Date 04/sep/19

Page :
Date: / /

UNIT – 3

1) Trigonometric and exponential fourier series
2) Discrete spectra
3) Waveform symmetry
5) Steady state response to periodic non-sinusodial input
6) Power factor, effective values
4) Fourier transform & continuous spectra
7) Unbalanced three phase circuit and power calculation.

Ques, What is fourier series?

The representⁿ of a circuit defined over a certain time interval in terms of linear combinⁿ of orthogonal function is called fourier series.

$$\cancel{f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t}$$

a_0, a_n, b_n are fourier coefficient

It is defined for periodic

$a_0 + a_1 \cos\omega_0 t + a_2 \sin\omega_0 t + \ldots$

dc value fundamental Harmonic

Q= What are the conditions for the existence of fourier series
(dirichlet condit^n)

Ans A fx^n f(t) is fourier series if it satisfied following condition

1) f(t) is single valued i.e it meets mathematical definition of a fx^n.
2) f(t) contains certain discontinuities in each time period.
3) f(t) has certain number of maxima and mini in each time-period.

4) f(t) is absolute integrable over one time-period.

$$\int_{t_0}^{t_0+T} |f(t)|\, dt < \infty$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$\int_0^T f(t)\, dt = \int_0^T a_0\, dt + \sum_{n=1}^{\infty} \int_0^T (a_n \cos n\omega_0 t)\, dt$$

$$\underbrace{}_{0}$$

1 Time P = 2π

Page:

Date: / /

$$+ \int_0^T b_n \sin n\omega_0 t \, dt = 0$$

$$\int_0^T f(t)\, dt = a_0 T$$

$$\boxed{a_0 = \frac{1}{T}\int_0^T f(t)\, dt}$$

Date 05/sep/19

Happy teacher's day

Standard integral identities

1) $\int_0^T \sin m\omega_0 t \, dt = 0$ for all m

2) $\int_0^T \cos m\omega_0 t \, dt = 0$ $\forall$ $m \neq 0$

3) $\int_0^T \cos m\omega_0 t \cdot \cos n\omega_0 t \, dt = 0$ $\forall$ $m \neq n$

4) $\int_0^T \sin m\omega_0 t \cdot \cos n\omega_0 t = 0$ $\forall$ $m \neq n$

Page :
Date: / /

5) $\int_0^T \sin^2 m\omega_0 t = \frac{T}{2} \quad \forall\, m$

6) $\int_0^T \cos^2 n\omega_0 t\, dt = \frac{T}{2} \quad \forall\, n$

Evaluation fourier coeff 'a_n'

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

multiply both sides of above quantity by $\cos m\omega_0 t$

$$\int_0^T f(t) \cos m\omega_0 t = \int_0^T a_0 \cos m\omega_0 t\, dt + \sum_{n=1}^{\infty} a_n \cos m\omega_0 t \cos m\omega_0 t\, dt$$

$$+ \sum_{n=1}^{\infty} b_n \int_0^T \sin m\omega_0 t \cdot \cos m\omega_0 t\, dt$$

$$\boxed{a_n = \frac{2}{T} \int_0^T f(t) \cdot \cos n\omega_0 t \cdot dt}$$

Evaluation fourier coeff 'b_n'

multiply eqn for $f(t)$ by term $\sin n\omega_0 t$ and integrate over one time period.

$$\boxed{b_n = \frac{2}{T}\int_0^T f(t)\cdot \sin n\omega_0 t \, dt}$$

Cosine form of fourier series

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2+b_n^2}\left[\frac{a_n}{\sqrt{a_n^2+b_n^2}}\cdot \cos n\omega_0 t + \frac{b_n}{\sqrt{a_n^2+b_n^2}} \sin n\omega_0 t\right]$$

Put $a_0 = A_0$ $\cos\theta_n = \frac{a_n}{\sqrt{a_n^2+b_n^2}}$ $\sin\theta_n = \frac{-b_n}{\sqrt{a_n^2+b_n^2}}$

$$\theta_n = \tan^{-1}\frac{b_n}{a_n}$$

Page :
Date: / /

$$\boxed{f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \theta_n)}$$

put $a_0 = A_0$ $\sin\theta_n = \frac{a_n}{\sqrt{a_n^2 + b_n^2}}$

$$\cos\theta_n = \frac{-b_n}{\sqrt{a_n^2 + b_n^2}}$$

$$\phi_n = \tan^{-1}\frac{a_n}{b_n}$$

$$\boxed{f(t) = A_0 + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t + \phi_n)}$$

The above eq^n is called cosine eq^n of f(t) the no A_n represts = Amplitude coeff / Harmonic amplitude / Spectral amplitude of the ~~f.s~~ fourier Series

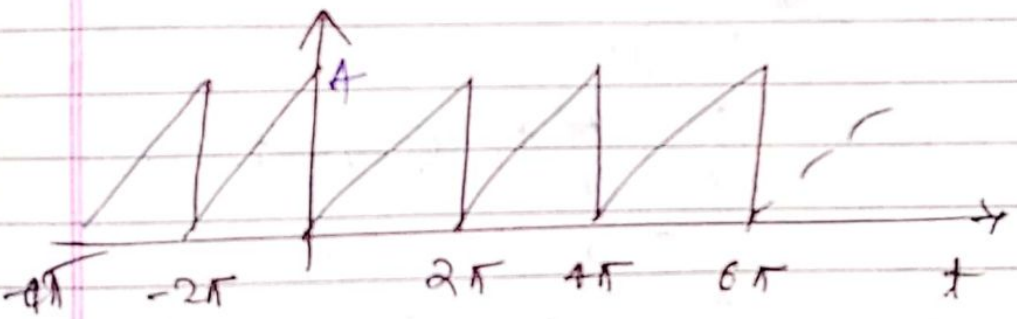
Page :
Date: / /

θ_n = the phase coef / phase angle / phase of the fourier series

The cosine form is also called the harmonic form of F.S.

OR polar form of F.S.

Ques Find the F.S of the waveform shown below.



Ans $T = 2\pi \quad \omega_0 = \frac{2\pi}{T} = 1$

$y = mx$

$f(t) = \frac{A}{2\pi} \cdot t$

$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$

Page :
Date: / /

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{A}{2\pi} \cdot t\, dt$$

$$a_0 = \left[\frac{t^2}{2 \times 4\pi}\right]_0^{2\pi}$$

$$= \frac{4\pi^2 A}{2 \times 4\pi^2} = \frac{A}{2}$$

Date
06/sep/19

~~Que Find the fouries series of the wavef~~

$$a_n = \frac{2}{2\pi}\int_0^{2\pi} \frac{A}{2\pi} t \cdot \cos m\omega_0 t\, dt$$

$$= \frac{A}{2\pi^2}\int_0^{2\pi} t \cdot \cos m\omega_0 t\, dt$$

$$\cancel{= \frac{A}{2\pi^2}\left[t \sin m\omega_0 t - 1 \frac{\sin m\omega_0 t}{m\omega_0}\right]_0^{2\pi}}$$

$$\cancel{= \frac{A}{2\pi^2}\left[2\pi - 0 - (0 - 0)\right]}$$

$$\cancel{= \frac{A \, 2\pi}{2\pi^2}} = A$$

$\cos n\pi = (-1)^{n+1}$

Page :
Date: / /

$\boxed{a_0 = \frac{0}{1}}$

$$= \frac{A}{2\pi^2} \int_0^{2\pi} t \cdot \cos nt \, dt$$

$$= \frac{A}{2\pi^2} \left[t \cdot \frac{\sin nt}{n} - 1(-1) \frac{\cos nt}{n^2} \right]_0^{2\pi}$$

$$= \frac{A}{2\pi^2} \left[0 + \frac{1}{n^2} - \frac{1}{n^2} \right]$$

$$= \boxed{0}$$

$$b_n = \frac{2}{2\pi} \int_0^{2\pi} \frac{A}{2\pi} \cdot t \sin nt \, dt$$

$$= \frac{A}{2\pi^2} \left[-t \cdot \frac{\cos nt}{n} - (1) \left(\frac{\sin nt}{n^2} \right) \right]_0^{2\pi}$$

$$= \frac{A}{\cancel{2}\pi^{\cancel{2}}} \left[-\frac{\cancel{2}\pi}{n} + 0 - (0) \right]$$

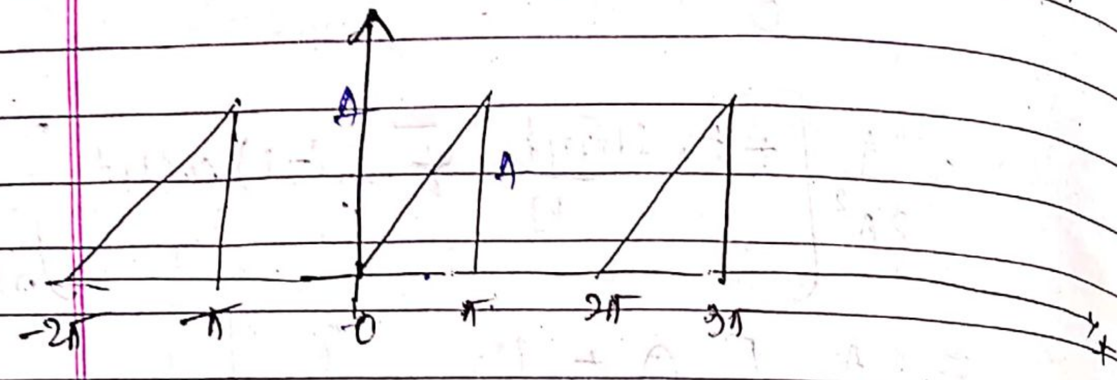
$$= \boxed{\frac{-A}{n\pi}}$$

$$f = \frac{A}{2} + \sum_{n=1}^{\infty} -\frac{A}{n\pi} \sin(nt)$$

$$\boxed{f = \frac{A}{2} - \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{\sin(nt)}{n}}$$

Page :
Date: / /

Ques. Find the fourier series of the waveform shown below.



Ans

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \text{ rad/sec}$$

$$f(t) = \begin{cases} \frac{A}{\pi} t & 0 \le t \le \pi \\ 0 & \pi \le t \le 2\pi \end{cases}$$

$$f = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} \frac{A}{\pi} t \, dt$$

$$= \frac{1}{2\pi} \cdot \frac{A}{\pi} \cdot \left[\frac{t^2}{2}\right]_0^{2\pi}$$

$$= \frac{A}{2\pi^2} \cdot \frac{\pi^2}{2} = \frac{A}{4}$$

$y = mx + 0$

$f(t) = mt$

$\tan\theta$

Page:

Date: / /

$\omega_0 = 1$

$$a_n = \frac{2}{2\pi}\int_0^{2\pi} f(t)\cos nt\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{A}{\pi} t\cos nt + 0$$

$$= \frac{1}{\pi}\cdot\frac{A}{\pi}\int_0^{\pi} t\cdot\cos nt$$

$$= \frac{A}{\pi^2}\left[t\cdot\frac{\sin nt}{n} + 1\cdot\frac{\cos nt}{n^2}\right]_0^{\pi}$$

$$= \frac{A}{\pi^2}\left[0 + \frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$$= \frac{A}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$a_n = \begin{cases} 0 & n = \text{even} \\ -\dfrac{2A}{n^2\pi^2} & n = \text{odd} \end{cases}$$

$$b_n = \frac{2}{2\pi}\int_0^{\pi} \frac{A}{\pi}\cdot t\cos nt\,dt$$

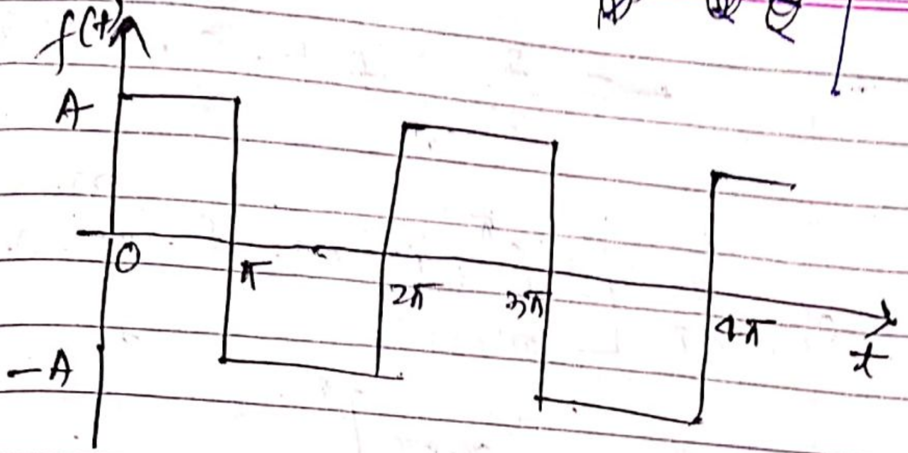
$$= \frac{A}{\pi^2}\int_0^{\pi} t\cdot\cos nt\,dt$$

$$= \frac{A}{\pi^2}\left[t\cdot\left(-\frac{\sin nt}{n}\right) - (1)\left(-\frac{\cos nt}{n^2}\right)\right]_0^{\pi}$$

$$b_n = \frac{A}{\pi^2}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right]$$

$b_n = \Big[$

Ques.



Ans.

$T = 2\pi$

$\omega_0 = \frac{2\pi}{T} = 1 \text{ rad/sec}$

$f(t) = A \quad \text{for } 0 \le t \le \pi$

$\quad\;\; -A \quad \text{for } \pi \le t \le 2\pi$

$a_0 = \frac{1}{T}\int_0^T f(t)\cdot dt$

$= \frac{1}{2\pi}\int_0^{\pi} A\cdot dt - \frac{1}{2\pi}\int_{\pi}^{2\pi} A\cdot dt$

$a_0 = 0$

Page :
Date: / /

$$a_n = \frac{2}{T}\int_0^{2\pi} f(t).\cos n\omega_0 t.dt$$

$$a_n = \frac{2}{2\pi}\left[\int_0^{\pi} A\cos nt.dt + \int_{\pi}^{2\pi} -A.\cos nt.dt\right]$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\int_0^{T} f(t)\sin nt.dt$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_0^{\pi} A\sin nt.dt + \frac{\cancel{2}}{\cancel{2}\pi}\int_{\pi}^{2\pi} A.\sin nt.dt$$

$$= -\frac{A}{n\pi}\left[\cos nt\right]_0^{\pi} + \frac{A}{n\pi}\left[\cos nt\right]_{\pi}^{2\pi}$$

$$= -\frac{A}{n\pi}\left[\cos n\pi - 1\right] + \frac{A}{n\pi}\left[\cos 2n\pi - \cos n\pi\right]$$

$$\begin{cases} \dfrac{2A}{n\pi} + \dfrac{2A}{n\pi} = \dfrac{4A}{n\pi} & n = \text{odd} \\ & 1,3,5 \\ 0 & n = \text{even} \\ & 0,2,4\ldots \end{cases}$$

Page:
Date: / /

Que) Exponential form of fourier series

It is most widely used form of fourier series in this $f(t)$ is express as a weighted sum of complex exponential fxn.

In trigonometric form of fourier series seprate expression is used to determine fourier coeff (a_0, a_n, b_n)

$$a_0 = \frac{1}{T} \int f(t) \cdot dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cdot \cos n\omega_0 t \cdot dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \cdot \sin n\omega_0 t \, dt$$

$$\cos(j\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

Page :
Date: / /

In expo form of fourier series only one coeff (F_n) is required to determine. It provide a convinient transition to fourier integral and fourier transform

put $\cos n\omega_0 t = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}$

$$\sin n\omega_0 t = \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) + b_n \left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right]$$

$$= a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n - jb_n}{2} \right) e^{jn\omega_0 t} + \left(\frac{a_n + jb_n}{2} \right) e^{-jn\omega_0 t}$$

$$\sin j\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

Page:
Date: / /

Let $a_0 = F_0$ $\qquad F_n = \frac{1}{2}(a_n - jb_n)$

$$F_{-n} = \left(\frac{a_n + jb_n}{2}\right)$$

$$f(t) = F_0 + \sum_{n=1}^{\infty} F_n e^{jn\omega_0 t} + F_{-n} e^{-jn\omega_0 t}$$

$$\sum_{n=0}^{\infty} F_n e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty} F_n e^{nj\omega_0 t}$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}} \longrightarrow \text{this is expo form of fourier series.}$$

F_n = fourier coeff

coefficient $F_n = \frac{1}{2}(a_n - jb_n)$

$$\frac{1}{2}\left[\frac{2}{T}\left\{\int_0^T f(t)\cos n\omega_0 t\,dt - j\int_0^T f(t)\cdot\sin n\omega_0 t\,dt\right\}\right]$$

$$F_n = \frac{1}{T}\int_0^T f(t)\left[\cos n\omega_0 t\,dt - j\sin n\omega_0 t\right]dt$$

→ line spectra / discrete spectra

$$\boxed{F_n = \frac{1}{T}\int_0^T f(t)\cdot e^{-jn\omega_0 t}\,dt}$$

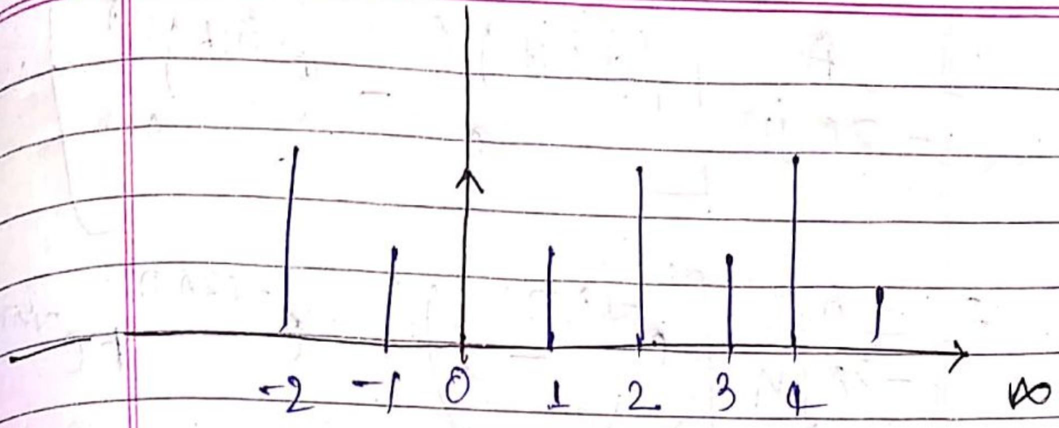
The coeff F_n is line or discrete spectra.

∴ The value of F_n occurs only for discrete values of 'n'.

It represent a complex spectrum & has both magnitude and phase spectra [Fourier spectrum]

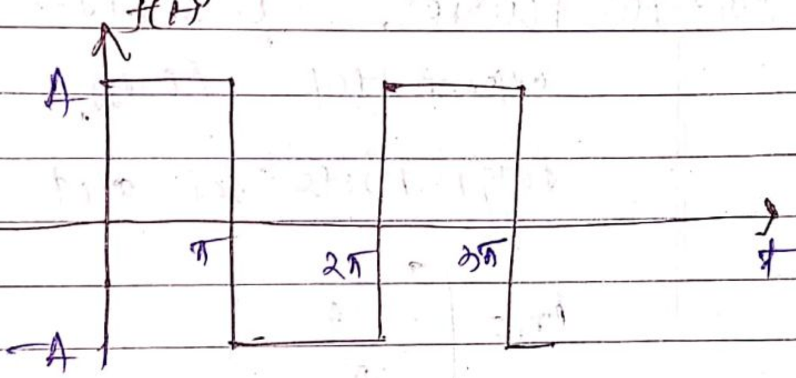
The magnitude spectrum is plot of amplitude of F_n VS frequency.

The spectra can be ploted for both +ve & -ve frequency ∴ it is called two sided spectra the amplitude spectrum of expo fourier series is symmetrical about the vertical axis. This is true for all periodic fxn.



magnitude / Amplitude

Ques Draw the fourier spectrum of the signal shown below.



find F_n & $|F_n|$

Ans

$T = 2\pi$

$\omega = 1$ rad/sec

$$F_n = \frac{1}{2\pi}\int_0^{2\pi} A \cdot e^{-jnt}\, dt + \frac{1}{2\pi}\int_{\pi}^{2\pi} A\, e^{-jnt}\, dt$$

$$= \frac{1}{2\pi}\left[\left(\frac{A \cdot e^{-jnt}}{-jn}\right)_0^{\pi} - \left(\frac{A e^{-jnt}}{-jn}\right)_{\pi}^{2\pi}\right]$$

Date 11/03/19

Page :
Date: / /

$$= \frac{A}{-2\pi j n}\left[\left(e^{-jnt}\right)_0^{\pi} - \left(e^{-jnt}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{A}{-2\pi j n}\left[\left(e^{-j\pi n} - 1\right) - e^{-j2\pi n} + e^{-j\pi n}\right]$$

$$F_n = \frac{A}{-2\pi j n}\left[2e^{-j\pi n} - e^{-2\pi j n} - 1\right]$$

Note- For Fourier spectra we use exponential form. ~~not use~~ trignometrical and cosine form

$$F_n = \begin{cases} \frac{-j2A}{n\pi} & n = \text{odd} \\ 0 & n = \text{even} \end{cases}$$

$$|F_n| = \frac{2A}{n\pi} \quad |F_{-n}| = \frac{2A}{n\pi}$$

NOTE :- The magnitude spectrum is symmetrical about vertical axis passing through origin & phase spectrum is anti-symmetrical about vertical axis passing origin. The magnitude spectrum shows even symmetry.

and phase spectrum shows odd symmetry.

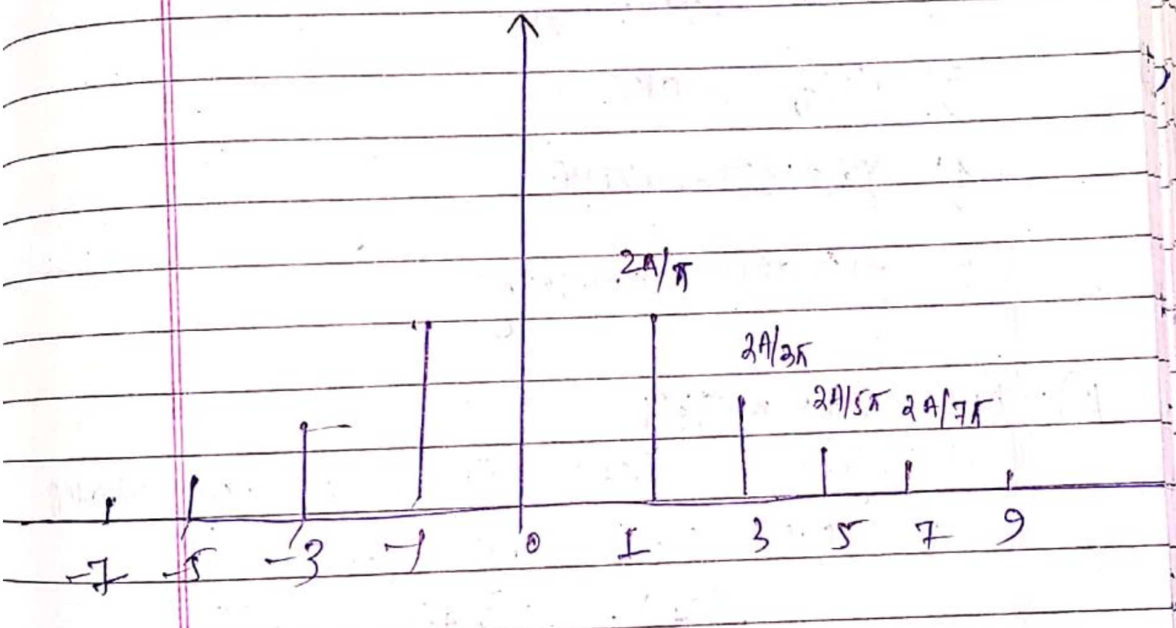
$F_0 = 0$

$F_1 = 2A/\pi$

$F_2 = 0$

$F_3 = 2A/3\pi$

$F_5 = 2A/5\pi$



$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$$

Page :
Date: / /

Wave form symmetry

Importance :-

- If the periodic signal f(t) has some form of symmetry then some of the fourier coeff become zero. And calculation of fourier coeff becomes simple.

There are following type of symmetry

1) Even symmetry (mirror symmetry)
2) odd symmetry
3) Half - wave
4) Quater - wave
5) Hidden symmetry

1) Even symmetry
A signal f(t) is said to have an even symmetry if

$$f(t) = f(-t) \text{ then}$$

$\boxed{b_n = 0}$ * and only a_0 and a_n are to be evaluated.

Page:
Date: / /

The summation of two or more even fx^n is always even. ~~these types of fx^n or signal~~ are always symmetry w.r.t vertical axis.

Date 12/09/19

$$a_0 = \frac{1}{T}\int_0^T f(t).dt$$

$$f(t) = f_e(t) + f_o(t)$$

for even $\quad f(t) = f_e(t)$

$$\boxed{f_o(t) = 0}$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f_e(t).dt$$

$$\boxed{a_0 = \frac{2}{T}\int_0^{T/2} f_e(t)\,dt}$$

$$a_n = \frac{2}{T}\int^T f_e(t).\cos n\omega_0 t\,dt$$

$$\boxed{a_n = \frac{4}{T}\int_0^{T/2} f_e(t)\cos n\omega_0 t\,dt}$$

Scanned by CamScanner

The product of odd $f x^n$ = even

Page:
Date: / /

2) Odd symmetry :-

A fx^n is said to have odd symmetry if

$$f(t) = -f(-t)$$

$$\boxed{f(t) = -f(t)}$$

If fx^n has odd symmetry

$\boxed{a_0 = 0}$ $\boxed{a_n = 0}$

only b_n is to be evaluated

$$\boxed{b_n = \frac{4}{T}\int_0^{T/2} f(t) . \sin n\omega_0 t \, dt}$$

3) Half wave symmetry :-

A signal is said to have half wave symmetry

if $f(t) = -f\left(t \pm T/2\right)$

This fx^n is neither purely odd nor purely even for such fx^ns

Page :
Date :

$\boxed{a_0 = 0}$ *

fourier series expansion of such type of periodic signal contains only odd harmonics.

For n = even

$\boxed{a_n = 0}$ $\boxed{b_n = 0}$

For n = odd

$$\boxed{a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t \, dt}$$

$$\boxed{b_n = \frac{4}{T}\int_0^{T/2} f(t) \sin n\omega_0 t \, dt}$$

4) Quater wave symmetry

A fxn $f(t)$ is said to have quater wave symmetry if $\underbrace{f(t) = f(-t)}_{\text{even}}$ or $\underbrace{f(t) = -f(-t)}_{\text{odd}}$

and

$$\underbrace{f(t) = -f\left(t \pm T/2\right)}_{\text{half wave}}$$

i.e fxn $f(t)$ have either even or odd symmetry along with half wave symmetry

Page :
Date: / /

is said to have quarter wave symmetry.

Case 1 :

$f(t) = f(-t)$ and $f(t) = -f(t \pm T/2)$

$\boxed{a_n = 0}$ *
$\boxed{a_0 = 0}$

$$b_n = \frac{8}{T}\int_0^{T/4} f(t).\sin n\omega_0 t.dt$$

(when $n = odd$)

case 2

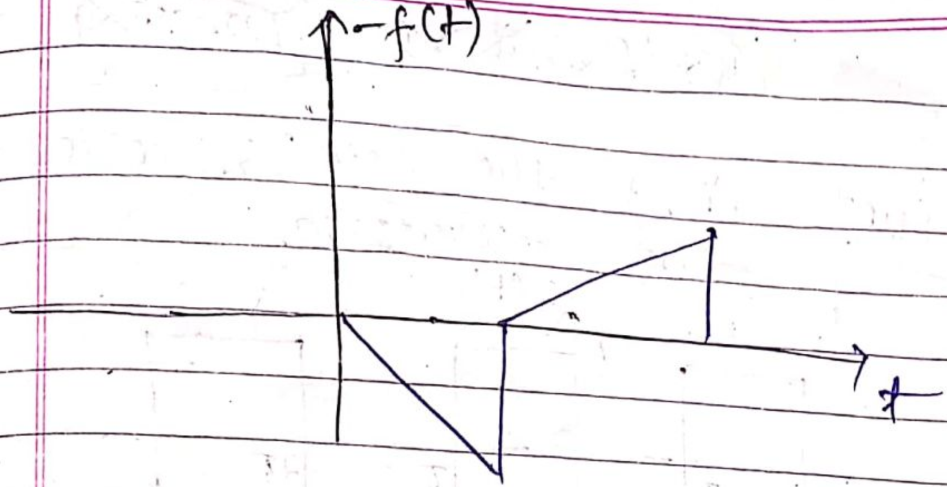
$f(t) = f(-t)$ and $f(t) = -f(t \pm T/2)$

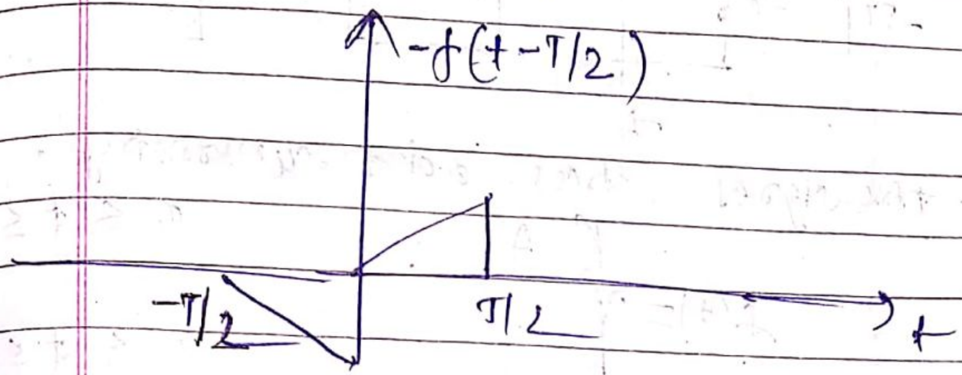
$\boxed{a_0 = 0}$ $\boxed{b_n = 0}$

$$\boxed{a_n = \frac{8}{T}\int_0^{T/4} f(t).\cos n\omega_0 t.dt}$$ *

$n = even$

NOTE :- a_0 is always zero for half wave symmetry.





Check half wave symmetry.

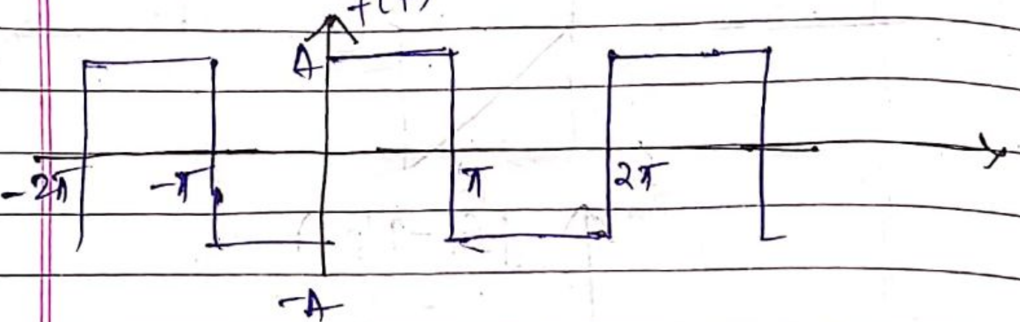
1) Take f(t) for 1 Time period
2) Invert the signal.
3) Shift the signal by ± T/2.
4) compare with original signal

Date 13/09/19

Page :
Date: / /

~~Quarter wave symmetry~~ :-

Que Find the fourier series of the waveform.



⇒ the signal has odd symmetry.

$$f(t) = \begin{cases} A & 0 \le t \le \pi \\ -A & \pi \le t \le 2\pi \end{cases}$$

$T = 2\pi$

$\omega_0 = 1$

$\boxed{a_0 = 0}$ $\boxed{a_n = 0}$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\cdot \sin(n\omega_0 t)\,dt$$

$$= \frac{4}{2\pi}\int_0^{\pi} A\cdot \sin(nt)\,dt$$

$$-\frac{2}{\pi}\int_{\pi}^{2\pi} A\cdot \sin(nt)\,dt$$

Page :
Date: / /

$$= -\frac{2}{\pi} \cdot A \left[\frac{\cos nt}{n} \right]_0^{\pi} + \frac{2}{\pi} A \left[\frac{\cos nt}{n} \right]_{\pi}^{2\pi}$$

$$= -\frac{2}{\pi}\frac{A}{n} \left[\cos(n\pi) - 1 \right] + \frac{2}{\pi}\frac{A}{n} \left[\cos 2\pi n - \cos n\pi \right]$$

$$= \frac{2}{\pi}\frac{A}{n} \left[-\cos(n\pi) + 1 + \cos(2\pi n) - \cos n\pi \right]$$

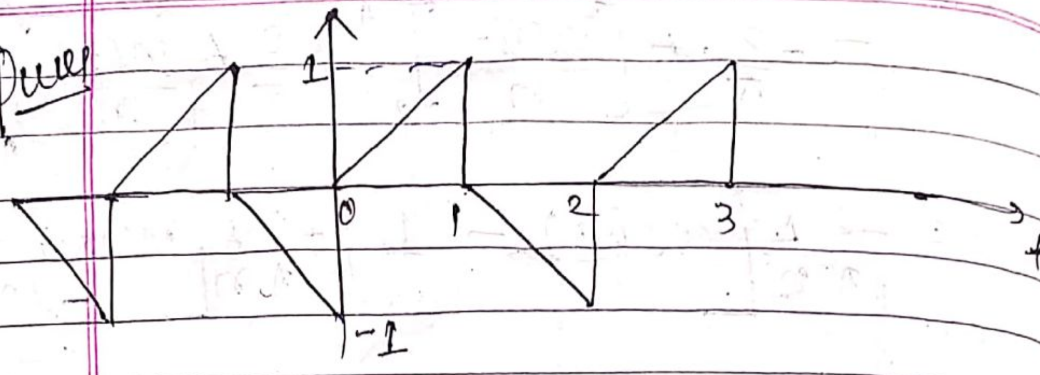
$$= \frac{2}{n}\frac{A}{\pi} \left[-2\cos n\pi + \cos(2\pi n) + 1 \right]$$

$$b_n = \begin{cases} 0 & n = \text{even} \\ \frac{4A}{n\pi} & n = \text{odd} \end{cases}$$

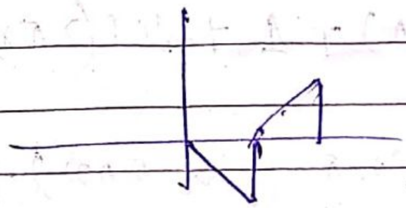
$$f(t) = \sum_{n=\text{odd}}^{\infty} \frac{4A}{n\pi} \cdot \sin nt$$

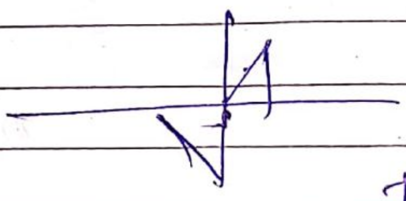
Page :
Date: / /

Ques



Ans





The given signal has half wave symmetry

n = even

$\boxed{a_0 = 0}$ $\boxed{a_n = 0}$ $\boxed{b_n = 0}$

n = odd

$$a_n = \frac{4}{T}\int_0^{T/2} f(t) \cdot \cos n\omega_0 t \, dt$$

Scanned by CamScanner

Page :
Date: / /

$T = 2$

$\omega_0 = 2\pi f = \frac{2\pi}{2} = \pi$

$\boxed{\omega_0 = \pi}$

$$f(t) = \begin{cases} 1 \cdot t & 0 \le t \le 1 \\ -t & 1 \le t \le 2 \end{cases}$$

$$a_n = \frac{\cancel{4}\,2}{\cancel{T}} \int_0^1 t \cdot \cos(n\pi t) \cdot dt$$

$$- \frac{\cancel{4}\,2}{\cancel{T}} \int_1^2 t \cdot \cos(n\pi t) \cdot dt$$

$$= 2 \left[t \cdot \frac{\sin(n\pi t)}{n\pi} + \frac{1 \cos(n\pi)t}{(n\pi)^2} \right]_0^1$$

$$-2 \left[t \cdot \frac{\sin n\pi t}{n\pi} + \frac{1 \cdot \cos(n\pi)t}{(n\pi)^2} \right]_1^2$$

$$=2 \left[\frac{\sin(n\pi)}{n\pi} + \frac{\cos(n\pi)}{(n\pi)^2} - 1 \right]$$

$$-2 \left[2 \cdot \frac{\sin(2n\pi)}{n\pi} + \frac{\cos 2n\pi}{(n\pi)^2} - \left(\frac{\sin n\pi}{n\pi} + \frac{\cos n\pi}{(n\pi)^2} \right) \right.$$

Page :
Date: / /

$$\cancel{\frac{2\sin n\pi}{n\pi}} + \frac{\cos(n\pi)}{(n\pi)^2} - 2$$

$$-4\frac{\sin(n\pi)}{(n\pi)} + 2\frac{\cos 2n\pi}{(n\pi)^2} + \cancel{2\frac{\sin n\pi}{(n\pi)}}$$

$$+ 2\frac{\cos n\pi}{(n\pi)^2}$$

$$= 3\frac{\cos n\pi}{(n\pi)^2} - \frac{2\cos(2n\pi)}{(n\pi)^2}$$

$$\frac{1}{(n\pi)^2}$$ $n = odd$

$n = even$

$$\frac{1}{(n\pi)^2}$$ $n = odd$

Date 16/09/19

Page:
Date: / /

Fourier transform:- is a transform technique which transforms a signal $f(t)$ from continuous time domain $f(t)$ into corresponding frequency domain. and vice -versa.

It is applicable to periodic as well as non-periodic signal.

The fourier transform of a continuous time signal $f(t)$ is given by.

$$F[f(t)] = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt = F(\omega)$$

and $f(t) = F^{-1}[F(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$

magnitude & phase spectrum of $F(\omega)$

$$F(\omega) = F_R(\omega) + j F_I(\omega)$$

$$|F(\omega)| = \sqrt{F_R^2(\omega) + F_I^2(\omega)}$$

Page :
Date: / /

$$\boxed{F(\omega) = \tan^{-1} \frac{F_I(\omega)}{F_R(\omega)}}$$

necessary conditions for the existence of fourier transform (Drichlit condition)

1) The fxn $f(t)$ is absolutely integral over $(-\infty$ to $\infty)$

i.e $\int_{-\infty}^{\infty} f(t) \cdot dt < \infty$

2) $f(t)$ has finite no. of discontinuity over finite time interval.
Further the discontinuity is also finite.

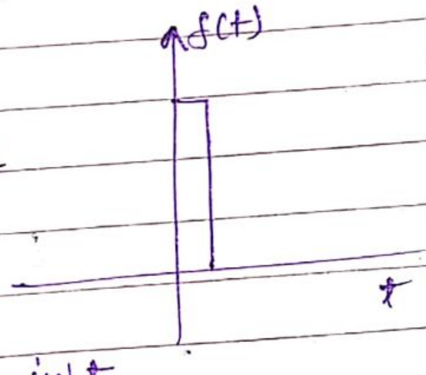
Scanned by CamScanner

3) f(t) has finite no. of maxima & minima in every finite time interval.

Fourier transform of standard signal.

Impulse signal $\delta(t)$
Impulse is a signal whose value is zero everywhere except at $t=0$.
At $t=0$ its value is infinite.

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



$$F(\delta(t)) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} \cdot dt$$

$$= e^{-j\omega t}\Big|_{t=0} = 1$$

Page :
Date: / /

Fourier transform of impulse
fxn

$$F[\delta(t)] = 1$$

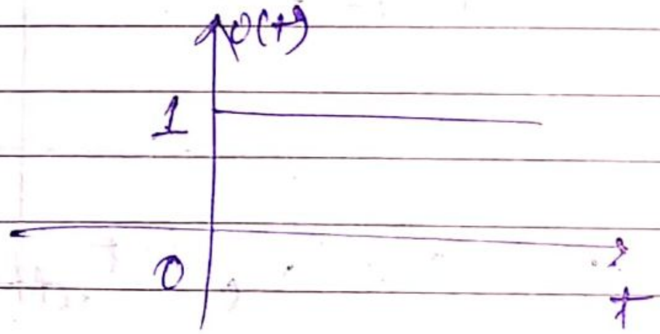
2) One-sided signal $e^{-at}u(t)$

$$f(t) = e^{-at} \cdot u(t)$$

$$F[f(t)] = F(\omega) = \int_{-\infty}^{\infty} e^{-at} \cdot u(t) e^{-j\omega t} \cdot dt$$

3) Unit step signal $u(t)$

$$u(t) = 1 \quad t \geq 0$$
$$\quad 0 \quad t < 0$$



NOTE:- whose value 0 to 1 in zero time
(value written above "value")

Page :
Date: / /

$$= \int_0^\infty e^{-at} \cdot e^{-j\omega t} dt$$

$$= \int_0^\infty e^{-(a+j\omega)t} \cdot dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^\infty$$

$$= \frac{1}{-(a+j\omega)} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{1}{-(a+j\omega)} [-1] = \frac{1}{(a+j\omega)}$$

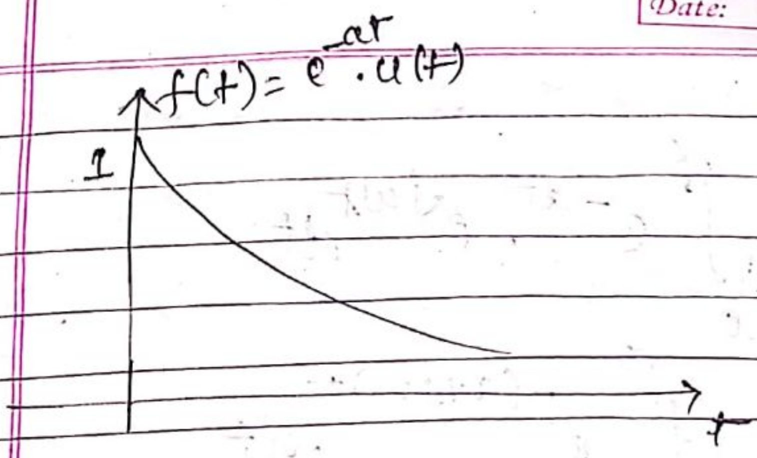
$$= \frac{a - j\omega}{a^2+\omega^2} = \frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}$$

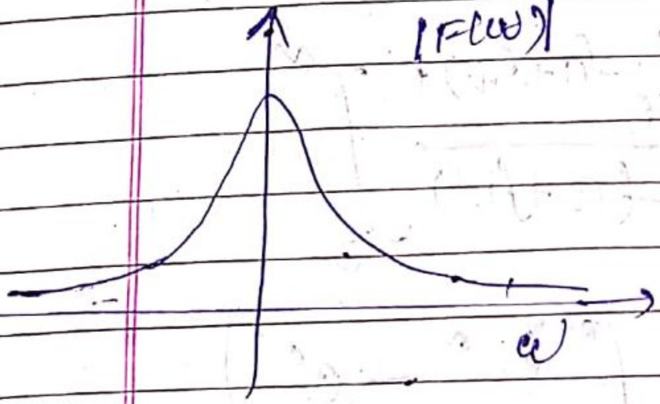
$$F_R = \frac{a}{a^2+\omega^2} \quad F_I = -\frac{\omega}{a^2+\omega^2}$$

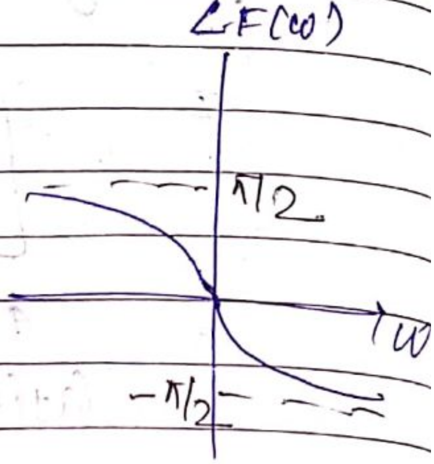
$$|F(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \quad \cancel{\angle\phi(\omega)}$$

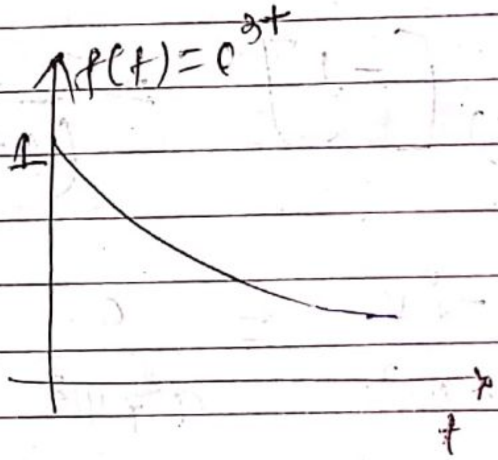
$$\angle\phi(\omega) = \tan^{-1}\left[\frac{F_I(\omega)}{F_R(\omega)}\right]$$

Page :
Date: / /









$a = -3$

Date 17/09/19

HBD MODI G

Page :
Date: / /

Properties of Fourier transform

1) Linearity property :-

If $f_1(t) \xleftrightarrow{FT} F_1(\omega)$ and $f_2(t) \xleftrightarrow{FT} F_2(\omega)$

then $a_1 f_1(t) + b_1 f_2(t) \xleftrightarrow{F.T} a_1 F_1(\omega) + b_1 F_2(\omega)$

where a_1 & b_1 are const

2) Time shifting property :-

$t - t_0 = P$

$t = P + t_0$

$dt = dP$

If $f_1(t) \xleftrightarrow{F.T} F_1(\omega)$ then

$$f_1(t - t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} F(\omega)$$

3) Freq. shifting

If $f_1(t) \xleftrightarrow{F.T} F_1(\omega)$ then

$$e^{j\omega_0 t} f(t) \xleftrightarrow{F.T} F(\omega - \omega_0)$$

$$F[f(t-t_0)] = \int_{-\infty}^{\infty} f(t - t_0)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} dP\, e^{-j\omega(P + t_0)} . dt$$

$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} F(P) . e^{-j\omega P} . dP$$

Page :
Date: / /

4) Time Reversal :-

If $f(t) \xleftrightarrow{FT} F(\omega)$

then $f(-t) \xleftrightarrow{F.T} F(-\omega)$

5) Time scaling :-

If $f(t) \xleftrightarrow{FT} F(\omega)$

then $f(at) \longleftrightarrow \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$

case:- $a > 0$

$$F[f(at)] = \frac{1}{a} F\left(\frac{\omega}{a}\right)$$

$a < 0$

$$F[f(at)] = \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$$

6) Differentiation in time domain

If $f(t) \xleftrightarrow{FT} F(\omega)$.

then $\frac{d f(t)}{dt} \xleftrightarrow{F.T} j\omega F(\omega)$

Page :
Date: / /

7) differentiation in freq domain

If $f(t) \longleftrightarrow F(\omega)$

then $\cancel{t}f(t) \longleftrightarrow j\frac{d}{d\omega}F(\omega)$

8) ~~Time Integral property~~

If $f(t) \xleftrightarrow{FT} F(\omega)$

then $\int_{-\infty}^{\infty} f(\tau)\,d\tau \xleftrightarrow{FT} \frac{1}{j\omega}F(j\omega)$

iff $f(0) = 0$

If $f(0) \neq 0$ then fourier ~~transform of~~

$$\int_{-\infty}^{\infty} f(\tau)\,d\tau \xleftrightarrow{FT} \frac{1}{j\omega}F(j\omega) + \pi f(0)\delta\omega$$

Scanned by CamScanner

9) Convolution property / Thm

If $f_1(t) \xleftrightarrow{FT} F_1(w)$ and

$f_2(t) \xleftrightarrow{FT} F_2(w)$

then $f_1(t) * f_2(t) \xleftrightarrow{FT} F_1(w) * F_2(w)$

10) Multiplication

If $f_1(t) \xleftrightarrow{FT} F_1(w)$ and

$f_2(t) \xleftrightarrow{FT} F_2(w)$

then $\boxed{f_1(t) \cdot f_2(t) \xleftrightarrow{FT} \frac{1}{2\pi} F_1(w) * F_2(w)}$

11) Duality property (b/w t, w)

If $f(t) \xleftrightarrow{FT} F(w)$

then $F(-t) \longleftrightarrow 2\pi f(-w)$

NOTES- For even fn $F(-w) = F(w)$

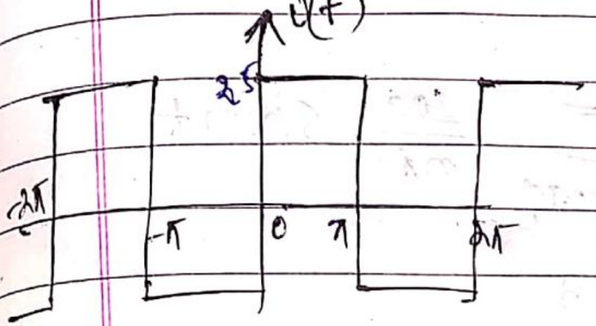
then $F(t) \longleftrightarrow 2\pi f(w)$

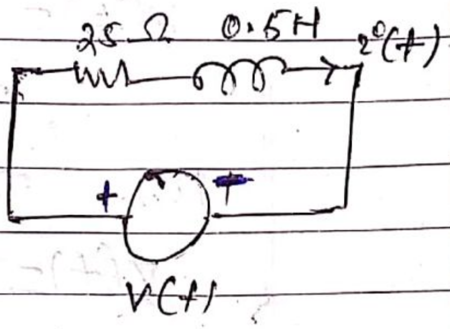
Harmonic- circuit of Heat develop

Page :

Date: / /

Que 1. The waveform shown below is applied to the circuit. Determine the third harmonic current frequency using Fourier series.





odd symmetry

$\boxed{a_0 = 0}$ $\boxed{a_n = 0}$ $\boxed{\omega_0 = 1}$

$T = 2\pi$

$$b_n = \frac{4}{T}\int_0^{T/2} v(t)\sin n\omega_0 t \, dt$$

$$v(t) = \begin{cases} 25 & 0 < t < \pi \\ -25 & \pi < t < 2\pi \end{cases}$$

$$b_n = \frac{4}{2\pi}\int_0^{\pi} 25 \sin nt \, dt - \frac{4}{2\pi}\int_{\pi}^{2\pi} 25 \sin nt \, dt$$

$$= \frac{50}{2\pi}\left[-\frac{\cos nt}{n}\right]_0^{\pi} + \frac{50}{2\pi}\left[\frac{\cos nt}{n}\right]_{\pi}^{2\pi}$$

$$\frac{50}{2\pi n}\left[-\cos n\pi + 1 + \cos 2\pi n - \cos n\pi\right]$$

Page :
Date: / /

$$b_n = \begin{cases} \frac{50 \times 4}{2\pi n} = \frac{100}{n\pi} & n = \text{odd} \\ 0 & n = \text{even} \end{cases}$$

$$V(t) = \sum_{n=\text{odd}} \frac{100}{n\pi} \cdot \sin nt$$

for $n = 3$

$$V(t) = \frac{100}{3\pi} \cdot \sin 3t$$

$$\boxed{V_{rms} = \frac{100}{3\pi} V}$$

$X_L = \omega L$

$X_L = 0.5 \times 1$

$X_L = 0.5$

$$Z = 25 + 0.5j$$

$$i(t)_{rms} = \frac{\cancel{100}\,4}{3\pi \cdot \cancel{25}.00 \angle 1.145}$$

$$= \frac{4}{3\pi} \angle -1.145$$

$$\boxed{i(t)_{rms} = 0.424 \angle -1.145 \text{ A}}$$

Practice
[illegible]/09/19

Page :
Date: / /

$$\int_a^b f(t)\cdot dt = -\int_b^a f(t)\cdot dt$$

$$\int_{-\infty}^{0} f(t)\cdot dt = \int_{0}^{\infty} f(-t)\cdot dt$$

⇒ Fourier transform of some imp functions.

Ques Find the fourier transform of following fxn

Imp (a) $f(t) = e^{3t} u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F[f(t)] = \int_{-\infty}^{\infty} e^{3t}\cdot u(t)\cdot e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{3t}\cdot e^{-j\omega t}\cdot dt$$

$$= \int_{0}^{\infty} e^{(3-j\omega)t}\cdot dt$$

$$= \left[\frac{e^{(3-j\omega)t}}{(3-j\omega)}\right]_0^{\infty} = \infty$$

The fourier transform of this fxn doesn't exist.

Page :
Date: / /

(b) $f(t) = 1$

$$F[f(t)] = \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t} \cdot dt$$

The fourier transform of Amp 1 can't be obtain directly so we take inverse fourier transform of $\delta(\omega)$.

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\omega t} \cdot dt$$

Now consider.

$$F(\omega) = \delta(\omega)$$

$$\delta(\omega) = 1 \quad \text{for } \omega = 0$$

$$0 \quad \text{for } \omega \neq 0$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega) \cdot e^{j\omega t} \cdot d\omega$$

$$F^{-1}[\delta(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega) \cdot e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \cdot (1)$$

Page :
Date: / /

$$\boxed{F^{-1}[2\pi\delta\omega] = 1}$$

(6) $f(t) = e^{-a|t|}$ for all t

$\downarrow$

$f(t) = e^{-a(-t)}$ for $-\infty < t < 0$

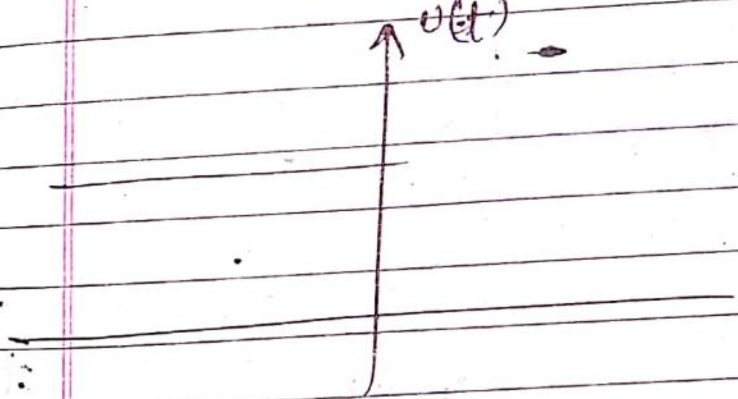
$f(t) = e^{-at}$ for $0 < t < \infty$

$u(t) = 1 \quad t \geq 0$

$\quad\quad\; 0 \quad\; 0 \quad t < 0$

$u(t) = 1 \quad t < 0$

$\quad\quad\quad 0 \quad t \geq 0$



$$f(t) = \int_{-\infty}^{0} e^{at} \cdot e^{-j\omega t} \cdot dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} \cdot dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{(a-j\omega)}\right]_{-\infty}^{0} + \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_{0}^{\infty}$$

$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

Page :
Date: / /

$\frac{1}{(a-jw)}\left[1-0\right] - \frac{1}{(a+jw)}\left[0-1\right]$

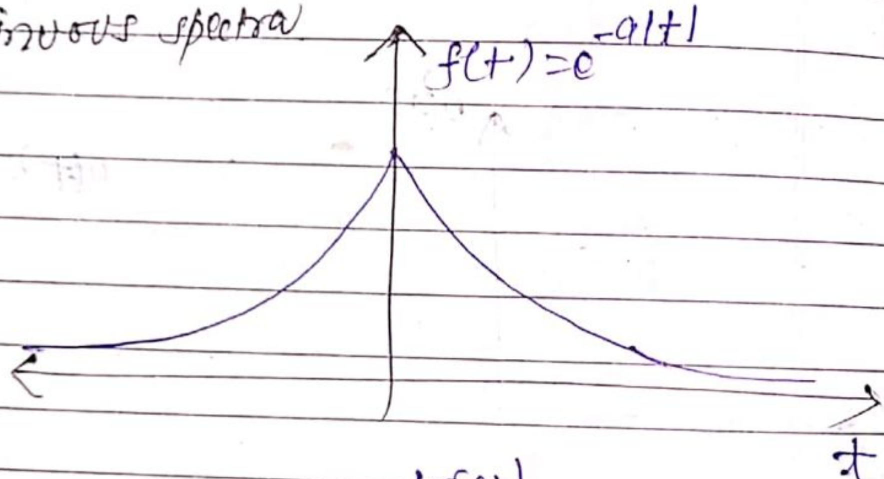
$= \frac{1}{a-jw} + \frac{1}{a+jw} = \frac{a+\cancel{jw}+a-\cancel{jw}}{a^2+w^2}$

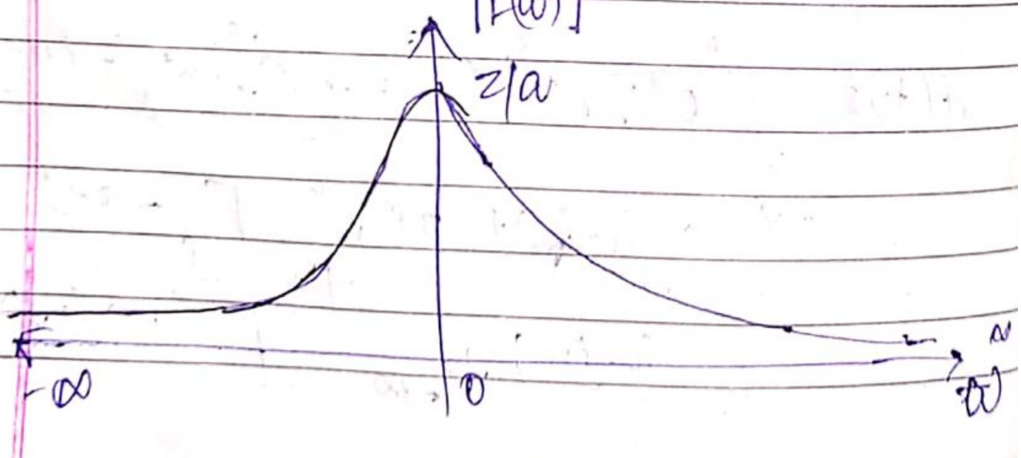
$F[e^{-a|t|}] = \frac{2a}{a^2+w^2}$

$|F[w]| = \frac{2a}{a^2+w^2}$

$\angle F(w) = 0$

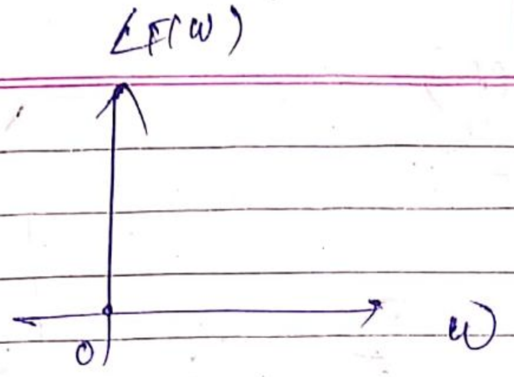
continuous spectra





$\left(\frac{2}{j\omega}\right)$

Page :
Date : / /



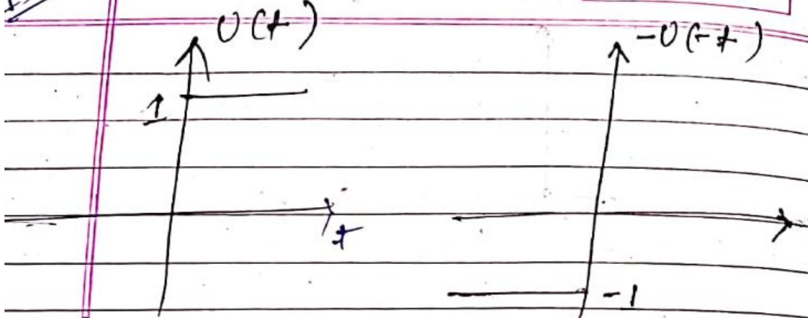
(d) Signum fxn sgn(t)

$$sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

It can't find directly so we take FT of fxn $e^{-a|t|} \cdot sgn(t)$ and limit $a \to 0$

$$e^{-a|t|} = -e^{at}u(-t) + e^{-at}u(t)$$

Date 19/09/19



$U(t) = 1 \quad t \geq 0$

$\quad\quad\; 0 \quad t < 0$

$sgn(t) = U(t) + [-U(-t)]$

$= U(t) - U(-t)$

F.T of sgn fx^n can't obtain directly However F.T of sgn fx^n obtain by multiply $\lim_{a \to 0} e^{-a|t|} sgn(t)$

$= \lim_{a \to 0} e^{-a|t|} [U(t) - U(-t)]$

$e^{at} \quad -\infty < t < 0$

$e^{-at} \quad 0 < t < \infty$

$= \lim_{a \to 0} [e^{-at} U(t) - e^{at} U(-t)] \quad S_2(t) = sgn(t)$

$$F[sgn(t)] = \lim_{a \to 0} \int_{-\infty}^{\infty} e^{-at} U(t) \cdot e^{-j\omega t} dt - \int_{-\infty}^{\infty} e^{at} U(-t) \cdot e^{-j\omega t} dt$$

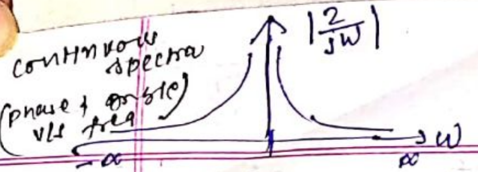
$$= \lim_{a \to 0} \int_0^{\infty} e^{-at} e^{-j\omega t} dt - \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt$$

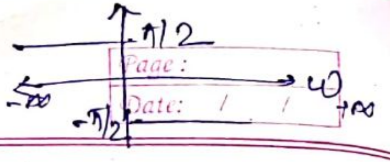
$$= \lim_{a \to 0} \int_0^{\infty} e^{-(a+j\omega)t} dt - \int_{-\infty}^{0} e^{(a-j\omega)t} dt$$

$$= \lim_{a \to 0} \int_0^{\infty} e^{-(a+j\omega)t} - \int_0^{\infty} e^{-(a-j\omega)t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_0^{\infty} - \left[\frac{e^{-(a-j\omega)t}}{-(a-j\omega)}\right]_0^{\infty}$$

$$= \frac{1}{-(a+j\omega)} [e^{-\infty} - 1] + \frac{1}{(a-j\omega)} [e^{-\infty} - 1]$$





$$\lim_{a \to 0} \frac{1}{(a+j\omega)} - \frac{1}{(a-j\omega)}$$

$$= \frac{\cancel{a}-j\omega-\cancel{a}-j\omega}{a^2+\omega^2} = \frac{-2j\omega}{a^2+\omega^2}$$

$$= \lim_{a \to 0} \left[\frac{-2j\omega}{a^2+\omega^2}\right]$$

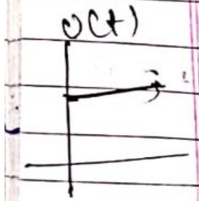
$$= \frac{-2j\omega}{\omega^2} = \frac{-2j}{\omega}$$

$$F[sgn(t)] = \boxed{\frac{2}{j\omega}}$$

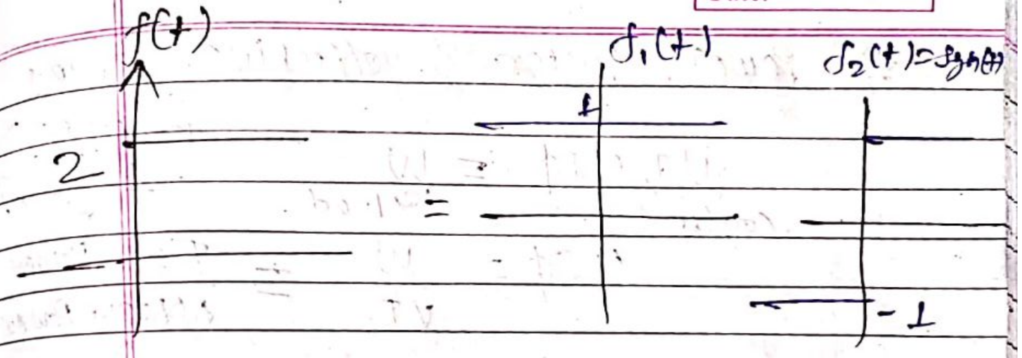
$\angle F(sgn\,t) = \pi/2 \quad \omega<0$

$-\pi/2 \quad \omega>0$

(e) $F[u(t)]$



The F.T of unit step can't obtain directly it can be represent as



$$2u(t) = 1 + sgn(t)$$

$$u(t) = \frac{1}{2} + \frac{1}{2} \cdot sgn(t)$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2}\pi\delta(\omega) + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{j\omega}$$

$$\boxed{F[u(t)] = \pi\delta(\omega) + \frac{1}{j\omega}}$$

P.F = angle b/w supplied volt & resulting current.

Page:
Date: / /

⇛ Power factor & effective values

$$\underbrace{VI}_{source} \cos\phi = \underbrace{W}_{load}$$ (av power delivers to load)

$$\cos\phi = \frac{W}{VI} = \frac{\text{True Power}}{\text{Apparent Power}}$$

measured only rms value

Date 20/09/19

Significance :-

For power factor low then for same load the value of 'VI' is more

$$\boxed{P.F = \frac{P_{av}}{V_{rms} \cdot I_{rms}}}$$

The effective value, I_{eff} of a periodic signal $i(t)$ is constant current 'I' which will produce the same power as is on an average power by a periodic signal $i(t)$.

The power in a resistor due to constant current 'I'

$$P = I^2R \quad \text{---} (1)$$

$I_{rms} = I_{effective}$

Page:
Date: / /

⇛ sinusoidal periodic signals $i(t)$, $v(t)$

$$P_{av} = [P_R(t)]_{av} = \frac{1}{2} I_m^2 R$$

as $P_{av} = P$ and $I = I_{eff}$

$$I_{eff}^2 R = \frac{1}{2} I_m^2 R$$

$$I_{rms} = \boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$$

$$V_{rms} = \boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$$

$$P = \frac{V^2}{R} \text{ and } V_m = I_m R$$

⇛ For non-sinusoidal periodic current $i(t)$ the ave power produce in a resistor 'R' is

$$P = I^2R \quad \text{---} (1)$$

$$P_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t) R \cdot dt$$

Page:
Date: / /

where $T = \frac{2\pi}{\omega}$

$P = P_{av}$

$$I^2 \cancel{R} = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,\cancel{R}\,dt$$

$$I_{eff}^2 = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt$$

$$\boxed{I_{rms} = I_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt}}$$

$$\boxed{V_{rms} = V_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt}}$$

⇒ Average power in term of fourier series

If $v(t)$ & $i(t)$ are periodic then they may be express as

Page:
Date: / /

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_m \cos(n\omega t - \phi_n)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_m \cos(n\omega t - \theta_n)$$

where time period of signal is $(2\pi/\omega)$.

In general the inst power $v(t) \cdot i(t)$ can assume both +ve & -ve values at various point during AC line cycle.

So the energy transmitted to the load over one time period is

$$W = \int_0^T v(t) \cdot i(t)\,dt$$

The avg power

$\frac{\text{Energy}}{\text{time}}$ $P_{avg} = \frac{W}{T} = \frac{1}{T}\int_0^T v(t) \cdot i(t)\,dt$

It can be shown that only contribution two integral comes from prod of 'V' & 'I' harmonics of same freq.

$$= \frac{1}{T}\int_0^T \left[V_0 + \sum_{n=1}^{\infty} \left[V_n \cos(n\omega t - \phi_n)\right]\right] \left[i_0 + \sum_{n=1}^{\infty} i_n \cos(n\omega t - \theta_n)\right]$$

* integral of cross prod term are zero. $\boxed{n \neq m}$

$$\boxed{P_{avg} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos(\phi_n - \theta_n)}$$

DC $\quad \boxed{m = n} \quad$ Harmonic

⟹ RMS value of wave form in term of fourier coefficient

The rms value of a periodic wave form $v(t)$ with time period

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T \left[V_0 + \sum_{n=1}^{\infty} \left[V_n \cos(n\omega t - \phi_n)\right]\right]^2}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \left[I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega t - \theta_n)\right]^2}$$

$$\boxed{I_{rms} = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}}$$

$$\boxed{V_{rms} = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}}}$$

Date
23/09/19

Ques) Find the effective value of the signal whose wave-form in shown below.

Ans

$$I_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2}\int_0^2 i^2(t)\,dt}$$

$$i(t) = \begin{cases} t & 0 \le t \le 1 \\ 1 & 1 \le t \le 2 \end{cases}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 t^2\,dt + \int_1^2 1.\,dt\right]}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\left(\frac{t^3}{3}\right)_0^1 + \left(t\right)_1^2}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{1}{3} + 1}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{4}{3}} = \frac{2}{3\sqrt{2}} = \sqrt{\frac{2}{3}}$$

$$\boxed{I_{rms} = \sqrt{\frac{2}{3}}}\text{ unit}$$

Ques) The voltage and current waveforms expressions are given as

$$v(t) = 1.2\cos(\omega t) + 0.33\,(3\omega t) + 0.2\cos(5\omega t)$$

$$i(t) = 0.6\cos(\omega t + 30^\circ) + 0.1\cos(5\omega t + 45^\circ)$$

$$\cancel{0.1\cos(7\omega t + 60^\circ)}$$

Find the average power.

Ans $$P_{av} = \underbrace{V_0 I_0}_{\text{zero}} + \sum_{n=1}^{\infty} \frac{V_n I_n \cos(\phi_n - \theta_n)}{2}$$

$$P_{av} = \frac{V_1 I_1}{2}\cos(30^\circ) + \frac{V_5 I_5}{2}\cos(45^\circ)$$

$$= \frac{1.2\times0.6\times\sqrt{3}}{4} + \frac{0.2\times0.1}{2}\cdot\frac{1}{\sqrt{2}}$$

$$= 0.3114 + 0.00707$$

$P_{av} = 0.318\, V_{out}$

NOTES :- Presence of Harmonic in a signal always Increases its RMS value.
In the case where voltage contains only fundamental while current contains fundamental as well as harmonic.
Then Harmonic increase the RMS value of current. while avg power delivered to load remains unchanged. This is undesirable bcz Harmonics do not lead to net delivery of power to the load. More over they increase the $I_{rms}^2 R$ losses in the system.

Date 24/09/19

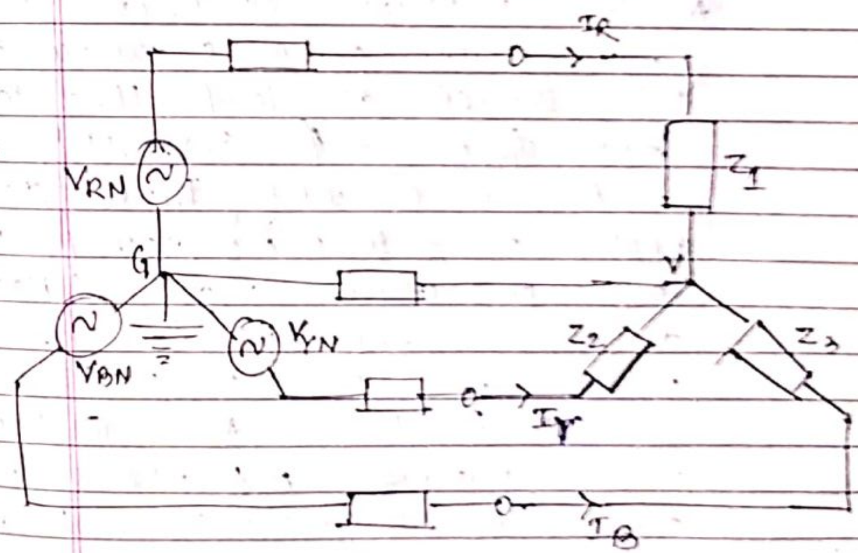
Power factor :-
It is figure of merit which measures how effectively power is transmitted b/w source & load in a n/w.

It is defined as

$$P.f = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{\text{Average Power}}{V_{rms} \cdot I_{rms}}$$

~~Note-~~

3-ϕ unbalanced circuit and power calculation :-



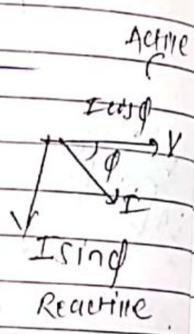
If load or supply are unbalanced then circuit is unbalanced.

$Z_1 \neq Z_2 \neq Z_3$

Active power $\rightarrow P$
Reactive power $\rightarrow Q$
Apparent power $\rightarrow S$

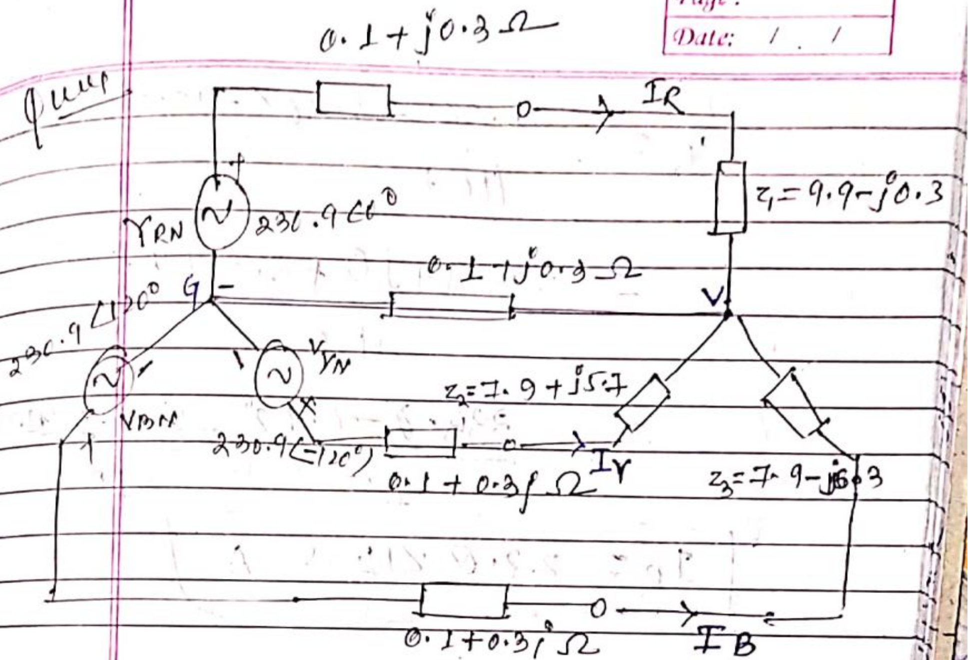
P.f $\quad S = \sqrt{P^2 + Q^2}$

$$\boxed{P.f = \frac{P}{S}}$$



A 3-φ unbalance circuit is one that contains atleast one source or load that does not have 3-φ symmetry.
A source with three phases supply magnitude are unequal and/or successive phase displacement different from 120° can make a circuit unbalance.
Similarly, A 3-φ load with unequal phase impd can make a circuit unbalance.

Ques



determine line vol & phase vol active & reactive power App power & power factor.

$I_N = I_R + I_Y + I_B$

Ans

Line voltage :-
(V_{RY}, V_{YB}, V_{BR})

$$\frac{V - 230.9\angle 0^\circ}{10} + \frac{V - 230.9\angle -120^\circ}{(8 + j6)} + \frac{V - 230.9\angle 120^\circ}{(8 - j6)} + \frac{V}{0.1 + j0.3} = 0$$

$$\boxed{V = 5.96 \angle -112.8\ V}$$

$V_{RY} = V_{RN} - V_{YN}$

Page :
Date: / /

$$I_R = \frac{230.9\angle 0^\circ - 5.96\angle -112.8}{(10)}$$

$$= \frac{230.9 + j0 + 2.30 + 5.42^\circ}{10}$$

$$= \frac{233.2 - 5.42^\circ}{10} = \frac{220.6\angle +1.35}{10}$$

$$\boxed{I_R = 22.0\angle +1.35 \text{ A}}$$

$$V_R = I_R Z_R = 22.0\angle 1.35$$

$$I_Y = \frac{230.9\angle 120^\circ - 5.96\angle -112.8}{0.1 + j0.3}$$

$$\cancel{= \frac{233.2 + 5.42^\circ}{0.8 + j0.6}}$$

$$\cancel{\angle(233.2 + 5.43)^\circ}$$

$$\cancel{= \frac{233.26\angle 1.326}{1.06\angle 36.86}}$$

$P = V_{RN} \times I_R \cos(\text{angle}$

$Q = V_{RN} \times I_R \sin(\text{angle}$

Date: / /

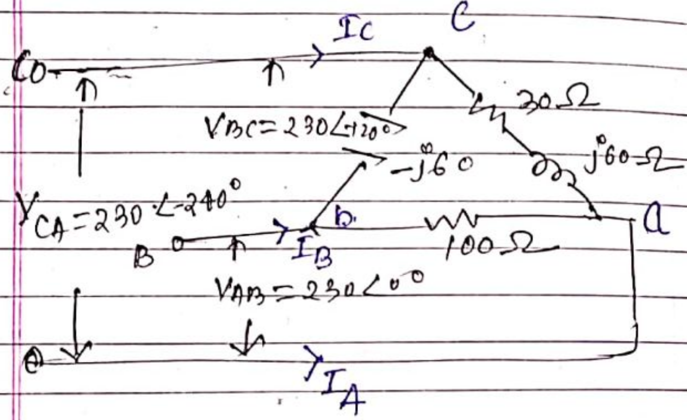
~~233.3~~

=

Date 25/09/19

$I_L = \sqrt{3} I_P$

Ques Determine.
a) phase current b) Line current for the unbalanced delta connected load.



A

Ans

$I_A = I_{ab} - I_{ca}$

$= \frac{230\angle 0^\circ}{100} - \frac{230\angle{-240^\circ}}{30 + j60}$

$= 2.3 - \frac{230\angle{-240}}{67.08\angle 63.4}$

$= 2.3 - 3.42\angle{-303.4}$

$= 2.3 - 1.88 - 2.05 i$

$= 0.42 - 2.05 j$

$= 1.741 \angle{-81.74}$

$I_P = \frac{I_L}{\sqrt{3}}$

$I_B = I_{bc} - I_{ab}$

$= \frac{230\angle{-120^\circ}}{-j60} - \frac{230\angle 0^\circ}{100}$

$= \frac{230\angle{-120^\circ}}{60\angle{-90}} - 2.3$

$= 3.83\angle{-30} - 2.3$

$= 3.31 - 1.915 j - 2.3$

$= 1.01 - 1.915 j$

$= 2.16\angle{-62.19}$

$I_C = I_{ca} - I_{bc}$

$= 3.42\angle{-303.4} - 3.83\angle{-30}$

$= 1.882 + 2.05 j - 3.31 + 1.915 j$

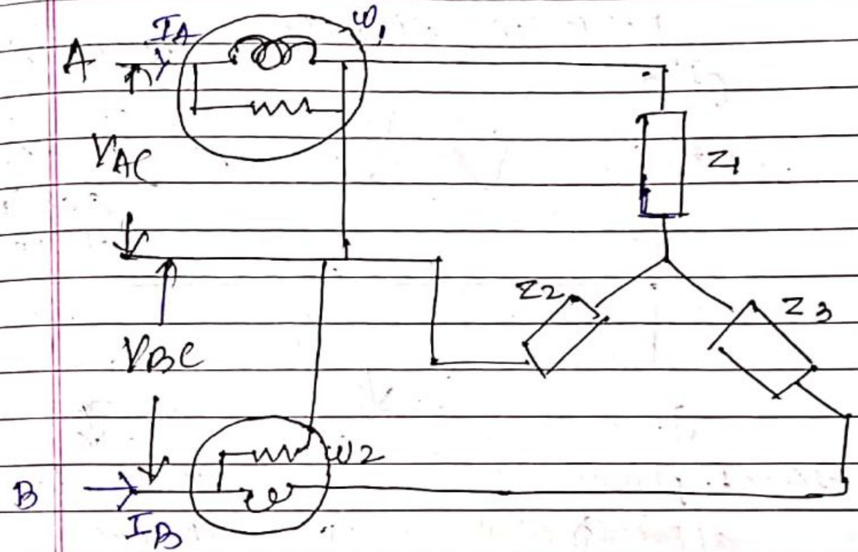
$= -1.428 + 4.765 j$

$= 4.97\angle 106.6$

Page :
Date: / /

Page :
Date: / /

⇒ Power measurement using Two wattmeter :-



$$W_1 = V_{AC} \times I_A = (V_A - V_C) I_A$$

$$W_2 = V_{BC} \times I_B = (V_B - V_C) I_B$$

$$I_A + I_B + I_C = 0 \implies I_A + I_B = -I_C$$

$$W_1 + W_2 = V_A I_A - V_C I_A + V_B I_B - V_C I_B$$

$$= V_A I_A + V_B I_B - V_C (I_A + I_B)$$

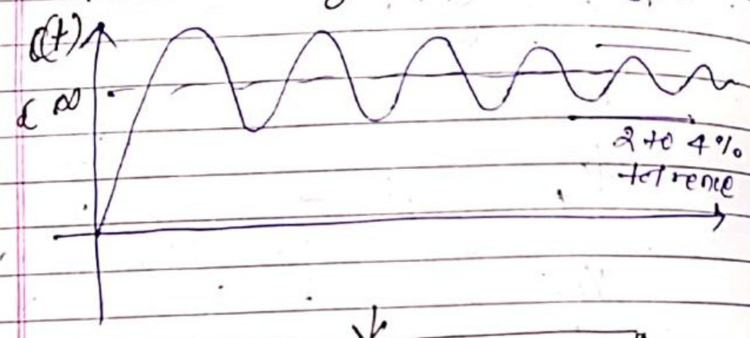
$$\boxed{W_1 + W_2 = V_A I_A + V_B I_B + V_C I_C}$$

Scanned by CamScanner

Date 26/09/19

when $t \to \infty$

→ Steady state response of a network due to non-sinusoidal periodic signals



↓ direct phasor approach — ↓ Spectrum prod approach

In direct phasor, phasor representⁿ of response is equal to the prod of Network function and phasor representating the excitation.

let the network be excited by non sinusodial periodic signal $V(t)$ the phasor response I

$$I = YV$$

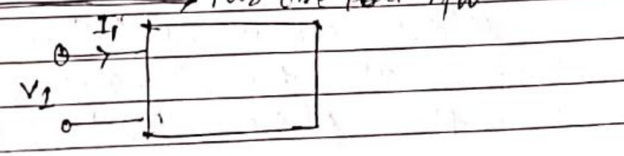
$I = YV$ ↳ admittance $\qquad Y(s) = Y(j\omega)$

$V = ZI$

Network function :-

A transform $f x^n$ relating current + voltages at differeent paies of n/w is called n/w $f x^n$.

→ For one port n/w



$$Y_1(s) = \frac{I_1(s)}{V_1(s)} \quad ; \quad Z_1 = \frac{V_1(s)}{I_1(s)}$$

→ Two port n/w

$$Z_{11} = \frac{V_1(s)}{I_1(s)} \qquad Y_{11}(s) = \frac{I_1}{V_1},$$

$$Z_{22} = \frac{V_2(s)}{I_2(s)} \qquad Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

The phasor V may be resolved into numbers of phasors such that

$$V = V_1 + V_2 \; - - - - \; V_n$$

then

$$I = Y(j\omega_0)V_1 + Y(j2\omega_0)V_2 + Y(j3\omega_0)V_3 - - \cdot Y(jn\omega_0)V_n$$

$$I_n = Y(jn\omega_0)V_n$$

where admittance "Y" is calculated at the frequency of the phasor "V_n" which is "$n\omega_0$".

Now we transform frequency domain to time domain which gives i_n corresponding to I_n

$$\boxed{i_n(t) = |I_n| \cos(n\omega_0 t + \theta_n)}$$ *

Algo :-

1) convert the given n/w and excitation in phaser
2) solve in phasor.
3) convert back to time domain.

2) Spectrum Product approach.

In this we use Prod amplitude + phase spectra of the excitation together with that for the n/w fxn.

$$\boxed{\text{Amplitude spectrum of response} = \left(\text{Magni of n/w fx}^n\right)\left(\text{Amplitude spectrum of excitat}^n\right)}$$

$$\boxed{\text{Phase spectrum of response} = \left(\text{phase of n/w fx}^n\right)\left(\text{Phase spectrum of excitation}\right)}$$

NOTE :- Fourier series must be in exponential form.

:- अगर sigo में हो तो cosine में convert कर लेना

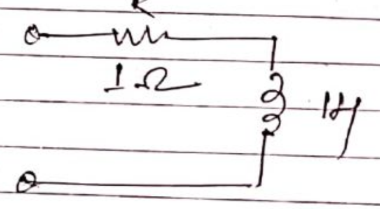
Ques. The series RL n/w to excited by non-sinusoidal periodic signal $v(t)$ whose fourier series is given as

$$v(t) = 1\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t$$

with $\omega_0 = 1$ rad/sec

Find steady state response $i(t)$

Ans



$V = V_1 + V_3 + V_5$

$Z = 1 + sL$

$$Y(s) = \frac{1}{1+sL}$$

$$Y(j\omega n) = \frac{1}{1+j\omega n}$$

$e^{-j\omega t} = \cos\omega t + j\sin n\omega t$

$$Y(j1) = \frac{1}{1+j} = \frac{1}{\sqrt{2}} e^{-j45}$$

$$Y(j3) = \frac{1}{1+j3} = \frac{1}{\sqrt{10}} e^{-71.57j^\circ}$$

$$Y(j5) = \frac{1}{1+j5} = \frac{1}{\sqrt{26}} e^{-78.69j^\circ}$$

$V_1 = 1 \cdot e^{0j^\circ}$ $\quad V_3 = -\frac{1}{3} = \frac{1}{3} e^{-j180^\circ}$

$$V_5 = \frac{1}{5} \cdot e^{j0}$$

$$I_1 = Y_1 V_1 = \frac{1}{\sqrt{2}} e^{-j45^\circ}$$

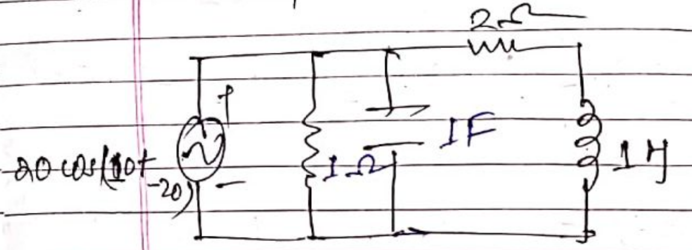
$$I_3 = Y_3 V_3 = \frac{1}{\sqrt{10}} \times \frac{1}{3} e^{-251.57j^\circ}$$

$$I_5 = Y_5 V_5 = \frac{1}{\sqrt{26}} \times \frac{1}{5} e^{-78.69j^\circ}$$

$$i(t) = 0.707\cos(t - 45^\circ) + 0.105\cos(3t - 251.57) + 0.039\cos(5t - 78.69)$$

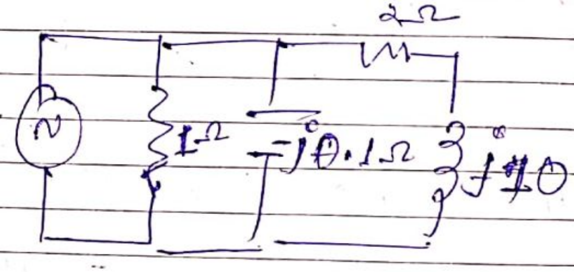
Date 27/09/19

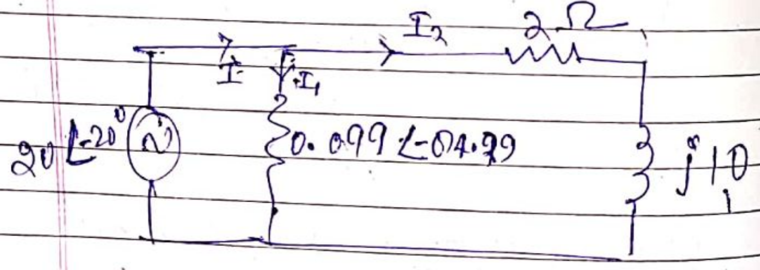
steady state of signal due to periodic sinusoidal signal



Find current through 2Ω resistor using phasor

Ans. convert into phasor





$$Z = \frac{(1)(-j0.1)}{1-j0.1}$$

$= 0.099 \angle -84.29$

$2(.0098 - 0.098j)$

$X_C = \frac{1}{j\omega C} = -j0.1\,\Omega$

$X_L = j\omega L = j \times 10 \times 1 = j10$

$$I = \frac{20 \angle -20^\circ}{Z}$$

$$Z = \frac{(2 + j10)(.0098 - 0.098j)}{2.0098 + 9.902j}$$

$$= \frac{(10.19 \angle 78.6)(.099 \angle -84.29)}{}$$

$$= \frac{(10.19 \times 0.99) \angle 78.6 - 84.29}{10.10 \angle 78.5}$$

$= 0.099 \angle -84.19$

$I = 202.02 \angle 64.19$

$$I_2 = \frac{0.099 \angle -84.29}{(2 + j10) + 0.0098 - 0.98j}$$

~~$\times 0.0099 \angle -84.19$~~

$202.02 \angle 64.19$

=

Page :
Date: / /

$$I_2 = \frac{19.99 \angle -20.1}{10.10 \angle 70.5}$$

$$\boxed{I_2 = 1.97 \angle -90.6}$$

$$I_2 = 1.97 \cos(10.1 - 90.6)$$

Page :
Date: / /