

### **Charge Neutrality:**

In thermal equilibrium, the semiconductor crystal is electrically neutral. The electrons are distributed among the various energy states creating negative and positive charges, but the net charge density is zero. This charge-neutrality condition is used to determine the thermal-equilibrium electron and hole concentrations as a function.

### **Compensated Semiconductors:**

A compensated semiconductor is one that contains both donor and acceptor impurity atoms in the same region. A compensated semiconductor can be formed, for example, by diffusing acceptor impurities into an n-type material, or by diffusing donor impurities into a p-type material. An n-type compensated semiconductor occurs when  $N_d > N_a$ , and a p-type compensated semiconductor occurs when  $N_a > N_d$ . If  $N_a = N_d$ , we have a completely compensated semiconductor, showing the characteristics of an intrinsic material. Compensated semiconductors are created quite naturally during device fabrication.

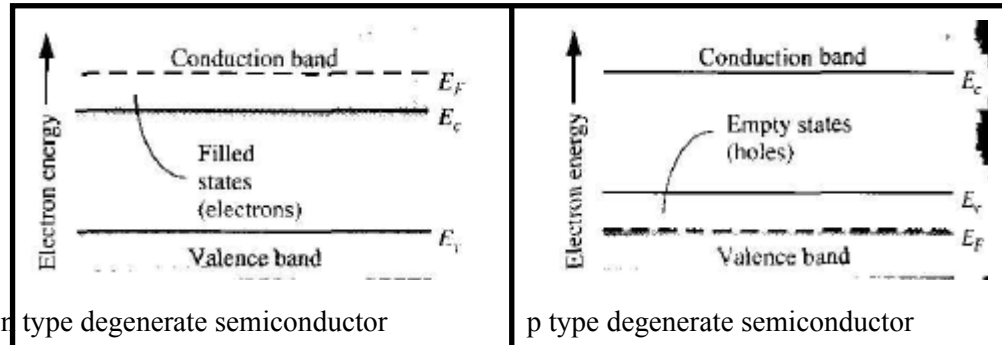
### **Degenerate and Non-degenerate Semiconductors:**

Addition of dopant atoms to a semiconductor, it is assumed that the concentration of dopant atoms added is small ( $\sim 10^{16}/\text{cm}^3$ ) compared to the density of host or semiconductor atoms ( $\sim 10^{22}/\text{cm}^3$ ). The small number of impurity atoms is spread far enough apart so that there is no interaction between donor electrons. We have assumed that the impurities introduce discrete, non-interacting donor energy states in the n-type semiconductor and discrete. These types of semiconductors are referred to as non-degenerate semiconductors.

If the impurity concentration increases, the distance between the impurity atoms decreases and a point will be reached when donor atoms (for n-type doping) will begin to interact with each other. When this occurs, the single discrete donor energy will split into a band of energies. As the donor concentration is further increased, the band of donor states widens and may overlap with the bottom of the conduction band. This overlap occurs when the donor concentration becomes comparable with the effective density of states. When the concentration of electrons in the conduction band exceeds the density of states  $N_C$ , the Fermi energy lies within the conduction band. This type of semiconductor is called a degenerate n-type semiconductor.

In a similar way, as the acceptor doping concentration increases in a p-type semiconductor, the discrete acceptor energy states will split into a band of energy and may overlap the top of the valence band. The Fermi energy will lie in the valence band when the concentration of holes exceeds the density of states  $N_V$ .

This type semiconductor is called a degenerate p-type semiconductor. Schematic models of the energy-band diagrams for a degenerate n-type and degenerate p-type semiconductor are shown in following figure.



### Carrier Transport:

The charge carriers in a solid are in motion, even at thermal equilibrium. At room temperature, for example, the thermal motion of an individual electron may be visualized as random scattering from lattice vibrations, impurities, other electrons, and defects. Since the scattering is random, there is no net motion of the group of  $n$  electrons/cm<sup>3</sup> over any period of time. The probability of the electron in returning to its starting point after some time  $t$  is negligibly small. However, if a large number of electrons is considered (e.g.,  $\sim 10^{16}$ /cm<sup>3</sup> in an n-type semiconductor), there will be no preferred direction of motion for the group of electrons and no net current flow.

If an electric field  $E_x$  is applied in the x-direction, each electron experiences a net force  $-qE_x$  from the field. The current density is proportional to the electric field, as we expect from Ohm's law:

$$\mathbf{J}_x = \sigma \mathbf{E}_x, \quad \text{where, the proportionality constant is } \sigma$$

The conductivity  $\sigma$  (ohm-cm)<sup>-1</sup> can also be written as

$$\sigma = qn\mu, \quad \mu \equiv \text{mobility of the carrier}; \mu_n \equiv \text{electron mobility}; \mu_p \equiv \text{hole mobility}$$

If both electrons and holes participate in conduction then,

$$\mathbf{J}_x = q(n\mu_n + p\mu_p)\mathbf{E}_x = \sigma \mathbf{E}_x$$

$$\mathbf{J}_x = \frac{\mathbf{I}}{\mathbf{A}} \text{ and } \mathbf{E}_x = \frac{\mathbf{V}}{\ell}; \text{ So, } \frac{\mathbf{I}}{\mathbf{A}} = \sigma \frac{\mathbf{V}}{\ell} \Rightarrow \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{\sigma} \frac{\ell}{\mathbf{A}} = \rho \frac{\ell}{\mathbf{A}}$$

$\ell$ : length of the semiconductor

$$\mathbf{R} = \frac{\rho \mathbf{L}}{\mathbf{A}} = \frac{\mathbf{L}}{\mathbf{wt}} \frac{1}{\sigma}$$

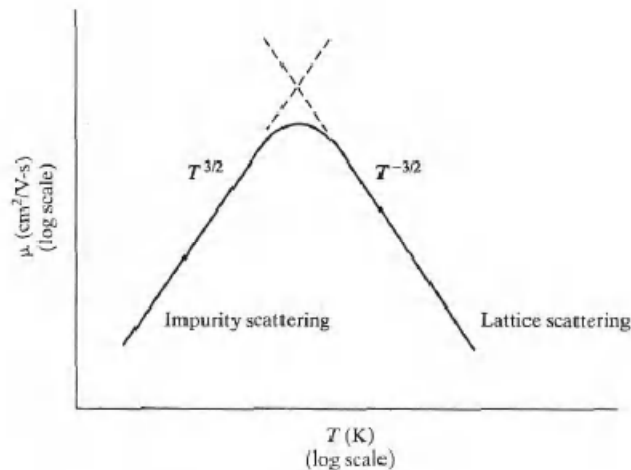
### Mobility (Effects of Temperature and Doping on Mobility)

The two basic types of scattering mechanisms that influence electron and hole mobility are *lattice scattering* and *impurity scattering*. In lattice scattering a carrier moving through the crystal is scattered by a vibration of the lattice, resulting from the temperature.<sup>1</sup> The frequency of such scattering events increases as the temperature increases, since the thermal agitation of the lattice becomes greater. Therefore, we should expect the mobility to decrease as the sample is heated. On the other hand, scattering from crystal defects such as ionized impurities becomes the dominant mechanism at low temperatures. Since the atoms of the cooler lattice are less agitated, lattice scattering is less important; however, the thermal motion of the carriers is also slower. Since a slowly moving carrier is likely to be scattered more strongly by an interaction with a charged ion than is a carrier with greater momentum,

Impurity scattering events cause a decrease in mobility with decreasing temperature. As Fig. indicates, the approximate temperature dependencies are  $T^{-3/2}$  for lattice scattering and  $T^{3/2}$  for impurity scattering.

Since the scattering probability is inversely proportional to the mean free time and therefore to mobility, the mobility due to two or more scattering mechanisms add inversely:

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3} + \dots$$



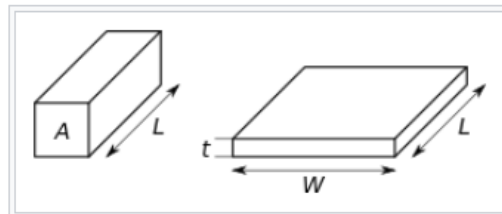
As the concentration of impurities increases, the effects of impurity scattering are felt at higher temperatures. For example, the electron mobility  $\mu_n$  of intrinsic silicon at 300 K is  $1350 \text{ cm}^2/(\text{V-s})$ . With a donor doping concentration of  $10^{17} \text{ cm}^{-3}$ , however,  $\mu_n$  is  $700 \text{ cm}^2/(\text{V-s})$ . Thus, the presence of the  $10^{17}$  ionized donors/ $\text{cm}^3$  introduces a significant amount of impurity scattering. This effect is illustrated in

Fig. which shows the variation of mobility with doping concentration at room temperature.

### Sheet resistance:

**Sheet resistance** is a measure of resistance of thin films that are nominally uniform in thickness. It is commonly used to characterize materials made by semiconductor doping, metal deposition, resistive paste printing, and glass coating. Examples of these processes are: doped semiconductor regions (e.g., silicon or polysilicon), and the resistors that are screen printed onto the substrates of thick-film hybrid microcircuits.

The utility of sheet resistance as opposed to resistance or resistivity is that it is directly measured using a four-terminal sensing measurement (also known as a four-point probe measurement) or indirectly by using a non-contact eddy current based testing device. Sheet resistance is invariable under scaling of the film contact and therefore can be used to compare the electrical properties of devices that are significantly different in size.



In a regular three-dimensional conductor, the resistance can be written as

$$R = \rho L / A = \rho L / (wt) = R_s (L/w)$$

$$\rho = R_s t$$

**1.3** (a) Find the concentrations of electrons and holes in a sample of silicon that has a concentration of donor atoms equal to  $5 \times 10^{15} \text{ cm}^{-3}$ . Is the semiconductor n-type or p-type?

$$\text{b. } n_0 = N_d - N_a = 5 \times 10^{15}$$

$$\text{majority carrier: } n_0 = 5 \times 10^{15} \text{ cm}^{-3}$$

minority carrier:

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{5 \times 10^{15}} = 4.5 \times 10^4 \text{ cm}^{-3}$$

## Excess Carriers in Semiconductors

### Diffusion of carriers

When excess carriers are created non-uniformly in a semiconductor, the electron and hole concentrations vary with position in the sample. Any such spatial variation (*gradient*) in  $n$  and  $p$  calls for a net motion of the carriers from regions of high carrier concentration to regions of low carrier concentration. This type of motion is called *diffusion*. The two basic processes of current conduction are diffusion due to a carrier gradient and drift due to applied electric field.

Consider,  $D_n \text{ cm}^2/\text{s}$  is the electron diffusion constant and  $D_p \text{ cm}^2/\text{s}$  is the hole diffusion constant. Consider, there is a decreasing concentration gradient of electron along x-axis. Then, the rate of electron flow in the +x-direction per unit area or “the electron flux density” can be written as

$$\phi_n(x) = -D_n \frac{dn(x)}{dx}$$

Similarly, if there is hole concentration gradient along +x-direction:

$$\phi_p(x) = -D_p \frac{dp(x)}{dx}$$

The diffusion current density (diffusion current crossing per unit area) is:

$$J_n(x) = -(-q)D_n \frac{dn(x)}{dx} = qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = -(+q)D_p \frac{dp(x)}{dx} = -qD_p \frac{dp(x)}{dx}$$

### Diffusion and Drift of Carriers; Built-in Fields:

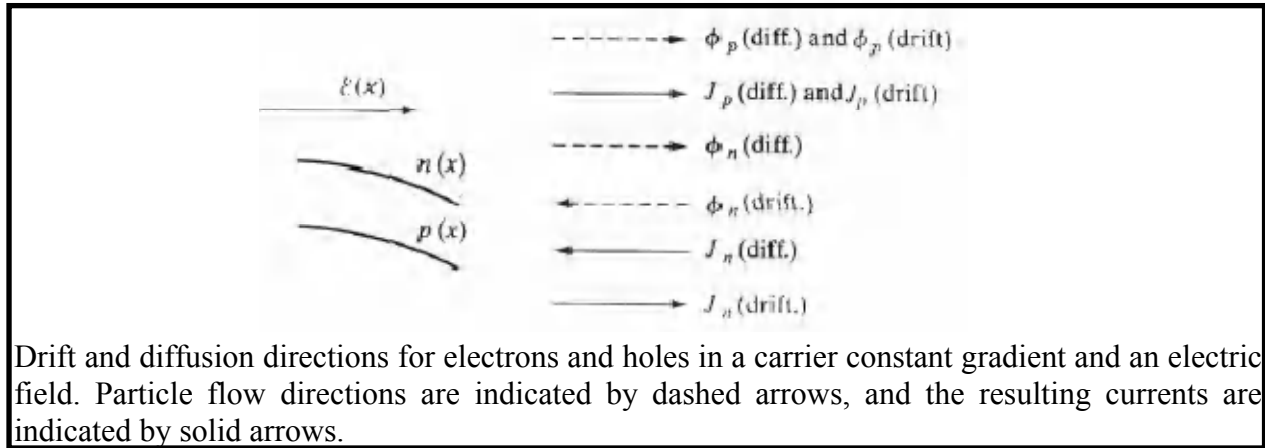
If an electric field is present in addition to the carrier gradient, the current densities will each have a drift component and a diffusion component

$$J_n(x) = \overbrace{q\mu_n n(x)E_x}^{\text{Drift}} + \overbrace{qD_n \frac{dn(x)}{dx}}^{\text{Diffusion}}$$

$$J_p(x) = q\mu_p p(x)E_x - qD_p \frac{dp(x)}{dx}$$

The total current density is the sum of the contributions due to electrons and holes:

$$J(x) = J_n(x) + J_p(x)$$



Now, from the definition of electric field

$$E_x = -dV(x)/dx$$

Now applied voltage introduces change in band diagram and band diagram describe electron potential energy. So, we can relate the electron potential energy in the band diagram by choosing some reference in the band for the electrostatic potential. Choosing  $E_i$  as a convenient reference, we can relate the electric field to this reference by

$$E_x = -\frac{dV(x)}{dx} = -\frac{d}{dx} \left[ \frac{E_i}{(-q)} \right] = \frac{1}{q} \frac{dE_i}{dx}$$

Now, we know

$$J_n(x) = q\mu_n n(x)E_x + qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = q\mu_p p(x)E_x - qD_p \frac{dp(x)}{dx}$$

At equilibrium,

$$J_x = 0 \Rightarrow J_p = 0, J_n = 0$$

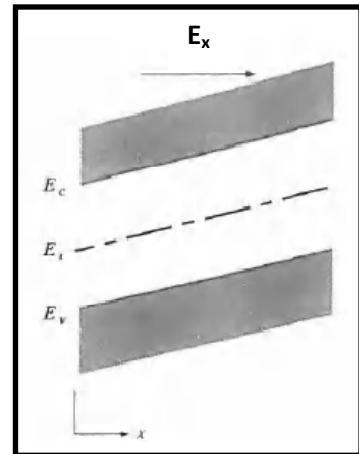
So, at equilibrium

$$q\mu_p p(x)E_x = qD_p \frac{dp(x)}{dx} \Rightarrow E_x = \frac{D_p}{\mu_p} \frac{1}{p(x)} \frac{dp(x)}{dx}$$

At equilibrium:

$$\begin{aligned} p(x) &= p_0 = N_v \exp[(E_F - E_v)/kT] = N_v \exp[(E_F + E_i - E_i - E_v)/kT] \\ &= N_v \exp[(E_i - E_v)/kT] \exp[(E_F - E_i)/kT] = p_i \exp[(E_F - E_i)/kT] \end{aligned}$$

$$\frac{dp(x)}{dx} = \frac{1}{kT} p_i \exp[(E_F - E_i)/kT] \left( \frac{dE_i}{dx} - \frac{dE_F}{dx} \right); \quad \text{where, at equilibrium } \frac{dE_F}{dx} = 0$$



$$E_x = \frac{D_p}{\mu_p} \frac{1}{p_i \exp[(E_F - E_i)/kT]} \frac{1}{kT} p_i \exp[(E_F - E_i)/kT] \left( \frac{dE_i}{dx} - \frac{dE_F}{dx} \right) = \frac{D_p}{\mu_p} \frac{1}{kT} \frac{dE_i}{dx}$$

Again

$$E_x = -\frac{dV(x)}{dx} = -\frac{d}{dx} \left[ \frac{E_i}{(-q)} \right] = \frac{1}{q} \frac{dE_i}{dx}$$


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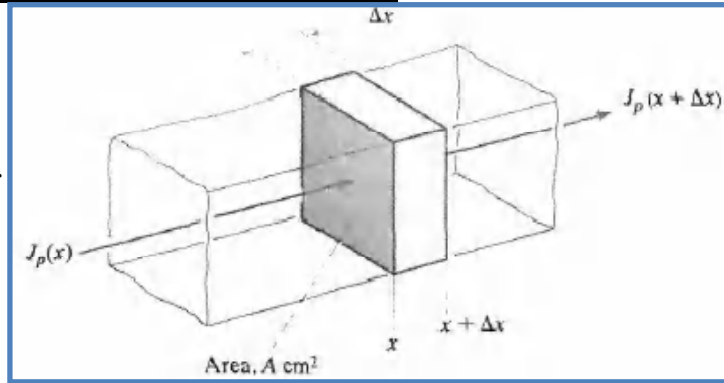
Combining these two equations we get

$$\frac{D_p}{\mu_p} \frac{1}{kT} \frac{dE_i}{dx} = \frac{1}{q} \frac{dE_i}{dx} \Rightarrow \frac{D_p}{\mu_p} = \frac{kT}{q}; \quad \frac{kT}{q} \approx 0.0258 \text{ V}$$

Similar relationship is application for electron also. The relationship  $D/\mu = kT/q$  is known as Einstein relation.

## Diffusion and Recombination; The Continuity Equation

In the previous discussions effect of recombination has been neglected. These effects must be included in a description of conduction processes, however, since recombination can cause a variation in the carrier distribution.



Consider a differential length  $\Delta x$  of a semiconductor sample with area  $A$  in the  $yz$ -plane. The hole current density leaving the volume,  $J_p(x + \Delta x)$ , can be larger or smaller than the current density entering,  $J_p(x)$ , depending on the generation and recombination of carriers taking place within the volume.

The net increase in hole concentration per unit time,  $\partial p / \partial t$ , is the difference between the hole flux per unit volume entering and leaving, minus the recombination rate. Hole current density can be converted to hole particle flux density by dividing  $J_p$  by  $q$ .

$$\left. \frac{\partial p}{\partial t} \right|_{x \rightarrow x + \Delta x} = \frac{1}{q} \frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

As,  $\Delta x \rightarrow 0$ , we can write the current change in derivative form

$$\frac{\partial p}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p}$$

The expression is called the *continuity equation* for holes. For electrons we can write

$$\frac{\partial \delta n}{\partial t} = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_p} \quad (\text{since, the electronic charge is negative})$$

When the current is carried strictly by diffusion (negligible drift),

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_p} \quad \left[ \text{As, } J_n(\text{diff.}) = q D_n \frac{\partial \delta n}{\partial x} \right]$$

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p} \quad \left[ \text{As for hole, } J_p(\text{diff.}) = -q D_p \frac{\partial \delta p}{\partial x} \right]$$

These equations are useful in solving transient problems of diffusion with recombination.