res(1:end, 1), res(1:end, 5),'.g-')
xlabel(G_h);
ylabel(G_p);
title([G_p, ' vs. ', G_h])
```

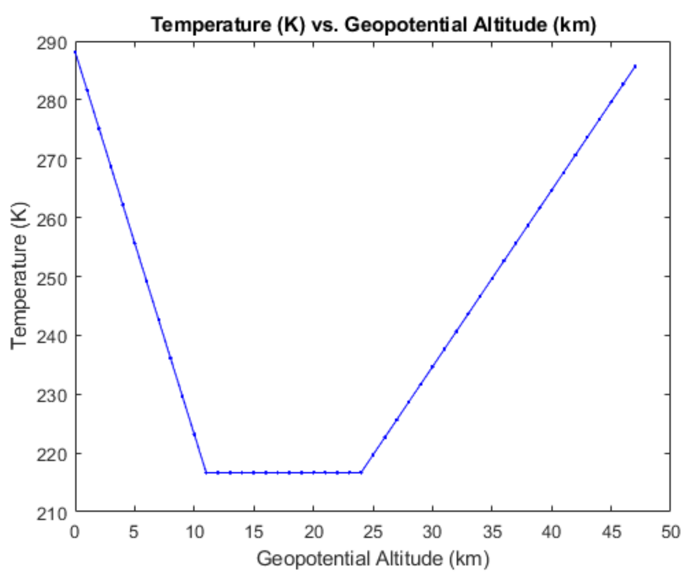
3.3 Standard Atmopshere Table

Geopotential	Geometric	Temperature	Density	Pressure
0	0	288.16	1.225	1.0133e+05
1	1.0002	281.66	1.1117	89880
2	2.0006	275.16	1.0066	79504
3	3.0014	268.66	0.90927	70120
4	4.0025	262.16	0.81932	61654
5	5.0039	255.66	0.73633	54036
6	6.0057	249.16	0.65993	47198
7	7.0077	242.66	0.58974	41078
8	8.0101	236.16	0.52542	35617
9	9.0127	229.66	0.4666	30760
10	10.016	223.16	0.41296	26453
11	11.019	216.66	0.36417	22648
12	12.023	216.66	0.31107	19345
13	13.027	216.66	0.26571	16524
14	14.031	216.66	0.22696	14115
15	15.035	216.66	0.19387	12057
16	16.04	216.66	0.1656	10299
17	17.045	216.66	0.14145	8796.8
18	18.051	216.66	0.12082	7514
19	19.057	216.66	0.1032	6418.3
20	20.063	216.66	0.088155	5482.4
21	21.069	216.66	0.0753	4683
22	22.076	216.66	0.06432	4000.1
23	23.083	216.66	0.054941	3416.8
24	24.091	216.66	0.046929	2918.6
25	25.098	219.66	0.039581	2495.7
26	26.107	222.66	0.033461	2138.6
27	27.115	225.66	0.028351	1836.4
28	28.124	228.66	0.024074	1580.1
29	29.133	231.66	0.020485	1362.2
30	30.142	234.66	0.017468	1176.6
31	31.152	237.66	0.014926	1018.2
32	32.162	240.66	0.012778	882.72
33	33.172	243.66	0.010961	766.62
34	34.182	246.66	0.0094197	666.94

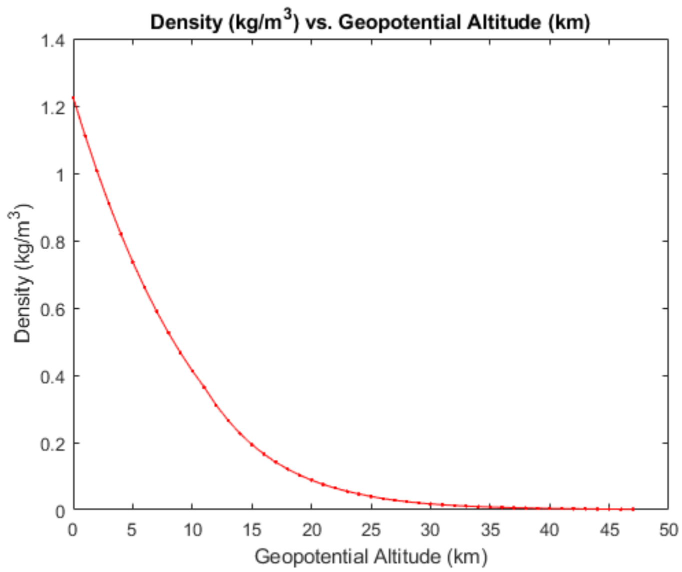
35	35.193	249.66	0.0081101	581.2
36	36.205	252.66	0.006995	507.31
37	37.216	255.66	0.0060438	443.53
38	38.228	258.66	0.0052309	388.37
39	39.24	261.66	0.0045348	340.6
40	40.253	264.66	0.0039378	299.15
41	41.266	267.66	0.0034248	263.13
42	42.279	270.66	0.0029833	231.78
43	43.292	273.66	0.0026027	204.44
44	44.306	276.66	0.002274	180.58
45	45.32	279.66	0.0019897	159.72
46	46.335	282.66	0.0017434	141.45
47	47.349	285.66	0.0015298	125.4

3.4 Plots

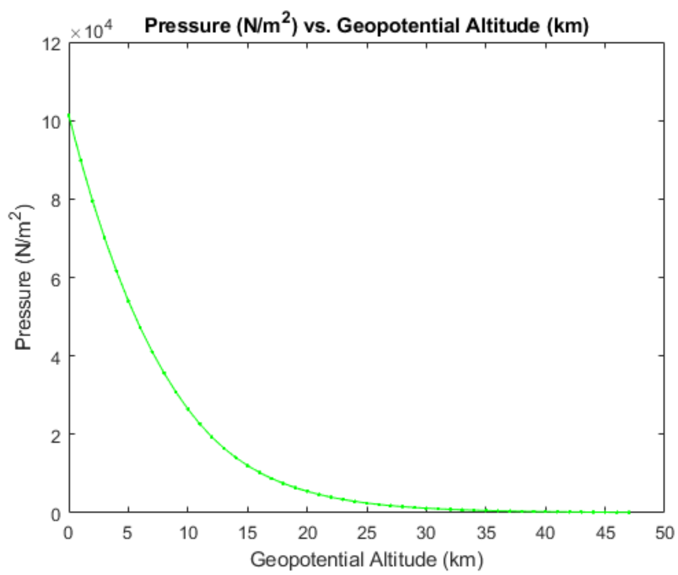
3.4.1 Temperature vs Geopotential Altitude



3.4.2 Density vs Geopotential Altitude



3.4.3 Pressure vs Geopotential Altitude



4 Discussion

4.1 Analysis

The geometric height is fairly close to the geopotential height, as throughout the table, the geometric height varies from the geopotential height by a small amount. As visible on the graphs, temperature changes differently from either density or pressure. Temperature starts at a constant value at sea level, then decreases linearly through the troposphere, hits a value of 216.66 K in the isothermal layer, and rises linearly through the stratosphere at a different lapse rate. The density and pressure both decrease fairly smoothly throughout the layers, albeit at different rates to each other, and different rates through the gradient and isothermal layers. The graphs overall show that the air temperature decreases the farther you get from the surface of the earth, but then warms as the atmosphere thins and the air is more directly heated by the sun. The density and pressure of the air both decrease sharply when coming off the Earth, but then decrease at a steadier rate the further off the Earth they get until they'd eventually reach a point where the atmosphere is negligible.

4.2 Challenges

This project, as beautiful as it may look, actually took me quite a bit of effort both in coding the MATLAB file, writing this LaTeX document, and uploading it to a GitHub repository. One of the challenges I faced was the formatting on the table of values while using the array2table() function. The spacing for the column headers had a smaller maximum width than the width of the names I had originally planned to use, so I had to create separate strings with shorter names that would fit into the table. I also had some issue with indexing, as my initial draft of the code read the initial row as row 0 when it should have been row 1, which was resolved by writing to the array at the value of the for loop plus one. When writing the LaTeX document, I had a lot of issues with properly formatting the equation and had to look up a good amount of how to type what I was typing. When working with the GitHub repository (available at `https://github.com/himam99/AERE161-Project1`), I was mostly reliant on GitHub's cheet sheet, and had some issues pushing to GitHub, so I had to Google that as well. Overall, aside from some minor coding issues, the project wasn't too difficult to complete, if a bit tedious.