Assignment-1 DSC

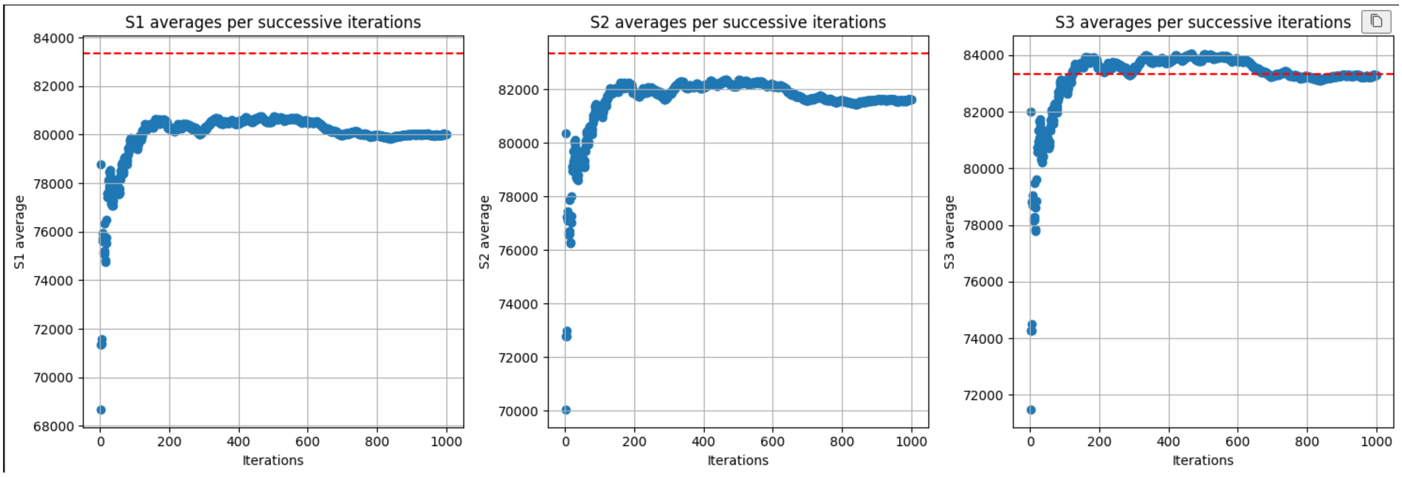
Q2.a)

We got true variance as follows :

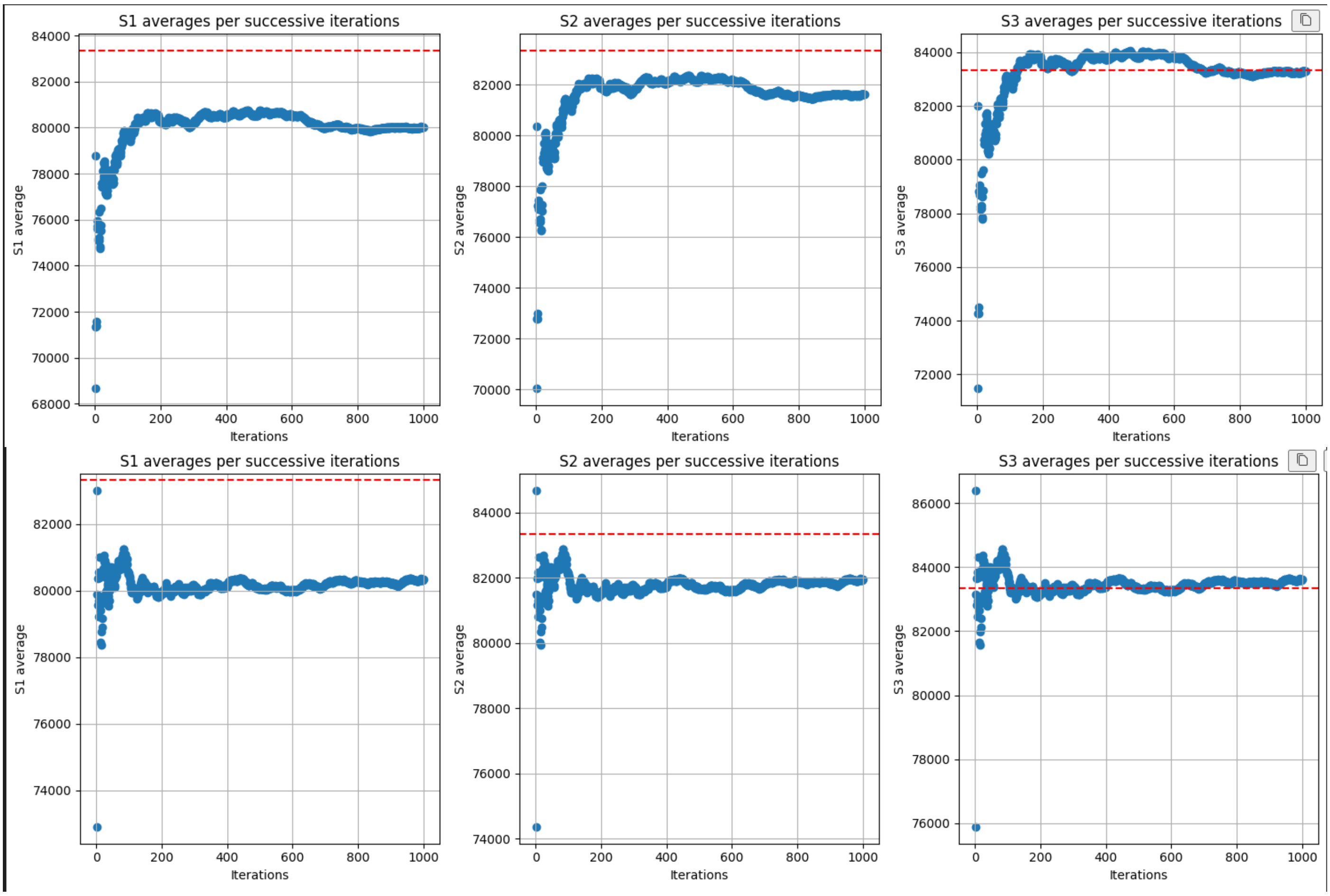
```
true_variance=var_calc(dataset)
print(true_variance)
✓ 0.0s
83333.333325
```

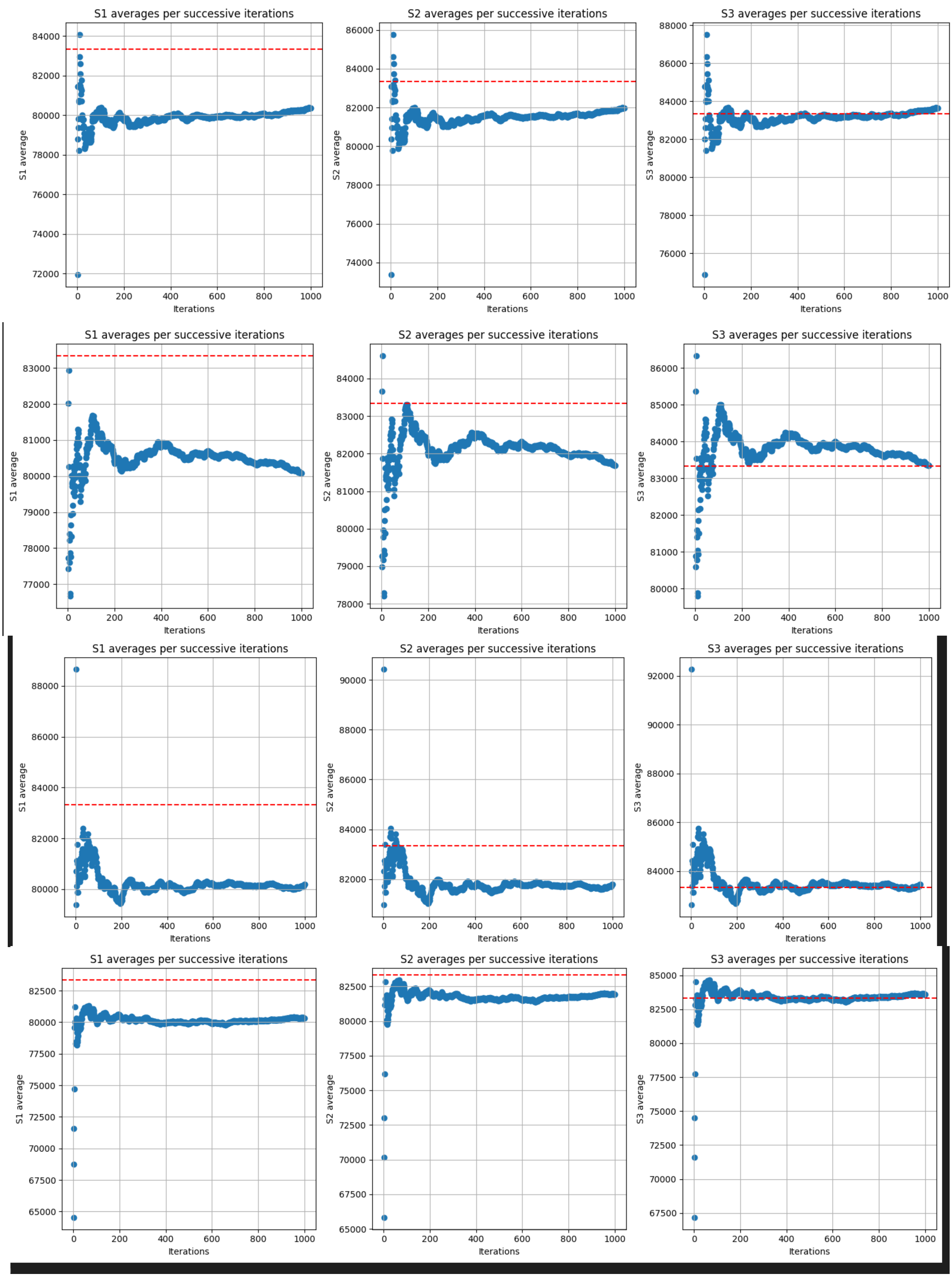
Q2.d)

We obtained the following scatter graphs for s1, s2, s3.



Q2.e)





It is quite clearly visible from the graphs that s3 reaches the true variance much more quickly than s2 which in turn is faster than s1. We also notice that s3 reaches much more near to true variance than s2 which also reaches near to true variance much more than s1.

s3 reaches near to true variance much faster because it is the unbiased variance unlike s2 and s1. It is unbiased because it accounts for degree of freedom being n-1 instead of n for the sample drawn from the dataset.

Q3.a)

Probability of seeing number √k on face = 1/k (unbiased die)

Let X = Geometric random variable equal to the number of times we need to roll the die to see √k

PMF(X) = (1-p)^(x-1) * p where x = no of iterations required to see √k

and p = 1/k

Expected number of times we need to roll for √k : E[X] = 1/p

= 1/(1/k) = k

We ned to roll die k times to expect to see √k.

Q3.b)

Probability to roll any face = 1/k (unbiased die)

Probability to roll unique i^{th} face = [k-(i-1)]/k

Let variable X_i represent geometric variable for number of times we need to roll die for seeing i^{th} unique number on die

PMF(X_i) = (1-p)^(x_i-1) * p where x_i = no of iterations required to see √k

And p = [k-(i-1)]/k

Expected number of times we need to roll for i^{th} unique number : E[X] = 1/p

= 1/([k-(i-1)]/k)

= k/(k+1-i)

Total Expected number of times we need to roll die to see all numbers = E[X_1]+ E[X_2]….+E[X_k]

= $\sum_{i=1}^{k} \frac{k}{k+1-i}$

= k * log(k)

Q3.c)

Let variable X_i represent geometric variable for number of times we need to roll die for seeing i^{th} unique face on die

For X_1, $p = ¼ + ¼ + ½ = 1$

Hence, $E[X_1] = 1$ (E[X]=1/p)

i.e. It will take only 1 roll to get our first unique face.

For X_2, cases:

- 1^{st} unique number was 1 or 3 :
  p for $X_{2\ case1} = 1 - ¼ = ¾$
  Hence, $E[X_{2\ case1}] = 4/3$ (E[X]=1/p)

  For X_3, cases again:
  - 2^{nd} unique number was 1 or 3 :
    p for $X_{3\ case1} = 1 - ¼ - ¼ = ½$
    Hence, $E[X_{3\ case1}] = 2$ (E[X]=1/p)
  - 2^{nd} unique number was 2 :
    p for $X_{3\ case2} = 1 - ¼ - ½ = ¼$
    Hence, $E[X_{3\ case2}] = 4$ (E[X]=1/p)

- 1^{st} unique number was 2 :
  p for $X_{2\ case2} = 1 - ½ = ½$
  Hence, $E[X_{2\ case2}] = 2$ (E[X]=1/p)

  2^{nd} unique number was 1 or 3 :
  p for $X_{3\ case3} = 1 - ½ - ¼ = ¼$
  Hence, $E[X_{3\ case3}] = 4$ (E[X]=1/p)

$E[X_2] = P(X_{2\ case1})*E[X_{2\ case1}] + P(X_{2\ case2})*E[X_{2\ case2}]$

$= P(1^{st}\text{ draw} = 1\text{ or }3)*E[X_{2\ case1}] + P(1^{st}\text{ draw} = 2)*E[X_{2\ case2}]$

$= ½ * 4/3 + ½ * 2$

$= 5/3$

$E[X_3] = P(X_{2\ case1})*(P(X_{3\ case1})*E[X_{3\ case1}] + P(X_{3\ case2})*E[X_{3\ case2}]) + P(X_{2\ case2})*P(X_{3\ case3})*E[X_{3\ case3}]$

$= ½ * (¼ /(¼ + ½)*2 + ½ /(¼ + ½)*4) + ½ * 1 * 4$

$= ½ * 10/3 + 2$

$= 11/3$

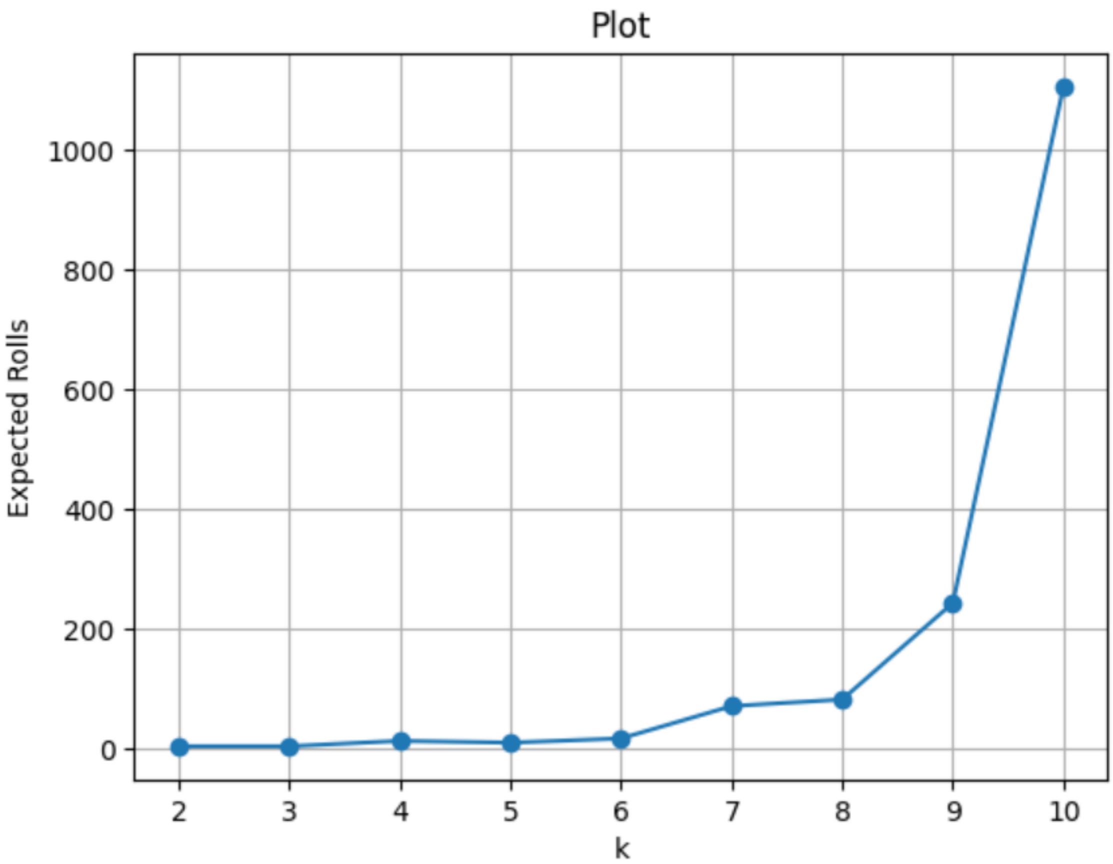
Total expected rolls required = $E[X_1]+E[X_2]+ E[X_3]$

$= 1+5/3+11/3$

$= 19/3$

Q3.d)

We got the following graph for number of rolls required for increasing k :



This plot matches with the total expected rolls values found in (c) part as the number of rolls required are nearly 5-6.