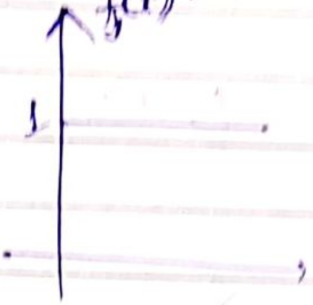
# Waveform synthesis

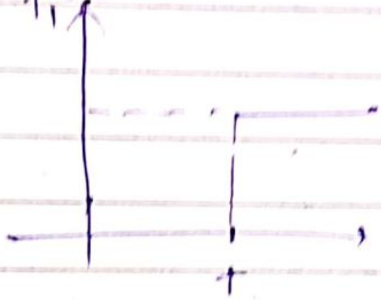
1) $f(t) = u(t)$




$$f(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

2) delayed unit

$f_1(t) = u(t-T)$



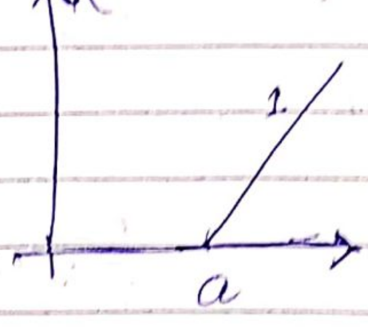

$$f_1(t) = \begin{cases} 1 & \text{for } t \geq T \\ 0 & t < T \end{cases}$$

3) $f_3(t) = r(t)$

$$f_3(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

delayed ramp

4) $r(t-T)$




$$f_4(t) = \begin{cases} (t-a) & t \geq a \\ 0 & t < a \end{cases}$$

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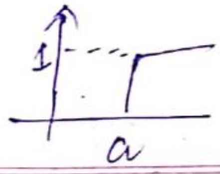
$u(t-a) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

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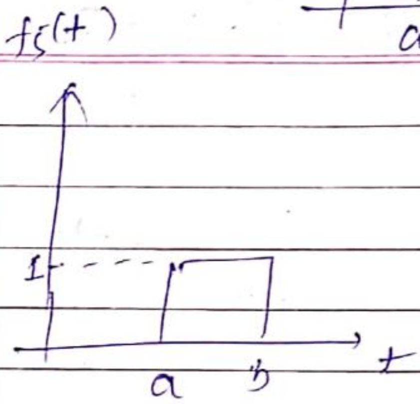
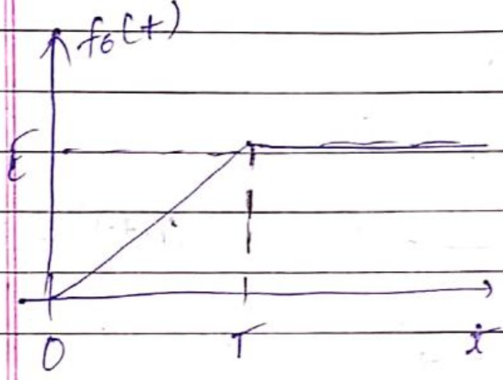
5)



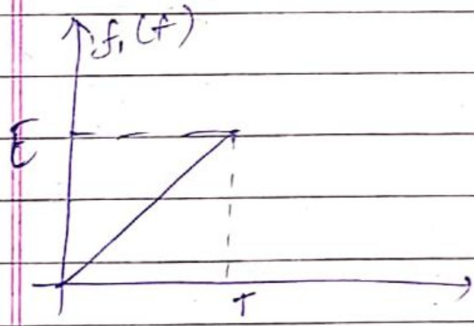

$$f_5(t) = u(t-a) - u(t-b)$$

6)




$$f_6(t) = \frac{E}{T}t \cdot u(t) - \frac{E}{T}(t-T) \cdot u(t-T)$$

7)




$$f_1(t) = \frac{E}{T}t$$

$$= \frac{E}{T}r(t) = \frac{E}{T} \cdot t \cdot u(t)$$

map - theorem

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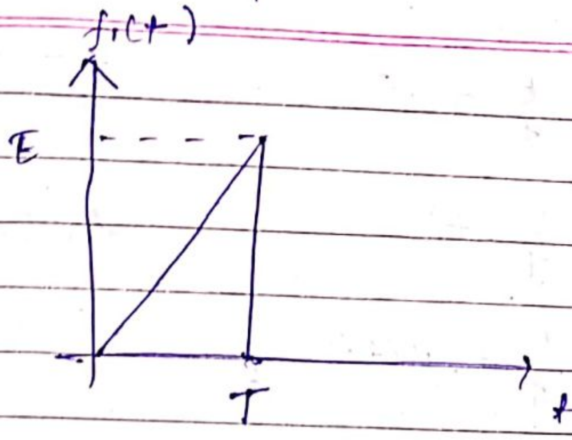
8)




$f_1(t) = \frac{E}{T}\cdot r(t) - \frac{E}{T}\cdot r(t-T) - E\cdot u(t-T)$

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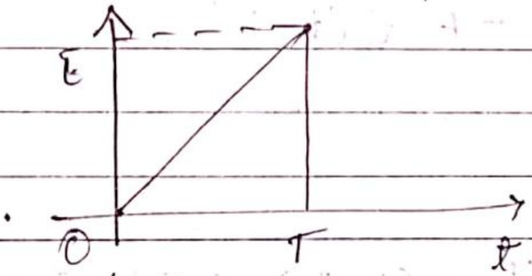
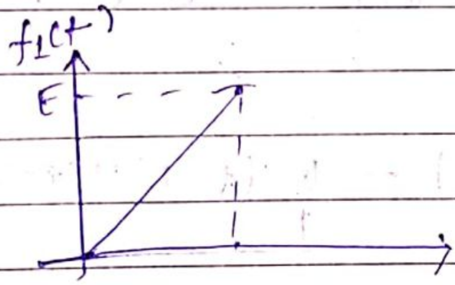
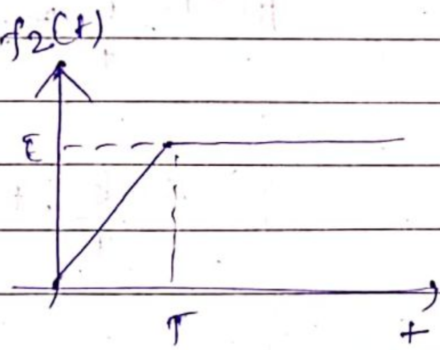
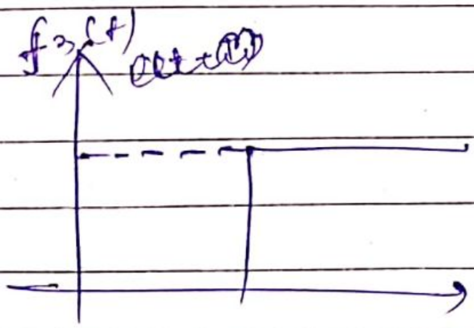
Ques Synthesis the given waveform in terms of unit step $f^{n}$ or ramp function.




Ans




$u(t-T) = E \quad t \geq T$

$= 0 \quad t \leq T$

$f_3(t) = -E\,u(t-T)$

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$$f_1(t) = \frac{E}{T} r(t) \qquad t \geq 0$$

$$= \frac{E}{T} \cdot t \qquad \text{for } t \geq 0$$

$$= \frac{E}{T} t \cdot u(t) \qquad \text{for } t \geq 0$$

$$f_2(t) = \frac{E}{T}(t) - \frac{E}{T} r(t-T)$$

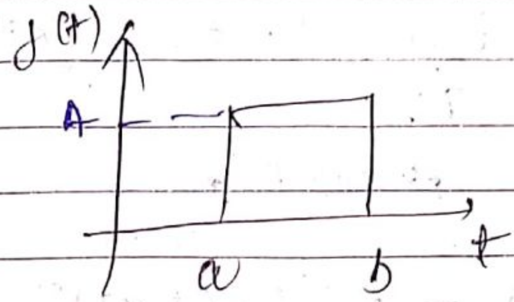
$$= \frac{E}{T} t - \frac{E}{T}(t-T)\, u(t-T)$$

$$f_3(t) = -E\, u(t-T) \qquad t \geq T$$

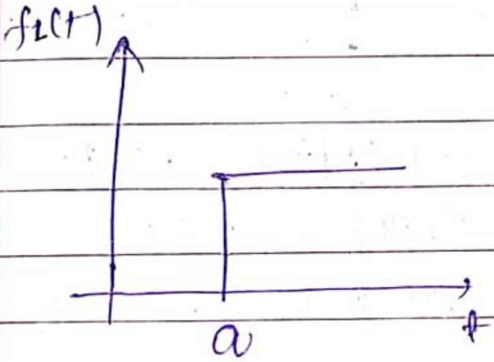
$$f(t) = \frac{E}{T} t - \frac{E}{T} r(t-T) - E\,u(t-T)$$

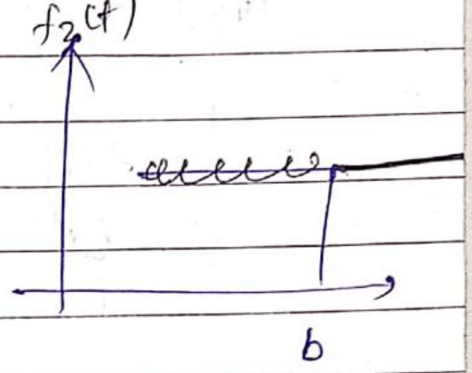
$$= \frac{E}{T} \cdot t\,u(t) - \frac{E}{T}(t-T)\,u(t-T) - E\,u(t-T)$$

**Ques** Synthesise the given pulse in terms of unit step fxn




**Ans**



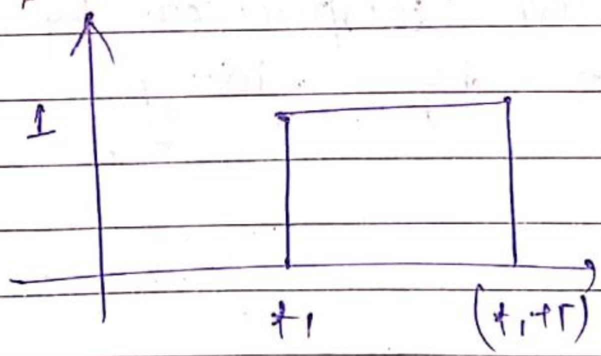




$u(t) = (t-a) \quad t \geq a$
$\qquad\quad 0 \quad t \leq a$

$u(t) = (t-b) \quad t \geq b$
$\qquad\quad 0 \quad t \leq 0$

$f(t) = A[u(t-a) - u(t-b)]$

Gate function :- $G_{t_1}(T)$



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gate is a rectangular pulse of unit height which starts at $t=t_1$ and last for a period $T$.

~~It is expressed as~~

$$g_{t_1}(t) = u(t-t_1) - u[t-(t_1+T)]$$

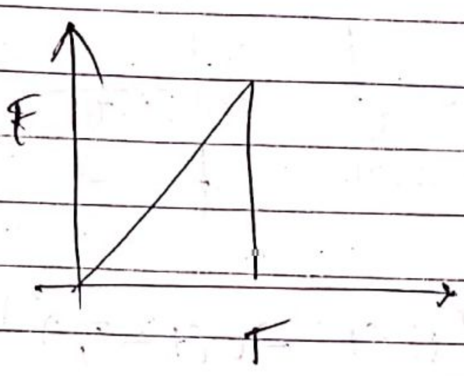
Any $fx^n$ multiplied with gate $fx^n$ could have none zero values in gate interval

$$t_1 < t < t_1+T$$

the value of $fx^n$ within the gate $fx^n$ would be uneffected.

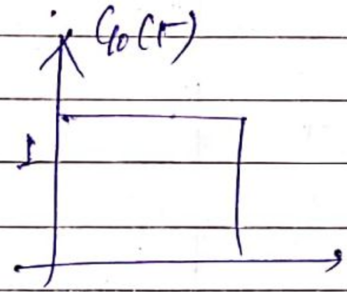
It is very useful in erasing the unwanted portions of a continuous $fx^n$ without graphical composition by shifting and by adding or substrating

**Ques** Squthesize the waveform




**Ans.** $f(t) = \frac{E}{T} r(t)\, G_0(T)$

$= \frac{E}{T} t\left[u(t-0) - u(t-T)\right]$




$= \frac{E}{T} t\, u(t) - \frac{E}{T}\left[(t-T+T)\, u(t-T)\right]$

$$f(t) = \frac{E}{T} t\, u(t) - \frac{E}{T}(t-T)\, u(t-T) - E\, u(t-T)$$

$$\boxed{F(s) = \frac{E}{T} \cdot \frac{1}{s^2} - \frac{E}{Ts^2} e^{-Ts} - \frac{E}{s} e^{-Ts}}$$

Laplace transform

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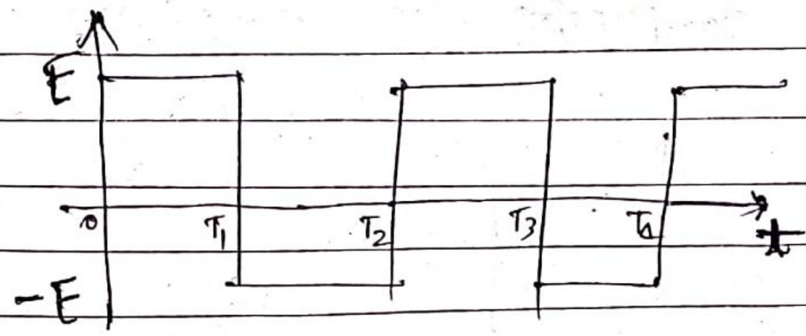
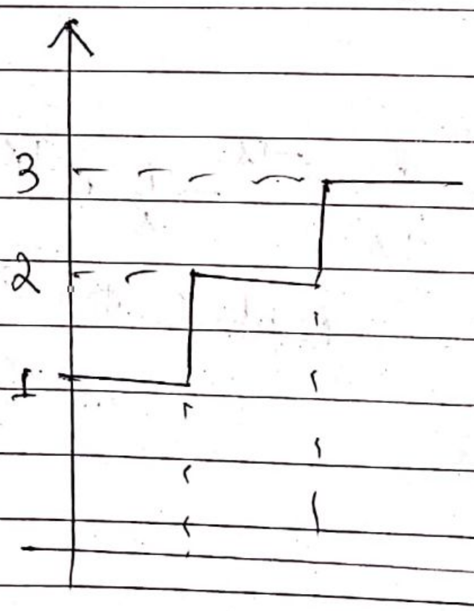
Ques




Ans

$$f(t) = E\left[u(t) - u(t-T_1)\right]$$
$$- E\left[u(t-T_1) - u(t-T_2)\right]$$
$$+ E\left[u(t-T_2) - u(t-T_3)\right]$$
$$- E\left[u(t-T_3) - u(t-T_4)\right] + \ldots$$

Ques



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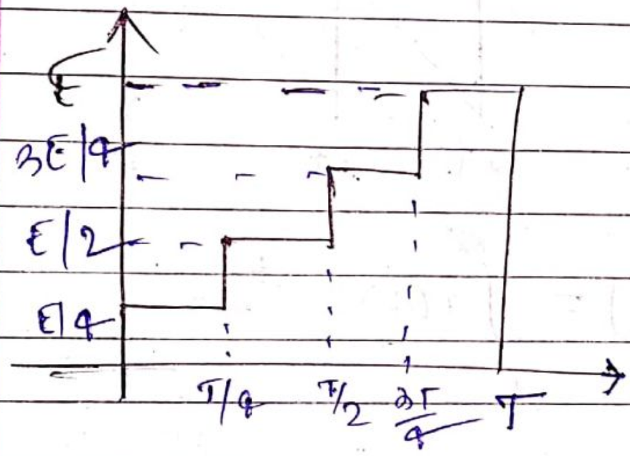
Ans

$$f(t) = u(t-0) - u(t-t_1) + 2[u(t-t_1) - u(t-t_2)] + 3[u(t-t_2)]$$

$$f(t) = u(t) + u(t-t_1) + u(t-t_2)$$

Que




Synthesis the given waveform.

Ans

$$f(t) = \frac{E}{4}[u(t) - u(t-T/4)] + \frac{E}{2}[u(t-T/4) - u(t-T/2)] + \frac{3E}{4}[u(t-T/2) - u(t-\frac{3T}{4})] + E[u(t-\frac{3T}{4}) - u(t-T)]$$

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$$= \frac{E}{4} U(t) + \frac{E}{4} \cdot U\left(t - \frac{T}{4}\right) + \frac{E}{4} U\left(t - \frac{T}{2}\right)$$
$$+ \frac{E}{4} U(t - T)$$

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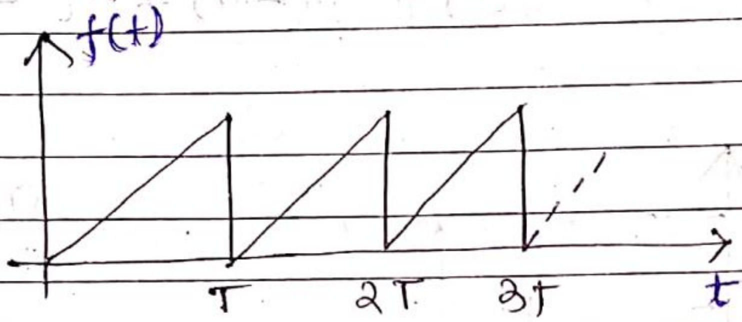
Laplace transform of Periodic signals




$$\mathcal{L}[f(t)] = \int_0^\infty f(t) e^{-st} \cdot dt$$

$$= \int_0^T f(t) e^{-st} \cdot dt + \int_T^{2T} f(t) e^{-st} \cdot dt$$
$$- - - - \int_{nT}^{(n+1)T} f(t) e^{-st} \cdot dt$$

$$= \int_0^T f(t) \cdot e^{-st} + e^{-sT} \int_0^T f(t) \cdot e^{-st} dt + e^{-s(2T)} \int_0^T f(t) e^{-st} dt$$

$$= \left[1 + e^{-sT} + e^{-s2T} - - -\right] \int_0^T f(t)\, e^{-st}\, dt$$

$$= \left[\frac{1}{1 + e^{-sT}}\right] \int_0^T f(t)\, e^{-sT}\, dt$$

$$= \left(\frac{1}{1 - e^{-sT}}\right) F_1(s)$$

$F_1(s)$ = LT of one cycle $f(t)$

If $f(t) \xleftrightarrow{LT} F(s)$

$f(t-T) \xleftrightarrow{LT} e^{-Ts} F(s)$

$$f(t) = f_1(t) + f_2(t) + f_3(t) \, - - -$$

$$f(t) = f_1(t) + f_1(t-T) + f_1(t-2T) \, - - - -$$

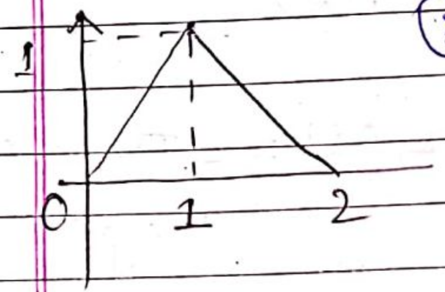
$$\mathcal{L}[f(t)] = F_1(s) + e^{Ts} F_1(s) + e^{-s2T} F_1(s) \, - - -$$

$$= F_1(s)\left[1 + e^{-sT} + e^{-s2T} - - -\right]$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}}$$

Que Write the waveform synthesis of the waveform in terms of either unit step or ramp fxn and find L.T

1)

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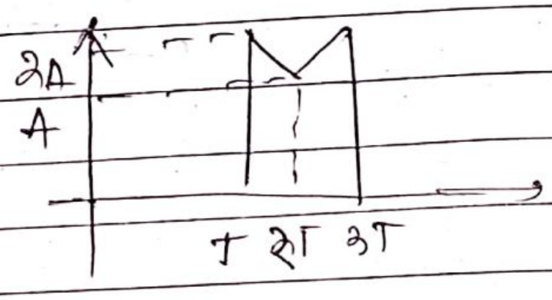


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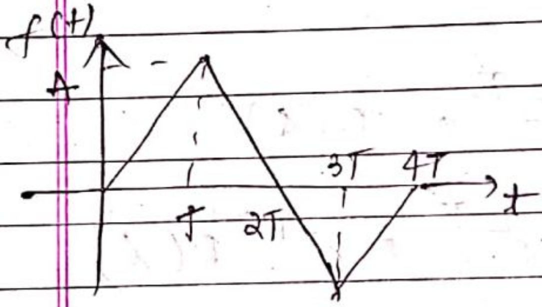


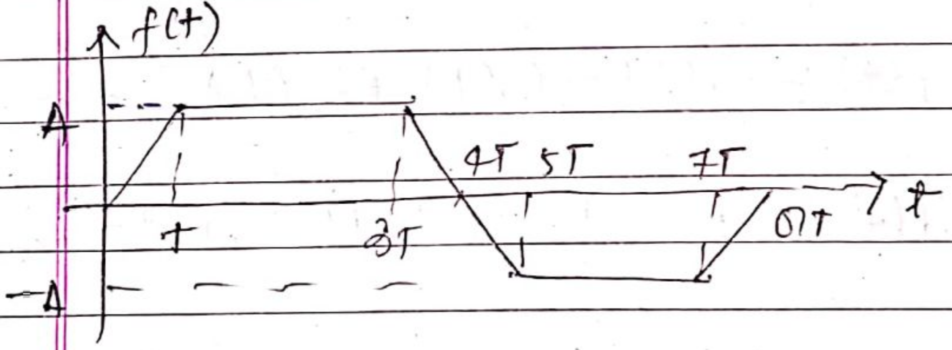


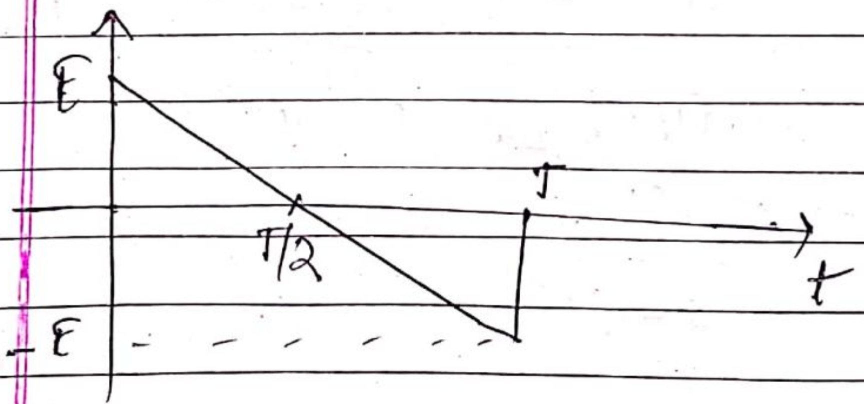

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$$\int_{T}^{2T} f(t-T)\, e^{-st} \cdot dt$$

$t - T = p$

$t = p + T$

$dt = dp$

$t = T \qquad p = 0$

$t = 2T \qquad p = T$

$$\int_{0}^{T} f(p) \cdot e^{-st} \cdot dp$$

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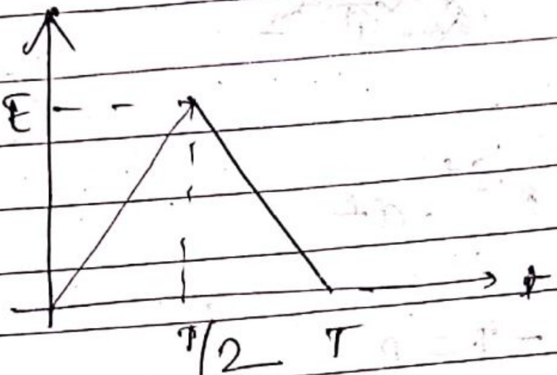
Ans (1)




$$f(t) = \frac{2E}{T} r(t) - \frac{2E}{T} r(t - T/2) - \frac{2E}{T} r(t - T/2) + \frac{2E}{T} r(t - T)$$

$$= \frac{2E}{T} t \cdot u(t) - \frac{4E}{T} (t - T/2)\, u(t - T/2) + \frac{2E}{T} (t - T)\, u(t - T)$$

taking laplace transform on both side above equation

$$\boxed{F(s) = \frac{2E}{T \cdot s^2} - \frac{4E}{Ts^2} e^{-T/2\, s} + \frac{2E}{Ts^2} e^{-Ts}}$$

$\frac{3T}{2}$ $\frac{T}{2}$ Page: $T/2$

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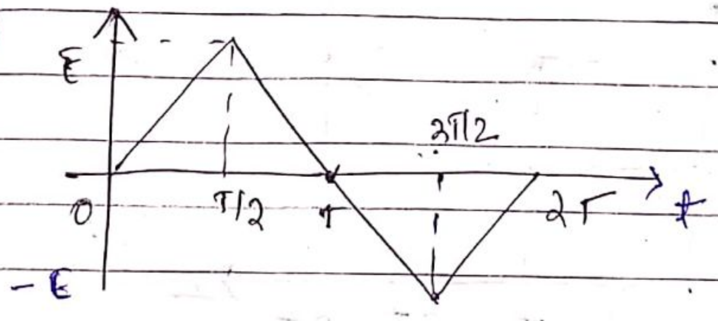
Ans (2)




Ans $f(t) = \frac{2E}{T} r(t) - \frac{2E}{T} r(t - T/2) - \frac{2E}{T} r(t - T/2)$

$+ \left[ -\frac{2E}{T} r\left(t - \frac{3T}{2}\right) \right] + \left[ -\frac{2E}{T} r\left(t - \frac{3T}{2}\right) \right]$

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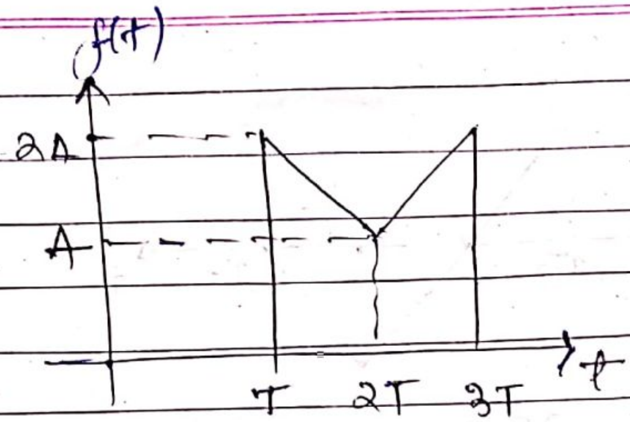
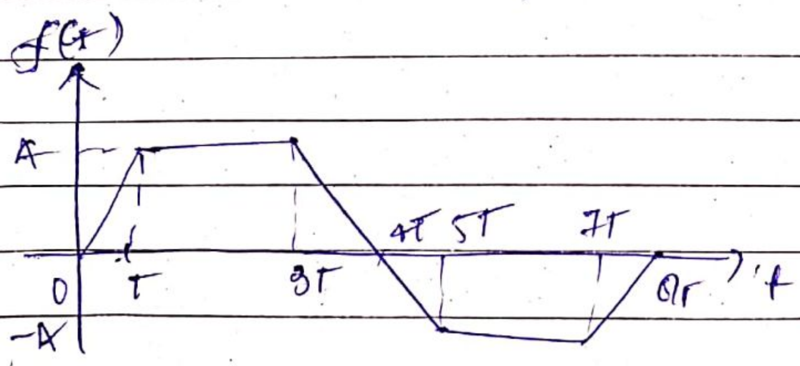
Ans (3)




$$f(t) = 2A\,u(t-T) - \frac{A}{T} r(t-T) + \frac{2A}{T} r(t-2T) - \frac{A}{T} r(t-3T) - 2A\,u(t-3T)$$

$$F(s) = \frac{2A}{s} e^{-sT} - \frac{A}{Ts^2} e^{-Ts} + \frac{2A}{Ts^2} e^{-2Ts} - \frac{A}{Ts^2} e^{-3Ts} - \frac{2A}{s} e^{-3Ts}$$

Ans (2)



$$\int_0^{\infty} 2AU(t-T)e^{-st}.dt = \int_T^{\infty} 2Ae^{-st}dt$$

4) $$f(t) = \frac{A}{T}r(t) - \frac{A}{T}r(t-T) - \frac{A}{T}r(t-3T)$$

$$+ \left[-\frac{A}{T}r(t-5T)\right] + -\frac{A}{T}(t-7T)$$

$$- \left[-\frac{A}{T}r(t-8T)\right]$$

x //

state convulation thm & and prove it.

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Date 21/10/19

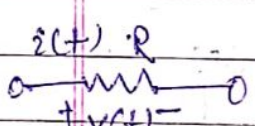
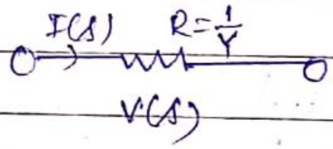
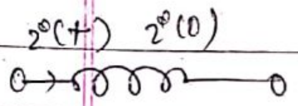
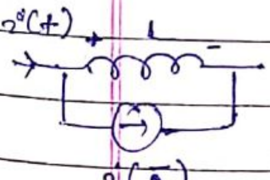
Sf(s)

1) Analysis of RL, RC and RLC network with and without initial conditions with laplace transform

2) Evaluation of initial conditions.

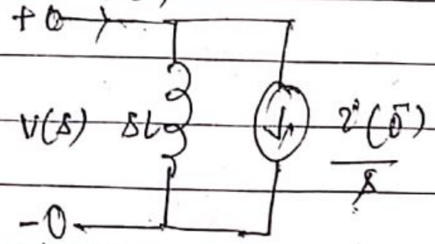
before t=0 ← $i(0^-)$, $\frac{d i(0)}{dt}$, $v(0)$, $\frac{dv(0)}{dt}$, $\frac{d^2 i(0)}{dt}$, $\frac{d^2 v(0)}{dt^2}$

$i(\infty)$, $\frac{d i(\infty)}{dt}$, $v(\infty)$, $\frac{dv(\infty)}{dt}$, $\frac{d^2 v(\infty)}{dt^2}$

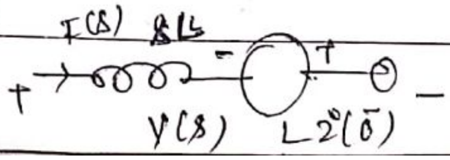
| Element | Time domain | s-domain |
|---|---|---|
|   | $v(t) = i(t) \cdot R$ | $V(s) = I(s) \cdot R$ |
| | $i(t) = \frac{v(t)}{R}$ | $I(s) = \frac{V(s)}{R}$ |
| no. initial condition bcz it dissipate energy | $= v(t) \cdot Y$ | $I(s) = V(s) \cdot Y$ |
| | |   |
|   | $V_L(t) = L \cdot \frac{d i(t)}{dt}$ | $V_L(s) = sL\,I(s) - L i(0^-)$ |
| | $i_L = \frac{1}{L} \cdot \int_{0^-}^{t} V(\tau) \cdot d\tau + i(0^-)$ | |
|   | | $I_L(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}$ |

current (initial)

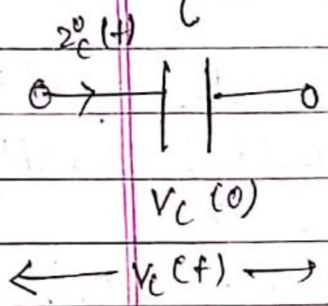
(KCL)




(KVL)



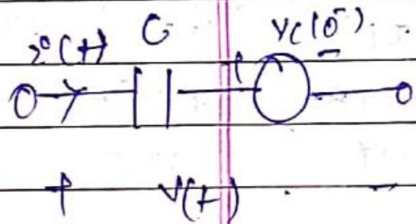
initial condition
charge, voltage

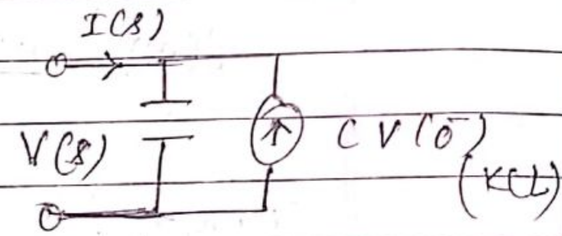


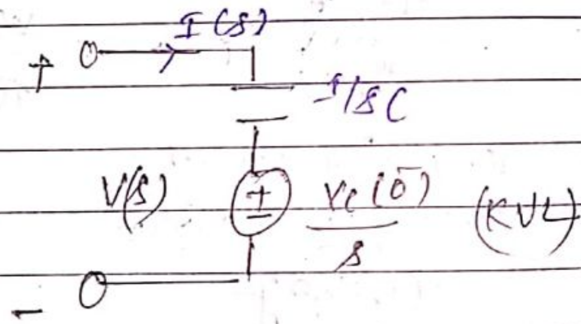

$$i_c(t) = C\frac{dv(t)}{dt} \qquad I(s) = sCV(s) - CV(0^-)$$

$$v(t) = \frac{1}{C}\int_0^t i(t)\,dt + V_c(0^-)$$

$$V(s) = \frac{1}{sC}I(s) + \frac{V_c(0^-)}{s}$$

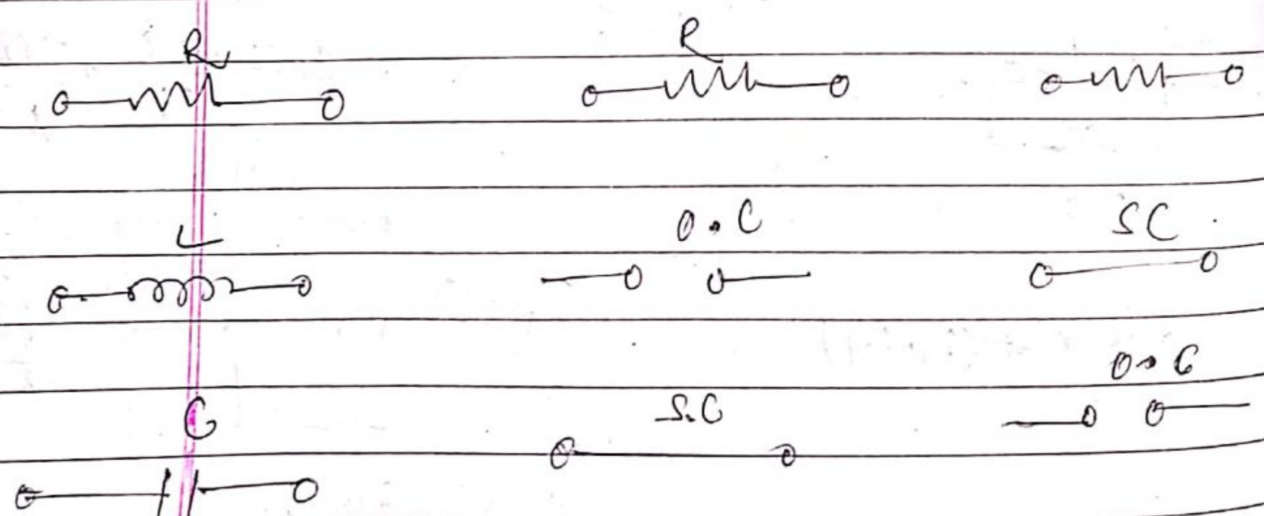


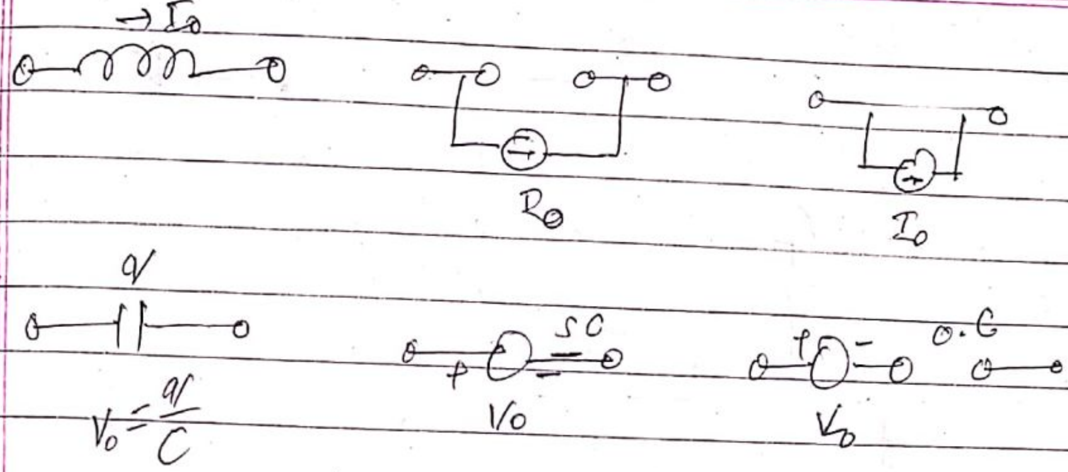

Date 22/10/17

| Element and their initial condn | Equivalent ckt at $t=0^+$ (initial value) | Equivalent ckt at $t=\infty$, $(t=0^-)$ |
|---|---|---|



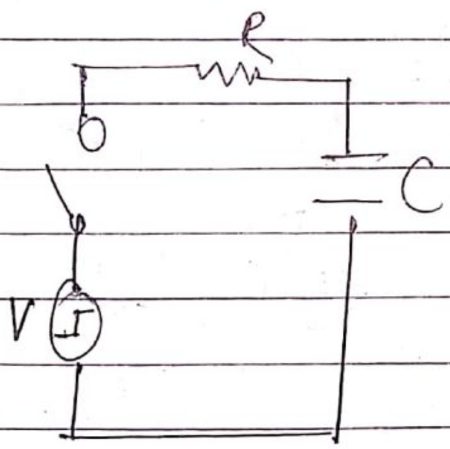
at (t = ∞) switch close very long time

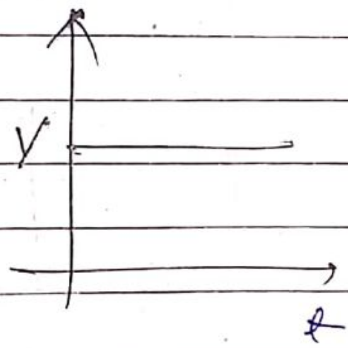





## RC

**Ques** The switch close at $t = 0$ find $i(0^+)$, $\frac{d\,i(0^+)}{dt}$ for RC ckt. There is no initial charge on capacitor the wave form of voltage as shown in figure.



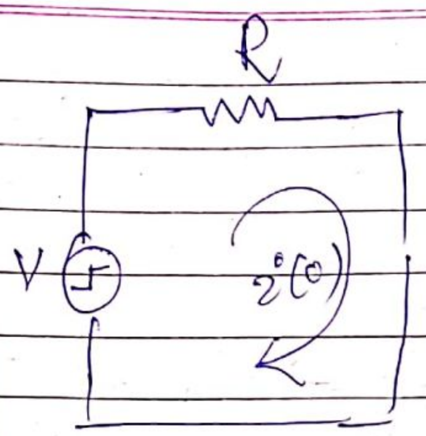

$V = V \cdot u(t)$

Fig 1

$\frac{d\,u(t)}{dt} = \delta(t)$ $\quad u(s) = \frac{1}{s}$

Page :

Date: $s\,(u/s) = 1$




$i(0^+) = \frac{V}{R}$

$V u(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt$

— General

differentiating w.r.t time

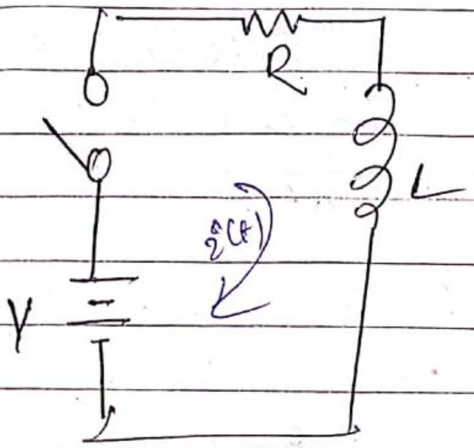
$V\delta(t) = R\frac{di(t)}{dt} + \frac{i(t)}{C}$

$0 = R\cdot\frac{d\,i(0^+)}{dt} + \frac{i(0^+)}{C}$

$-R\cdot\frac{d\,i(0^+)}{dt} = \frac{V}{RC}$

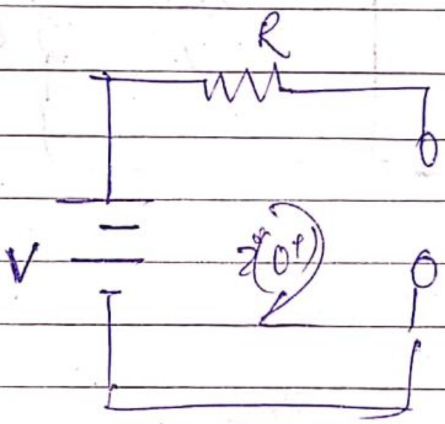
$$\boxed{\frac{d\,i(0^+)}{dt} = \frac{V}{-R^2C}}$$

**Ques**




there is no current in inductor

**Ans**




$i(0^+) = 0$

$$V = R\,i(t) + L\frac{d\,i(t)}{dt}$$

at $t = 0^+$

$$V = \underbrace{R\,i(0^+)}_{0} + L\frac{d\,i(0^+)}{dt}$$

$$\boxed{\frac{d\,i(0^+)}{dt} = \frac{V}{L}}$$

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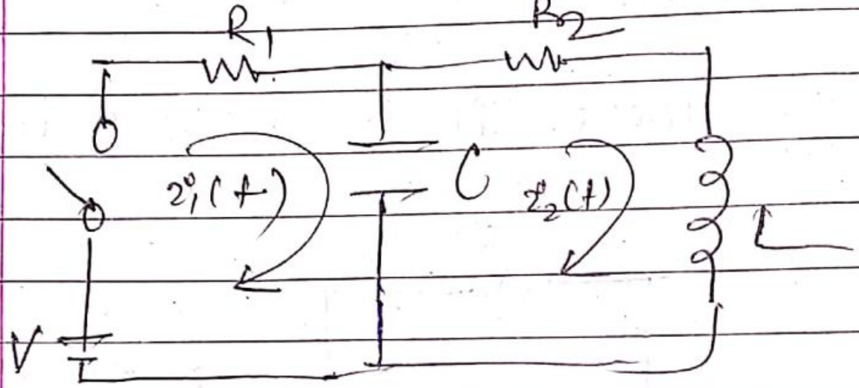
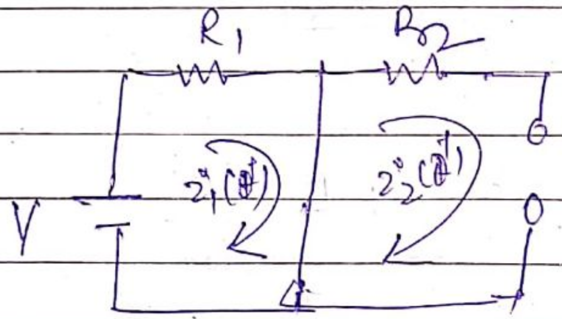
Ans The switch close at $t=0$ $i_1(0^+)$, $i_2(0^+)$

$\frac{d i_1(0^+)}{dt}$, $\frac{d i_2(0^+)}{dt}$

there are no




Ans at $t=0^+$




$$i_1(0^+) = \frac{V}{R_1}$$

$$i_2(0^+) = 0$$

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$$V - R_1 i_1(t) - \frac{1}{C}\int \left(i_1(t) - i_2(t)\right) dt = 0 \quad \text{--- (1)}$$

$$- R_2 i_2(t) - L\frac{d i_2(t)}{dt} - \frac{1}{C}\int \left(i_2(t) - i_1(t)\right) dt = 0 \quad \text{--- (2)}$$

$V - R_1 i(t)$ .