AerE 161 Project 2: Flight Path of a Projectile

Due date: Friday, March 23rd, 2018 by 6 pm.

Problem Statement

In this project, we will calculate the flight path of a projectile with and without air resistance. For simplification, we will assume that the projectile is small and spherical in shape.

Your task is to write a MATLAB programs that calculate the flight path, range and flight time for several values of coefficients of resistance of the projectile, and plot the results.

Submission requirements:

Submit Electronically on Canvas (assignment - Project 2):

- Your Project Report (pdf document)
- All 5 of your MATLAB program files (m-files)

Name your Project report pdf file as follows: *LastName_FirstName_Project2.pdf*

Projects turned in after the due date will not be graded and will receive a 0 grade

Presentation requirements:

- Create a Project Report (PDF document) as a complete solution.
- Include a complete write-up (Cover Sheet, Table of Contents, Problem Statement, Theory, Solution, Discussion).
- In cover sheet, include your name (with affiliation), title of your report, the course number and the due date.
- The table of contents should provide a brief overview of the organizational structure of your report. All major parts of the reports should be listed here along with their corresponding page numbers.
- In the Theory part: include all equations (use MathType or Equation Editor) used in your program.
- In the Solution part include: copies of the well-commented computer codes, and all plots.
- In the Discussion part: include the analysis and interpretation of your results. Also, briefly discuss the challenges faced by you and the solutions that helped you complete this project.

Theory

If we neglect the air resistance, the flight path of a projectile launched at an initial speed v_0 and an angle θ of departure relative to the horizontal is a parabola (see Fig. 1).

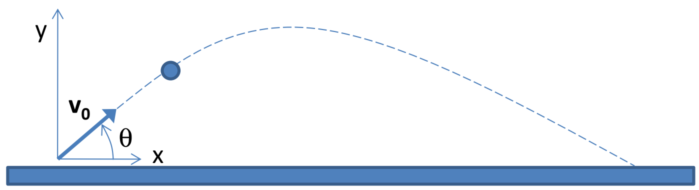

Figure 1: Flightpath of a projectile.

If we assume that the projectile is launched from the surface of the Earth, i.e., at $y_0 = 0$, then the altitude of the projectile as a function of time is

$$y(t) = -\frac{1}{2}gt^2 + Vt\,, \tag{1}$$

where $g = 9.81$ m/s^2 is the gravitational constant, t is time (in seconds), and $V = v_0 \sin\theta$ (in meters per second). The distance of the projectile from the launching position as a function of time is

$$x(t) = Ut\,, \tag{2}$$

where $U = v_0 \cos\theta$. The velocity of the projectile in the horizontal direction, i.e., parallel to the x-axis, is

$$u(t) = U\,, \tag{3}$$

and the velocity in the vertical direction, i.e., parallel to the y-axis, is

$$v(t) = -gt + V. \tag{4}$$

If we include the effects of air resistance, then the altitude of the projectile as a function of time can be approximated as

$$y(t) = -\frac{gt}{k} + \frac{kV+g}{k^2}(1 - e^{-kt})\,, \tag{5}$$

where k is the coefficient of resistance (with the unit 1/s). The distance of the projectile relative to the initial position as a function of time is

$$x(t) = \frac{U}{k}(1 - e^{-kt})\,. \tag{6}$$

The velocity of the projectile in the horizontal direction is

$$u(t) = Ue^{-kt}\,, \tag{7}$$

and in the vertical direction

$$v(t) = Ve^{-kt} + \frac{g}{k}(e^{-kt} - 1). \tag{8}$$

Deliverables:

1. A MATLAB function, flightpath.m, capable of calculating the flight path of a projectile, with and without air resistance, given an initial speed v_0, an angle θ of departure relative to the horizontal, and a coefficient of resistance k.
   Use Eqns. (1)-(8).

2. A script, main_flightpaths.m, which uses the function flightpath.m to calculate and the function plot_flightpaths.m to plot the following:
   a. Altitude, y, as a function of distance, x (see Fig. 2)
   b. Altitude, y, as a function of time, t
   c. Horizontal velocity, u, as a function of time, t
   d. Vertical velocity, v, as a function of time, t

   For these plots, assume v_0 = 600 m/s, θ = 60 deg., and k = [0 0.005 0.01 0.02 0.04 0.08] s^{-1}.

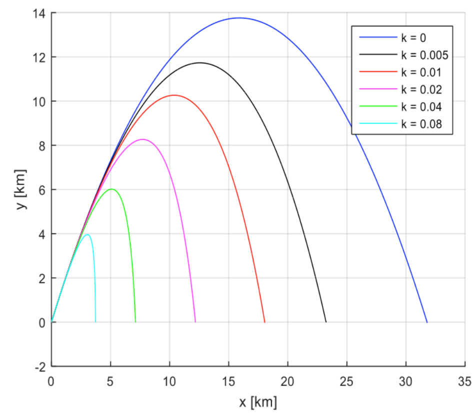

Figure 2: Altitude versus distance.

3. A script, main_range.m, which uses the function flightpath.m to calculate and function plot_range.m to plot the following:
   a. Range as a function of k
   b. Total flight time as a function of k

   For these plots, assume v_0 = 600 m/s, θ = 60 deg., and k from 0 to 0.08 s^{-1} with step size of 0.001 s^{-1}.

4. A detailed report describing the problem, your solution approach, and your results. Be sure to discuss and comment on your results. Show your MATLAB codes and the output from the code when you execute it. Remember to include appropriate comments in the MATLAB code. The following table summarizes the codes you need to write:

m-file	Description
flightpath.m	A function that calculates the characteristics of the projectile flightpath. Input: v_0, θ, k Output: Range, flight time, speed/position as a function of time (vectors: t, u, v, x, y)
main_flightpaths.m	A script which initializes parameters and calls flightpath.m and plot_flightpaths.m
plot_flightpaths.m	A function which plots several plots (see part 2)
main_range.m	A script which initializes parameters and calls flightpath.m and plot_range.m
plot_range.m	A function which plots several plots (see part 3)

Note: if you are using any MATLAB built-in functions to solve nonlinear equations in your project, then in your report you must explain in detail how these functions work and the method/theory behind them.