Assignment 04

1. $F(s) = \dfrac{s-1}{(s+1)(s^2+2s+5)} = \dfrac{s-1}{(s+1+2j)(s+1-2j)(s+1)}$

$F(s) = \dfrac{K_1}{s+1} + \dfrac{K_2}{s+1+2j} + \dfrac{K_2^*}{s+1-2j}$

$K_1 = (s+1)F(s)\Big|_{s=-1} = \dfrac{\cancel{(s+1)}(s-1)}{\cancel{(s+1)}(s+1+2j)(s+1-j2)}\Big|_{s=-1}$

$K_1 = \dfrac{-1-1}{(\cancel{-1}+\cancel{1}+2j)(\cancel{-1}+\cancel{1}-2j)} = -\dfrac{2}{4} = -\dfrac{1}{2}$; $\boxed{K_1 = -\frac{1}{2}}$

$K_2 = (s+1+2j)F(s)\Big|_{s=-1-2j} = \dfrac{\cancel{(s+1+j2)}(s-1)}{(s+1)\cancel{(s+1+2j)}(s+1-j2)}\Big|_{s=-1-j2}$

$= \dfrac{(-1-j2-1)}{(-1-j2+1)(-1-j2+1-j2)} = \dfrac{-2-j2}{(-j2)(-j4)} = \dfrac{-2(1+j)}{-8} = \dfrac{1}{4}+\dfrac{j}{4}$

$\boxed{K_2 = \frac{1}{4}+\frac{j}{4}}$; $\boxed{K_2^* = \frac{1}{4}-\frac{j}{4}}$

$F(s) = -\dfrac{1}{2}\dfrac{1}{s+1} + \dfrac{\left(\frac{1}{4}+\frac{j}{4}\right)}{s+1+j2} + \dfrac{\frac{1}{4}-\frac{j}{4}}{s+1-j2}$

$f(t) = -\dfrac{1}{2}e^{-t} + \left(\dfrac{1}{4}+\dfrac{j}{4}\right)e^{-(1+j2)t} + \left(\dfrac{1}{4}-\dfrac{j}{4}\right)e^{-(1-j2)t}$

$= -\dfrac{1}{2}e^{-t} + \dfrac{1}{4}e^{-t}e^{-j2t} + \dfrac{j}{4}e^{-t}e^{-j2t} + \dfrac{1}{4}e^{-t}e^{j2t} - \dfrac{j}{4}e^{-t}e^{j2t}$

$= -\dfrac{1}{2}e^{-t} + \dfrac{1}{4}e^{-t}e^{j2t} + \dfrac{1}{4}e^{-t}e^{-j2t} + \dfrac{j}{4}e^{-t}\left(e^{-j2t}-e^{j2t}\right)$

$= -\dfrac{1}{2}e^{-t} + \dfrac{1}{4}e^{-t}\left(e^{j2t}+e^{-j2t}\right) - \dfrac{j}{4}e^{-t}\left(e^{j2t}-e^{-j2t}\right)$

$= -\dfrac{1}{2}e^{-t} + \dfrac{1}{2}e^{-t}\left(\dfrac{e^{j2t}+e^{-j2t}}{2}\right) + \dfrac{1}{2}e^{-t}\left(\dfrac{e^{j2t}-e^{-j2t}}{2j}\right)$

$= -\dfrac{1}{2}e^{-t} + \dfrac{1}{2}e^{-t}\cos 2t + \dfrac{1}{2}e^{-t}\sin 2t$

$\boxed{f(t) = \frac{1}{2}e^{-t}\left[-1+\cos 2t+\sin 2t\right]}$

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2. $\frac{d^2y(t)}{dt^2} + \frac{5\,dy(t)}{dt} + 6y(t) = u(t)$

Taking Laplace Transform on both sides of the differential equation

$$s^2Y(s) - s\,y(\bar{0}) - \frac{dy(\bar{0})}{dt} + 5[sY(s) - y(\bar{0})] + 6Y(s) = \frac{1}{s}$$

or, $[s^2 + 5s + 6]Y(s) - 2 - 5 - s = \frac{1}{s}$

or, $[s^2 + 5s + 6]Y(s) = \frac{1}{s} + s + 2 + 5$

$$\text{or, } Y(s) = \underbrace{\frac{1}{s(s^2+5s+6)}}_{\substack{\text{Response due} \\ \text{to input} \\ \text{(forced Response)}}} + \underbrace{\frac{s+2+5}{s^2+5s+6}}_{\substack{\text{Response due to} \\ \text{initial} \\ \text{condition} \\ \text{(Free response)}}}$$

$Y(s) = Y_1(s) + Y_2(s)$

Now $Y_1(s) = \frac{1}{s(s^2+5s+6)} = \frac{1}{s(s+2)(s+3)} = \frac{K_0}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$

or, $Y_1(s) = \frac{1}{6}\frac{1}{s} - \frac{1}{2}\frac{1}{(s+2)} + \frac{1}{3(s+3)}$

$y_1(t) = \frac{1}{6}u(t) - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}$

or, Forced Response is

$y_1(t) = \left[\frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t}\right]u(t)$ for $t \geq 0$

Now $Y_2(s) = \frac{s+7}{(s+2)(s+3)} = \frac{K_2'}{s+2} + \frac{K_3'}{s+3}$

$Y_2(s) = \frac{5}{s+2} - \frac{4}{s+3}$; $y_2(t) = (5e^{-2t} - 4e^{-3t})u(t)$ for $t \geq 0$

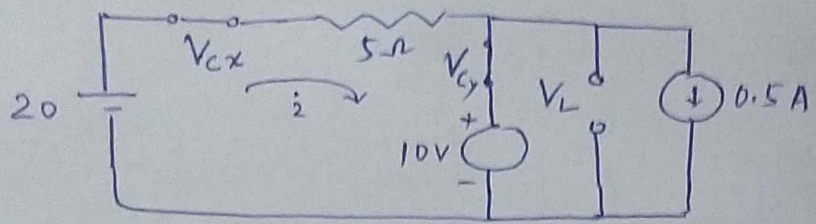
$\boxed{y_2(t) = 5e^{-2t} - 4e^{-3t}}$

Free response of the system $y_2(t) = (5e^{-2t} - 4e^{-3t})u(t)$ for $t \geq 0$

Total response of the system = Forced Response + Free response

$y(t) = \left(\frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t} + 5e^{-2t} - 4e^{-3t}\right)u(t)$ for $t \geq 0$

4. at $t = 0^+$, the circuit becomes



$V_{cx} = 0$

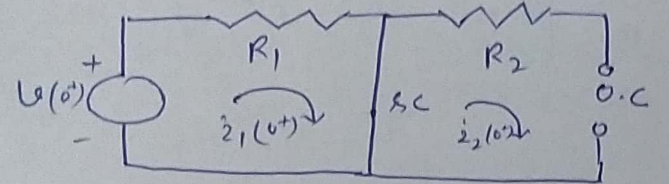
$V_S = 5 \times 2 = 10$ Volt

$V_L = 10$ Volt

$i_L = 0.5$ Amp

$$i = \frac{20-10}{5} = 2 \text{ Amp}$$

5. at $t = 0^+$, the circuit becomes



$i_1(0^+) = \frac{v(0^+)}{R_1}$; $i_2(0^+) = 0$

$$\frac{di_2(0^+)}{dt} = 0$$

<u>at time t</u> the equilibrium equation

$$v(t) = R_1 i_1(t) + \frac{1}{C}\int [i_1(t) - i_2(t)].dt \quad \text{---(i)}$$

$$R_2 i_2(t) + L\frac{di_2(t)}{dt} + \frac{1}{C}\int [i_2(t) - i_1(t)].dt = 0 \quad \text{---(ii)}$$

, at $t = 0^+$, the voltage across capacitor is zero

i.e. $\frac{1}{C}\int_{0^-}^{0^+} (i_2 - i_1).dt = 0$ and $i_2(0^+) = 0$

$R\, i_2(0^+) = 0$

Hence $L.\frac{di_2(0)}{dt} = 0$ or, $\boxed{\frac{di_2(0^+)}{dt} = 0}$

Differentiating equation (i)

$$R_1\frac{di_1(t)}{dt} + \frac{1}{C}[i_1(t) - i_2(t)] = \frac{dv(t)}{dt} \quad \text{-----(iii)}$$

or, $R_1\frac{di_1(0)}{dt} + \frac{1}{C}[i_1(0^+) - i_2(0^+)] = \frac{dv(0^+)}{dt}$

or, $R_1\frac{di_1(0^+)}{dt} = \frac{dv(0^+)}{dt} - \frac{1}{C}i(0^+)$

or, $\boxed{\frac{di_1(0^+)}{dt} = \frac{1}{R_1}\left[\frac{dv(0^+)}{dt} - \frac{v(0^+)}{R_1C}\right]}$ ---(iv)

Differentiate equation (iii)

$$R_1 \frac{d^2 i_1(t)}{dt^2} + \frac{1}{C}\left[\frac{di_1(t)}{dt} - \frac{di_2(t)}{dt}\right] = \frac{d^2 v(t)}{dt^2}$$

at $t = 0^+$

$$R_1 \frac{d^2 i_1(0^+)}{dt^2} + \frac{1}{C}\left[\frac{di_1(0^+)}{dt} - \frac{di_2(0)}{dt}\right] = \frac{d^2 v(0^+)}{dt^2}$$

or, $R_1 \frac{d^2 i_1(0^+)}{dt^2} + \frac{1}{C}\left[\frac{1}{R_1}\left\{\frac{dv(0^+)}{dt} - \frac{v(0^+)}{R_1 C}\right\}\right] = \frac{d^2 v(0^+)}{dt^2}$ (since $\frac{di_2(0)}{dt} = 0$)

or, $$\boxed{\frac{d^2 i_1(0^+)}{dt^2} = \frac{1}{R_1}\frac{d^2 v(0^+)}{dt^2} - \frac{1}{R_1 C}\left[\frac{1}{R_1}\frac{dv(0)}{dt} - \frac{v(0^+)}{R_1^2 C}\right]}$$

Differentiating equation (ii)

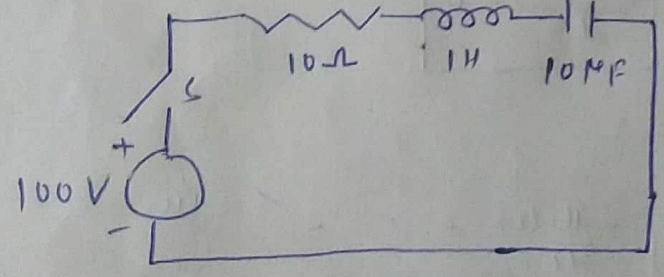
$$R_2 \frac{di_2(t)}{dt^2} + L\frac{d^2 i_2(t)}{dt^2} + \frac{i_2(t) - i_1(t)}{C} = 0$$

$$L\frac{d^2 i_2(t)}{dt} = \frac{i_1(t) - i_2(t)}{C} - R_2\frac{di_2(t)}{dt}$$

at $t = 0^+$ $L\frac{d^2 i_2(0)}{dt^2} = \frac{i_1(0)}{C} - 0 = \frac{v(0)}{R_1 C}$

or, $$\boxed{\frac{d^2 i_2(0^+)}{dt^2} = \frac{v(0^+)}{LCR_1}}$$

6.



at time t when switch is closed

$$100 = 10\, i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \quad \text{---(i)}$$

at $t = 0^+$, the inductor behaves as open circuit and capacitor behaves as short circuit

$$i(0^+) = 0$$

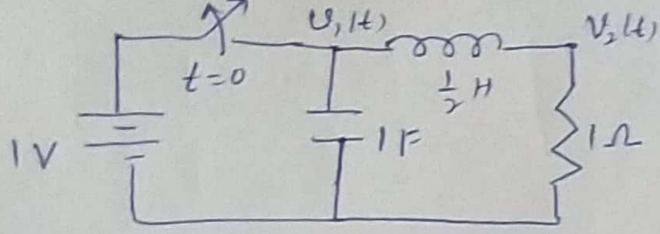
or, $100 = 0 + L\frac{di(0^+)}{dt} + 0$ or, $\frac{di(0^+)}{dt} = \frac{100}{L} = 100$ amp/sec

Differentiating equation (i)

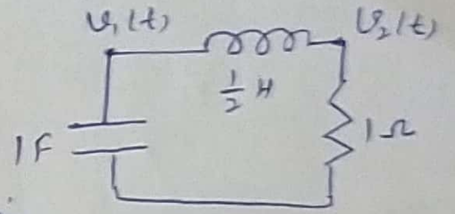
$$0 = R\frac{di_1(t)}{dt} + L\frac{d^2 i(t)}{dt^2} + \frac{1}{C} i(t) = 0$$

or, $L\frac{d^2 i_2(t)}{dt^2} = -R\frac{di_1(t)}{dt}$ or, $\frac{d^2 i_2(0)}{dt^2} = -\frac{R}{L}\cdot\frac{di_1(0)}{dt}$

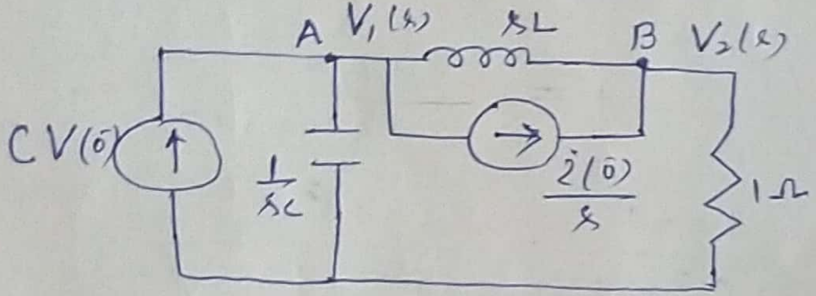
$$\boxed{\frac{d^2 i_2(0)}{dt^2} = -\frac{10}{1}\times 100 = -1000 \text{ A/sec}^2}$$

7. 

at $t=0$, the switch is opened, the circuit becomes



The s-domain circuit is



$C = 1F$, $L = \frac{1}{2}H$

$i(\bar{0}) = 1$ Amp

$v(\bar{0}) = 1$ Volt

<u>Apply KCL at node A</u>

$$CV(\bar{0}) = \frac{V_1(s)}{1/sC} + \frac{V_1(s) - V_2(s)}{sL} + \frac{i(\bar{0})}{s}$$

$$1 = sV_1(s) + \frac{2V_1(s) - 2V_2(s)}{s} + \frac{1}{s}$$

or, $\boxed{2V_2(s) = s^2 V_1(s) + 2V_1(s) + 1 - s}$ ------ (i)

<u>Apply KCL at node B</u>

$$\frac{V_2(s)}{1} = \frac{V_1(s) - V_2(s)}{sL} + \frac{i(\bar{0})}{s}$$

or, $V_2(s) = \frac{2V_1(s) - 2V_2(s)}{s} + \frac{1}{s}$

or, $sV_2(s) = 2V_1(s) - 2V_2(s) + 1$

or, $\boxed{V_2(s) = \frac{2V_1(s) + 1}{s+2}}$ ------ (ii)

From equation (i) & (ii)

$$\frac{4V_1(s) + 2}{s+2} = s^2 V_1(s) + 2V_1(s) + 1 - s$$

or, $4V_1(s) + 2 = s^2(s+2)V_1(s) + 2(s+2)V_1(s) + s + 2 - s^2 - 2s$

$= s^3 V_1(s) + 2s^2 V_1(s) + 2sV_1(s) + \cancel{4V_1(s)} - s + \cancel{2} - s^2$

or, $\cancel{4V_1(s)} + \cancel{2} + s^3 V_1(s) + 2s^2 V_1(s) + 2sV_1(s)$

or, $s(s+1) = s^3 V_1(s) + 2s^2 V_1(s) + 2sV_1(s)$

or, $s+1 = [s^2 + 2s + 2]V_1(s)$

or, $\boxed{V_1(s) = \frac{s+1}{s^2+2s+2}}$ ----(iii)

$$V_2(s) = \frac{2V_1(s)+1}{s+2}$$

or, $(s+2)V_2(s) = 2.V_1(s)+1$

or, $(s+2)V_2(s) = \frac{2(s+1)}{s^2+2s+2}+1 = \frac{2s+2+s^2+2s+2}{s^2+2s+2}$

or, $V_2(s) = \frac{s^2+4s+4}{(s+2)(s^2+2s+2)} = \frac{(s+2)^{\cancel{2}}}{(\cancel{s+2})(s^2+2s+2)}$

or, $\boxed{V_2(s) = \frac{s+2}{s^2+2s+2}}$ ----(iv)

From equation (iii)

$$V_1(s) = \frac{s+1}{s^2+2s+2} = \frac{s+1}{(s+1)^2+1}$$

or, $\boxed{v_1(t) = e^{-t}\cos t}$

$$V_2(s) = \frac{s+2}{s^2+2s+2} = \frac{s+1}{s^2+2s+2} + \frac{1}{s^2+2s+2}$$

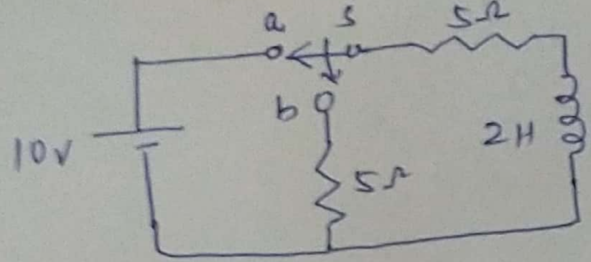
or, $V_2(s) = \frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}$

$v_2(t) = e^{-t}\cos t + e^{-t}\sin t$

Ans: $v_1(t) = e^{-t}\cos t . u(t)$ for $t \geq 0$

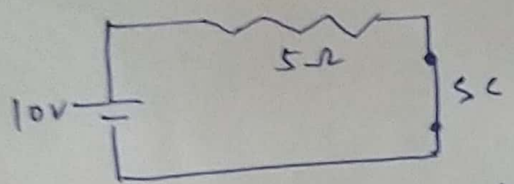
$v_2(t) = [e^{-t}\cos t + e^{-t}\sin t]u(t)$ for $t \geq 0$

(8)



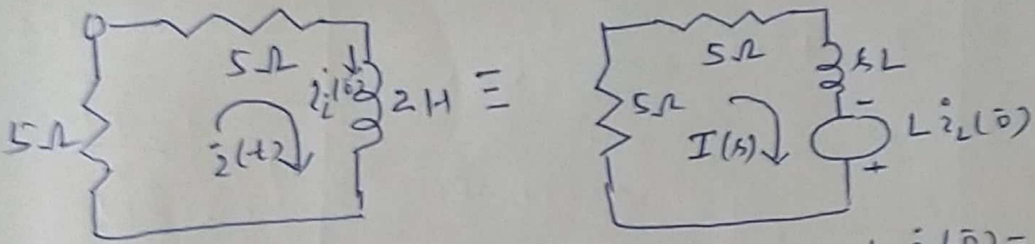
The switch S is at position a for a very long time. Determine $i(t)$ when the switch is moved at position b at $t=0$

When switch is at position a for very long time



$$i_L(0^-) = \frac{10}{5} = 2 \text{ amp}$$

When the switch is at position b



$$-10 I(s) - sL I(s) + L i(0^-) = 0 \quad \text{or, } L i(0^-) = (10 + sL) I(s)$$

$$\text{or, } I(s) = \frac{L i(0^-)}{sL+10} = \frac{2\times 2}{2s+10} = \frac{4}{2(s+5)} =$$

$$\text{or, } \boxed{i(t) = 2e^{-5t} u(t)} \text{ for } t \geq 0$$

Alternate method

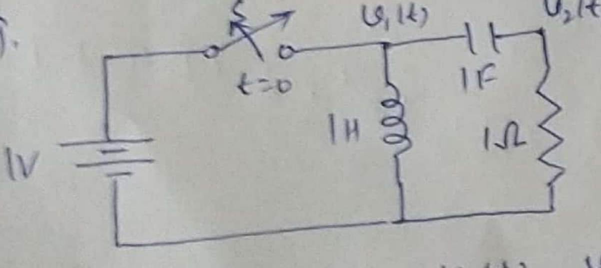
$$10 i(t) + L \cdot \frac{di(t)}{dt} = 0$$

$$\text{or, } 10 I(s) + sL I(s) - L i(0^-) = 0$$

$$\text{or, } I(s) = \frac{L i(0^-)}{sL+10} = \frac{4}{2(s+5)}$$

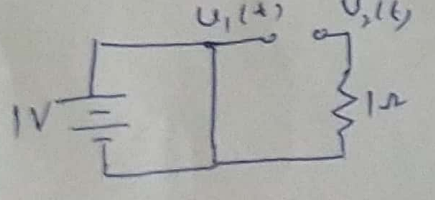
$$\text{or, } i(t) = 2e^{-5t} u(t) \text{ for } t \geq 0$$

9.

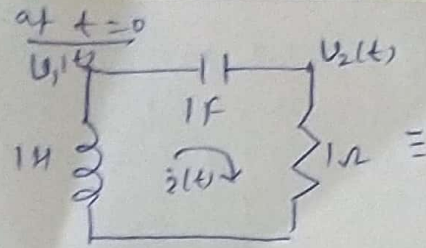


The switch is opened at $t=0$. Assume that prior to the opening of the switch, the circuit had been in steady state, find the node voltages $v_1(t)$ and $v_2(t)$

at $t=\infty$



at $t=0$



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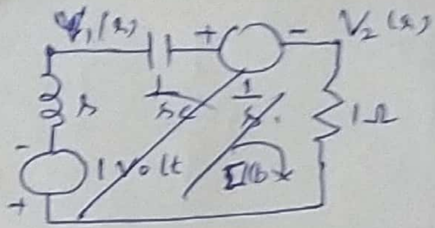
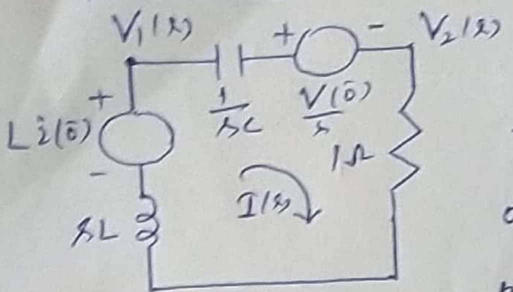




$$-sLI(s)+Li(\bar{0})-\frac{1}{sC}I(s)-\frac{V(\bar{0})}{s}-I(s)=0$$

or, $-sI(s)-\frac{1}{s}I(s)-I(s)+1-\frac{1}{s}=0$

or, $\left(s+\frac{1}{s}+1\right)I(s)=\frac{s-1}{s}$

or, $I(s)\left(\frac{s^2+s+1}{s}\right)=\frac{s-1}{s}$

or, $I(s)=\frac{s-1}{s^2+s+1}$

$$=\frac{s-1}{\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(s-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)}$$

$s^2+s+1=0$

$s=\frac{-1\pm\sqrt{1-4}}{2}$

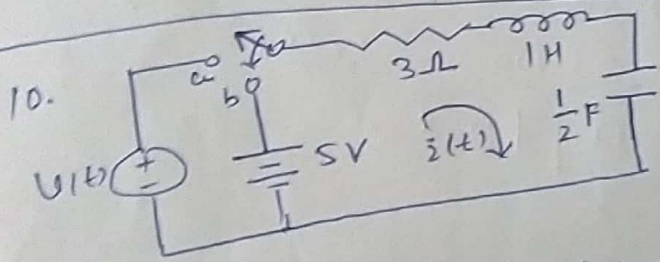
$=\frac{-1\pm j\sqrt{3}}{2}$

$=\frac{-1+j\sqrt{3}}{2}$

$\frac{-1-j\sqrt{3}}{2}$

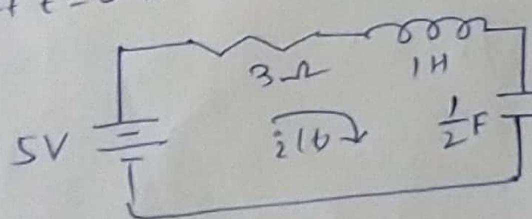
$$I_s(s)=\frac{K_1}{s+\frac{1}{2}+j\frac{\sqrt{3}}{2}}+\frac{K_1^*}{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}}$$

10.



The switch is thrown from position a to b at $t=0$. Prior to the opening of switch the circuit had been in steady state find the current $i(t)$. $V_c(\bar{0})=2V$, $i(\bar{0})=2A$

at $t=0$ when switch is at position b



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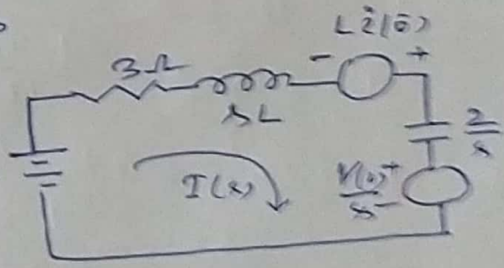

or, $\frac{5}{s}=\left(3+s+\frac{2}{s}\right)I(s)-2+\frac{2}{s}$

or, $I(s)=\frac{2s+3}{(s+1)(s+2)}=\frac{K_1}{s+1}+\frac{K_2}{s+2}$

or, $I(s)=\frac{K_1}{s+1}+\frac{K_2}{s+2}$

or, $i(t)=\left(1e^{-t}+e^{-2t}\right)u(t)$ for $t\geq 0$