Joseph Fourier developed a technique to analyse non-sinusoidal waveforms.

Exponential form :- Fourier demonstrated that a periodic function $f(t)$ can be expressed as sum of sinusoidal functions.

As per Fourier representation,

$$f(t) = a_0 + \sum_{n=1}^{\infty} M_n \cos(n\omega_0 t + \theta_n) \quad \cdots (1)$$

where $\omega_0 = 2\pi/T_0$,

$T_0 \rightarrow$ time period

A function $f(t)$ is periodic if $f(t) = f(t+T_0)$

when $n=1$, one cycle covers T_0 seconds while $M_1 \cos(\omega_0 t + \theta_1)$ is termed as fundamental. Taking $n=2$, T_0 represents two cycles in T_0 seconds and term ~~$M_2 \cos$~~ $M_2 \cos(2\omega_0 t + \theta_2)$ is called the 2nd harmonic and so on i.e. for $n=k$, k cycles are covered in T_0 seconds and $M_k \cos(k\omega_0 t + \theta_k)$ is called the kth harmonic term.

Using Euler's identity,

$$f(t) = a_0 + \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$n \neq 0$, $c_n \rightarrow$ complex Fourier coefficients

.. find the waveform symmetry of the waveforms

$$= a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \quad \text{---} (2)$$

C_n can be evaluated as

$$\because \boxed{f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$ Exponential Form of Fourier Series

multiplying both sides by $e^{-jk\omega_0 t}$ and integrating over the interval t_1 to $(t_1 + T_0)$ we get

$$\int_{t_1}^{t_1+T_0} f(t)\, e^{-jk\omega_0 t}\, dt = \int_{t_1}^{t_1+T_0} \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \cdot e^{-jk\omega_0 t}\, dt$$

$$= C_k T_0$$

$$\because \int_{t_1}^{t_1+T_0} e^{j(n-k)\omega_0 t}\, dt = \begin{cases} 0 & \text{for } n \neq k \\ T_0 & \text{for } n = k \end{cases}$$

∴ Fourier coefficients are defined by the expression

$$C_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t)\, e^{-jn\omega_0 t}\, dt \quad \text{---} (3)$$

expression (3) represents the exponential form of Fourier series.

C_n can also be written as

$$\boxed{C_n = \frac{1}{T_0} \int_0^{T_0} f(t)\, e^{-jn\omega_0 t}\, dt}$$

Trigonometric form of Fourier Series :-

From eqn (3), we can write

$$2C_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\left[\cos n\omega_0 t - j \sin n\omega_0 t\right] dt$$

$$\text{or } 2C_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t) \cos n\omega_0 t \;- j\,\frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t)\sin n\omega_0 t\, dt \quad \text{--- (4)}$$

$\because$ C_n is complex coefficient

$$\therefore\ 2C_n = a_n - j b_n \quad \text{--- (5)}$$

Comparing eqns (4) & (5)

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t) \cos n\omega_0 t\, dt \quad \text{--- 6(a)}$$

$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} f(t) \sin n\omega_0 t\, dt \quad \text{--- 6(b)}$$

Also from eqn (3) C_0 being written as a_0 and is given by

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)\, dt \quad \text{--- (7)}$$

$a_0 \rightarrow$ average value and can be directly evaluated from the waveform.

Since the periodic function can be represented as sum of sinusoidal functions, we can rewrite the eqn (1) as below

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where $a_0 \rightarrow$ constant

$a_1, a_2, a_3 \dots a_n$ and $b_1, b_2, b_3 \dots b_n$ are amplitudes of different harmonics.

$x =$ variable and $n =$ an integer 1, 2, ...

Fourier series can also be expressed in terms of either sine or cosine terms.

Let $M = a_n \cos nx + b_n \sin nx$

$$= \sqrt{a_n^2 + b_n^2} \left[\frac{a_n}{\sqrt{a_n^2+b_n^2}} \cos nx + \frac{b_n}{\sqrt{a_n^2+b_n^2}} \sin nx \right]$$

let $\frac{a_n}{\sqrt{a_n^2+b_n^2}} = \sin \phi_n$ and $\frac{b_n}{\sqrt{a_n^2+b_n^2}} = \cos \phi_n$

$$\therefore M = \sqrt{a_n^2+b_n^2} \left[\sin\phi_n \cos nx + \cos\phi_n \sin nx \right]$$

$$M = K_n \sin(nx + \phi_n)$$

$\boxed{\text{here } \tan\phi_n = \frac{a_n}{b_n} \quad \dots (a)}$

where $K_n = \sqrt{a_n^2 + b_n^2}$

Thus finally we can write

$$f(t) = a_0 + K_1 \sin(x+\phi_1) + K_2 \sin(2x+\phi_2) + \dots + K_n \sin(nx+\phi_n) \quad \dots (B)$$

On the other hand if we put

$$\frac{a_n}{\sqrt{a_n^2+b_n^2}} = \cos\phi_n \quad \text{and} \quad \frac{b_n}{\sqrt{a_n^2+b_n^2}} = \sin\phi_n$$

then $M = \sqrt{a_n^2+b_n^2}\,[\cos\phi_n \cos n\alpha + \sin\phi_n \sin n\alpha]$

$M = K_n \cos(n\alpha - \phi_n)$

i.e. $f(t) = a_0 + K_1\cos(\alpha-\phi_1) + K_2\cos(2\alpha-\phi_n) + K_3\cos(3\alpha-\phi_n) + \cdots + K_n\cos(n\alpha-\phi_n)$

Ex: Find the period of the function

$f(t) = \cos t/3 + \cos t/4$

Soln :- since $f(t)$ is periodic

$f(t) = f(t+T)$

$\cos t/3 + \cos t/4 = \cos\frac{1}{3}(t+T) + \cos\frac{1}{4}(t+T)$

we know $\cos\theta = \cos(\theta + 2n\pi)$

on equating corresponding terms on both sides

$t/3 + 2m\pi = \frac{1}{3}(t+T)$

$\Rightarrow 2m\pi = \frac{T}{3} \Rightarrow T = 6m\pi$ and

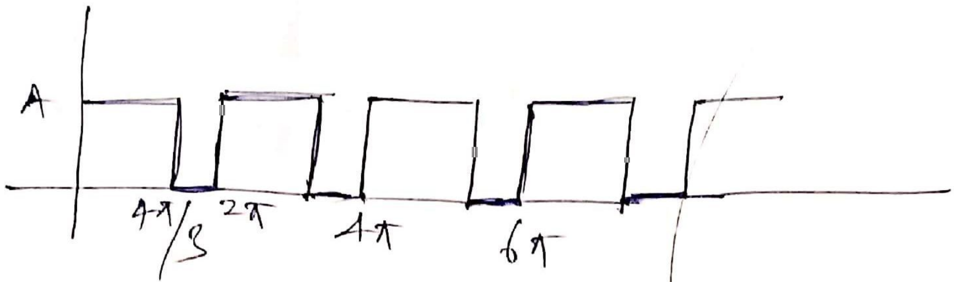
$\Rightarrow T = 6\pi, 12\pi, 18\pi, 24\pi, 30\pi, \ldots$

$\frac{t}{4} + 2n\pi = \frac{1}{4}(t+T) \Rightarrow T = 8n\pi$

$\Rightarrow T = 8\pi, 16\pi, 24\pi, 32\pi.$

therefore, smallest period, $T = 24\pi$

Ex 15.1 :- Obtain the coefficient of the exponential Fourier series for the waveform show below:



Soln :-

$$C_n = \frac{1}{T_0}\int_0^{T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{2\pi}\int_0^{4\pi/3} A\, e^{-jn\left(2\pi \times \frac{1}{T_0}\right)t}\, dt$$

$$= \frac{1}{2\pi}\int_0^{4\pi/3} A\, e^{-jnt}\, dt$$

$$= \frac{1}{2\pi}\int_0^{4\pi/3} A\, e^{-jnt}\, dt$$

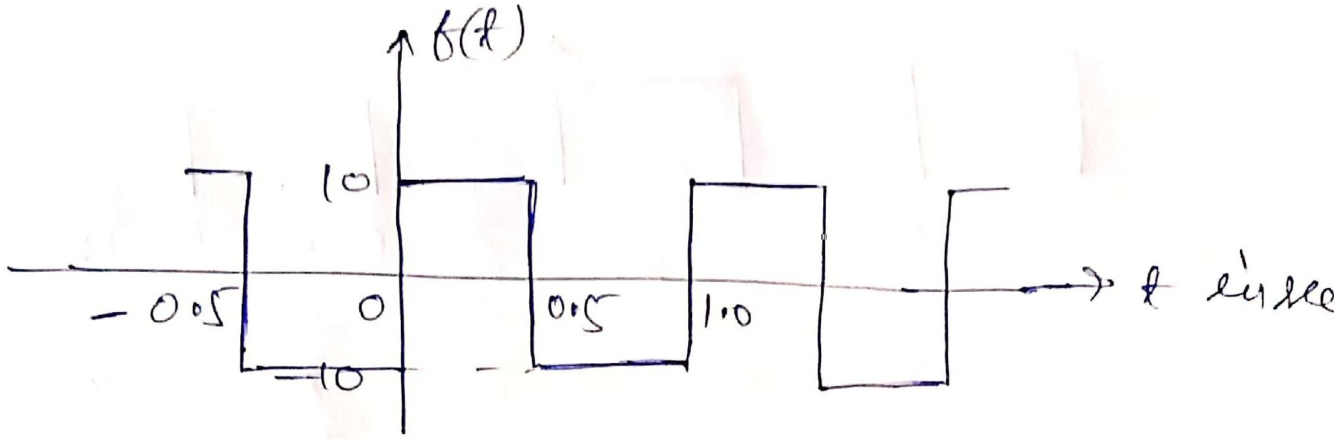
$$\cancel{\frac{jA}{2\pi n}\left[e^{-j\left(\frac{4\pi n}{3}\right)} - 1\right]}$$

$$= \frac{A}{2\pi}\int_0^{4\pi/3} e^{-jnt}\, dt$$

$$= \frac{A}{2\pi}\left[\frac{e^{-jnt}}{-jn}\right]_0^{4\pi/3}$$

$$= -\frac{A}{j2\pi n}\left[e^{-jnt}\right]_0^{4\pi/3} = \frac{jA}{2\pi n}\left[e^{-\frac{j4\pi n}{3}} - 1\right]$$

— :- Find the exponential Fourier series for the square wave below: (4)



Sol^n :- from the fig

$T = 1$ sec $\therefore \omega = \frac{2\pi}{T} = 2\pi$

Mathematical eq^n of f(t) is given by

$f(t) = 10$ for $0 < t < 0.5$

$= -10$ for $0.5 < t < 1$

The exponential series is given by

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

where $C_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega t} dt$

$$C_n = \frac{1}{1}\left[\int_0^{0.5} 10 . e^{-jn\omega t} dt + \int_{0.5}^{1.0} (-10) . e^{-jn\omega t} dt \right.$$

$$C_n = \frac{j10}{\pi n} [e^{-jn\pi} - 1]$$

$$\therefore f(t) = \sum_{-\infty}^{\infty} \frac{j10}{\pi n} [e^{-jn\pi} - 1] e^{jn\omega t} = \sum_{-\infty}^{\infty} \frac{j10}{\pi n} (e^{-jn\pi} - 1) e^{jn\omega t}$$

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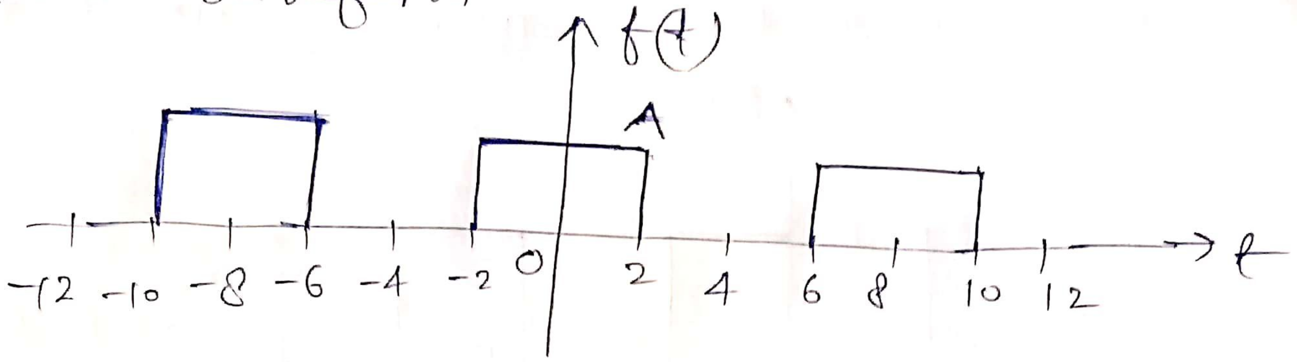
(13)

ex :- Find the exponential Fourier series for the waveform



Sol^n :- $T = 8$ $\quad \therefore \omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$

$f(t) = A$ for $-2 < t < 2$

$= 0$ for $2 < t < 6$

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega t}\, dt = \frac{1}{8}\left[\int_{-2}^{2} A e^{-jn\omega t}\, dt + \int_2^6 0\, dt\right]$$

$$= \frac{A}{2}\left[\frac{\sin n\pi/2}{n\pi/2}\right]$$

$$\therefore C_n = \frac{A}{2}\,\frac{\sin n\pi/2}{n\pi/2} \quad \text{for } n \neq 0$$

$$= A/2 \quad \text{for } n = 0$$

* Let $C_n = \frac{j5}{n\pi}$

$\therefore a_n = \frac{j5}{n\pi} + \frac{j5}{n\pi}$

for $n = 0$

$$C_0 = \frac{1}{T}\int_0^T f(t)\, e^0\, dt = \frac{1}{8}\left[\int_{-2}^{2} A\, dt + \int_2^6 0 \cdot dt\right] = \frac{A}{2}$$

$$\therefore f(t) = \sum_{-\infty}^{\infty} \frac{A}{2}\,\frac{\sin n\pi/2}{n\pi/2}\, e^{jn\pi/4\, t}$$

Relation between Trigonometric exponential Fourier coefficients :-

$a_0 = C_0$, $\quad a_n = C_n + C_{-n}$ & $b_n = j(C_n - C_{-n})$

for C_{-n} put $n = -n$ in C_n*

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Symmetry in Fourier series :- When a signal exhibit symmetrical properties, simplification procedure can be adopted in Fourier series. There are three types of symmetries :

(a) Even function symmetry
(b) Odd function symmetry
(c) Half wave symmetry

(a) **Even function symmetry** :- A function is said to be even if

$$f(-t) = f(t)$$

An even function is symmetrical about the vertical axis. In order to determine the coefficients of the Fourier series, the conditions of symmetry being applied, let $t_1 = -T_0/2$

$$\therefore \; a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} f(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{-T_0/2+T_0} f(t)\,dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\,dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{0} f(t)\,dt + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

Let us now change the variable of the first integral so that $t = -x$ in the first integral.

Then $f(-x) = f(x)$, $dt = d(-x) = -dx$ while the range of integration is from $x = T_0/2$ to 0.

$$\therefore \quad a_0 = \frac{1}{T_0}\int_{T_0/2}^{0} f(x)\,(-dx) + \frac{1}{T_0}\int_0^{T_0/2} f(t)\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} f(x)\,dx + \frac{1}{T_0}\int_0^{T_0/2} f(t)\,dt$$

$$\therefore \quad \boxed{a_0 = \frac{2}{T_0}\int_0^{T_0/2} f(t)\,dt}$$

Similarly

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{0} f(t)\cos n\omega_0 t\,dt + \frac{2}{T_0}\int_0^{T_0/2} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\int_{T_0/2}^{0} f(x)\cos(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_0 f(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{T_0}\int_0^{T_0/2} f(x)\cos(n\omega_0 x)\,dx + \frac{2}{T_0}\int_0^{T_0/2} f(t)\cos(n\omega_0 t)\,dt$$

$$\boxed{a_n = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos n\omega_0 t\,dt}$$

quency sp...

Also for the other coefficient (6) (13) (9)

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{0} f(t)\sin(n\omega_0 t)\,dt + \frac{2}{T_0}\int_0^{T_0/2} f(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}\int_{T_0/2}^{0} f(x)\sin(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_0^{T_0/2} f(t)\sin(n\omega_0 t)\,dt$$

$$= -\frac{2}{T_0}\int_0^{T_0/2} f(x)\sin(n\omega_0 x)\,dx + \frac{2}{T_0}\int_0^{T_0/2} f(t)\sin n\omega_0 t\,dt$$

$\therefore\ b_n = 0$

Thus we see that for even function symmetry

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} f t\,dt$$

$$a_n = \frac{4}{T_0}\int f(t)\cos(n\omega_0 t)\,dt$$

& $b_n = 0$

cosine wave is a even function and the sum or product of two or more even functions is a even function. with addition of a constant, the even nature is still present.

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5) Odd function symmetry :- A function is said to be odd if

$$f(-t) = -f(t)$$

we have seen that

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{0} f(t)\,dt + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

for $t = -x$, the first integral becomes $\frac{1}{T_0}\int_{T_0/2}^{0} -f(x)\,(-dx)$

$$\therefore a_0 = \frac{1}{T_0}\int_{T_0/2}^{0} -f(x)\,(-dx) + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

$$= -\frac{1}{T_0}\int_{0}^{T_0/2} f(x)\,dx + \frac{1}{T_0}\int_{0}^{T_0/2} f(t)\,dt$$

$$\therefore a_0 = 0$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{0} f(t)\cos(n\omega_0 t)\,dt + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}\int_{T_0/2}^{0} -f(x)\cos(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T_0}\int_{T_0/2}^{0} f(x)\cos(n\omega_0 x)\,dx + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos(n\omega_0 t)\,dt$$

$$= -\frac{2}{T_0}\int_{0}^{T_0/2} f(x)\cos(n\omega_0 x)\,dx + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\cos(n\omega_0 t)\,dt$$

$$\therefore a_n = 0$$

and $b_n = \frac{2}{T_0}\int_{-T_0/2}^{0} f(t)\sin(n\omega_0 t)\,dt + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin(n\omega_0 t)\,dt$ (3)

$= \frac{2}{T_0}\int_{T_0/2}^{0} -f(x)\sin(-n\omega_0 x)(-dx) + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin(n\omega_0 t)\,dt$ (9)

$= -\frac{2}{T_0}\int_{T_0/2}^{0} f(x)\sin(n\omega_0 x)\,dx + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega_0 t\,dt$

$= \frac{2}{T_0}\int_{0}^{T_0/2} f(x)\sin(n\omega_0 x)\,dx + \frac{2}{T_0}\int_{0}^{T_0/2} f(t)\sin n\omega_0 t\,dt$

$= \frac{4}{T_0}\int_{0}^{T_0/2} f(t)\sin(n\omega_0 t)\,dt$

Thus for odd symmetry

$a_0 = 0$

$a_n = 0$

& $b_n = \frac{4}{T_0}\int_{0}^{T_0/2} f(t)\sin(n\omega_0 t)\,dt$

Examples of even and odd symmetry are shown below

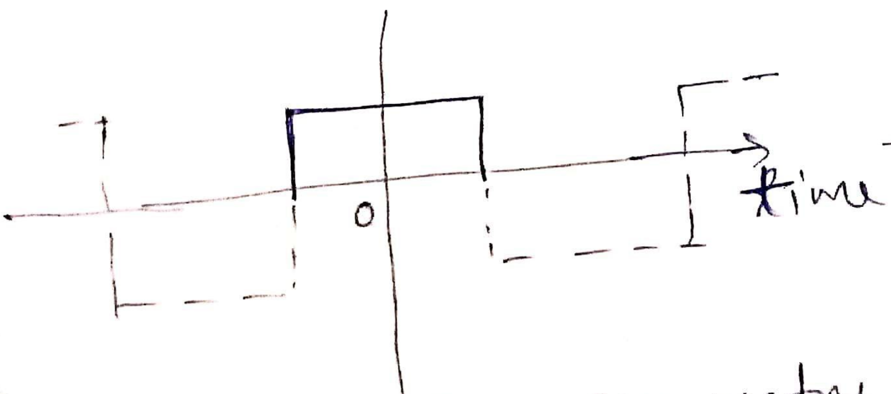


fig (a): Even symmetry

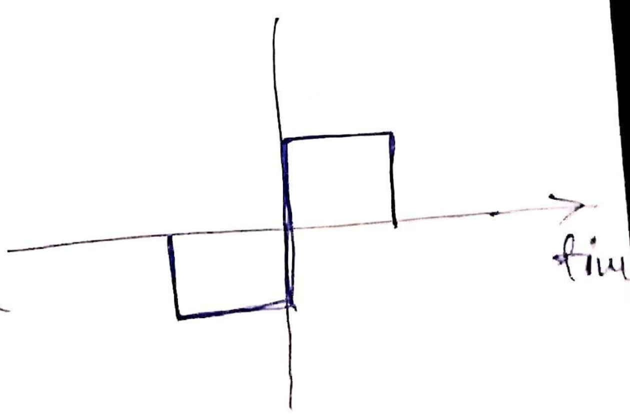
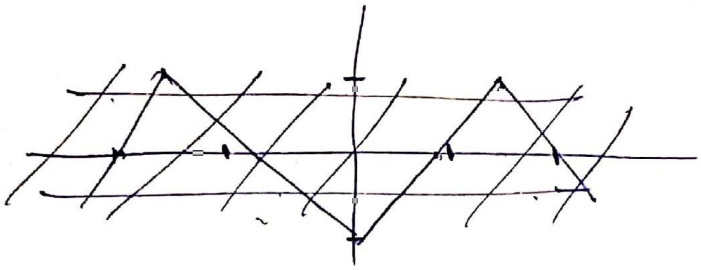


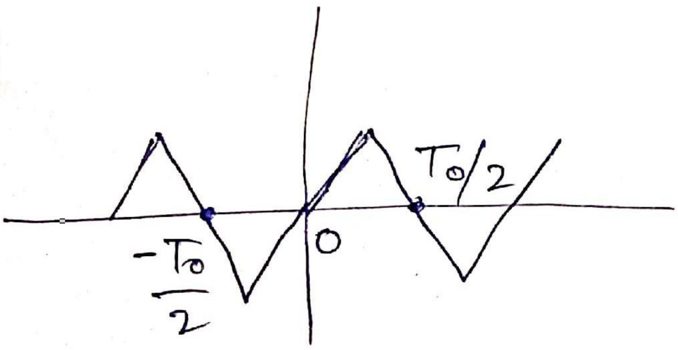
fig (b): odd symmetry

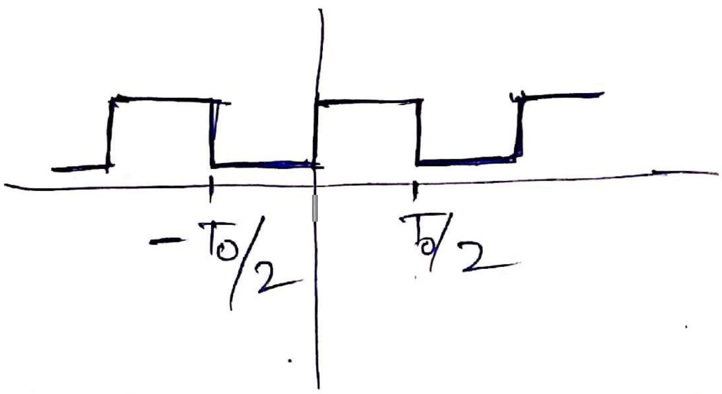
said to have half wave symmetry if

$$-f\left(t-\frac{T_o}{2}\right)=f(t)$$

i.e. the half cycle is an inverted version of the adjacent half cycle.
i.e. if the waveform from $-T_o/2$ to 0 is inverted then it becomes identical to the waveform from 0 to $T_o/2$.







Mathematically it can be shown that by changing the first variable of the Fourier coefficient expression for $t=x+T_o/2$ and

$$f(t-T_o/2)=-f(t)$$

$a_0=0$, $a_n=0$ & $b_n=0$ when n is even.

and $a_n=\frac{4}{T_o}\int_0^{T_o/2} f(t)\cos(n\omega_o t)\,dt$ when n is odd

$b_n=\frac{4}{T_o}\int_0^{T_o/2} f(t)\sin(n\omega_o t)\,dt$ when n is odd.

F[illegible] spectrum /

Q.15.2.

(8)

It may be noted that sum of two or more odd functions is an odd function, but unlike to the property of even function, the addition of a constant removes the odd nature of the function. The product of two odd functions is an even function.

Ex 15.2 :- find the waveform symmetry of the following waveforms

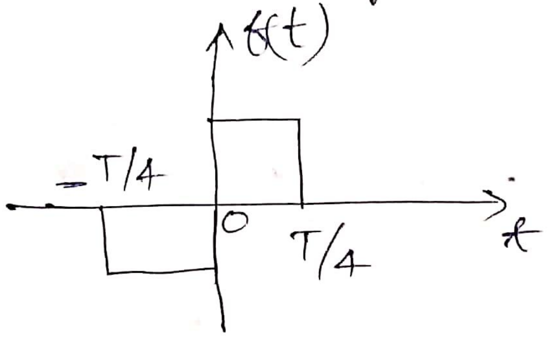


fig (a)

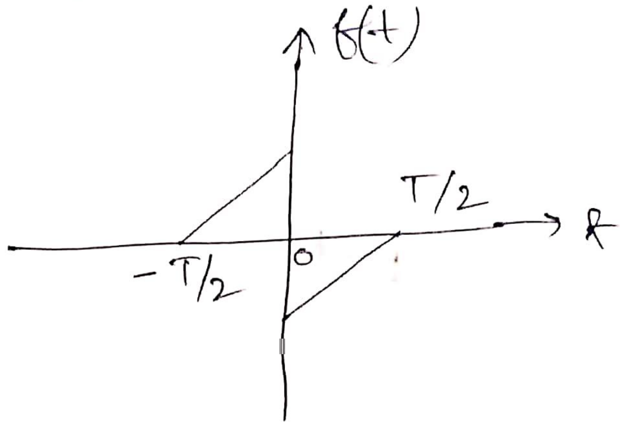


fig (b)

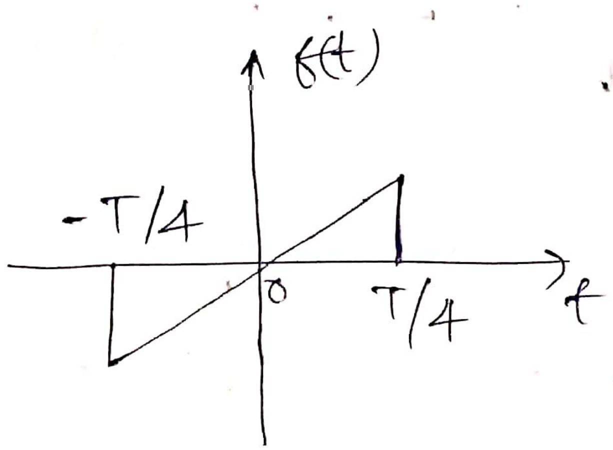


fig (c)

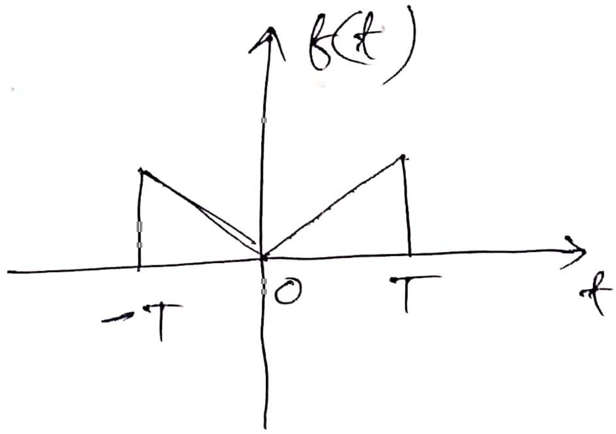


fig (d)

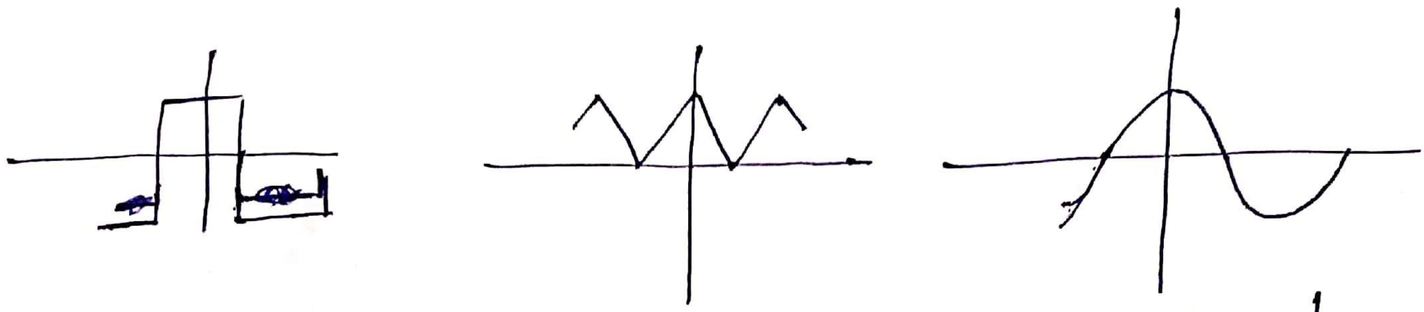
Sol^n :- By observation fig (a) is odd sym

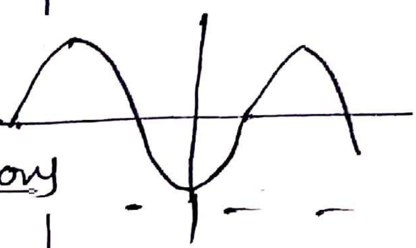
fig (b) is odd sym

fig (c) is odd sym

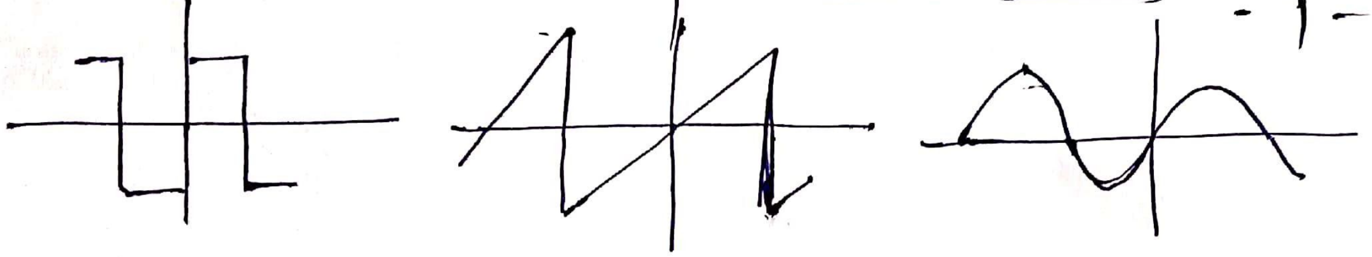
fig (d) is even symm

ex:- Identify even, odd & half wave symmetry in following:

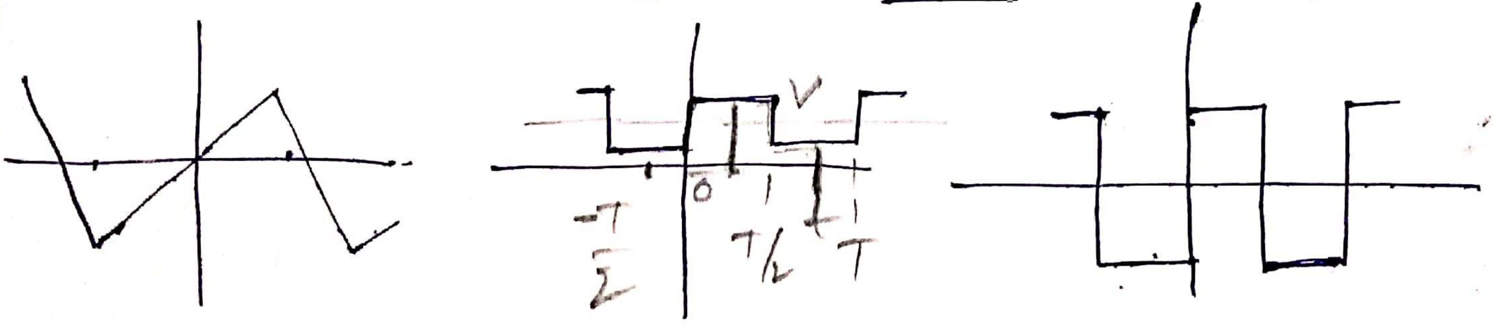




Even symm Functions



Odd symm Functions

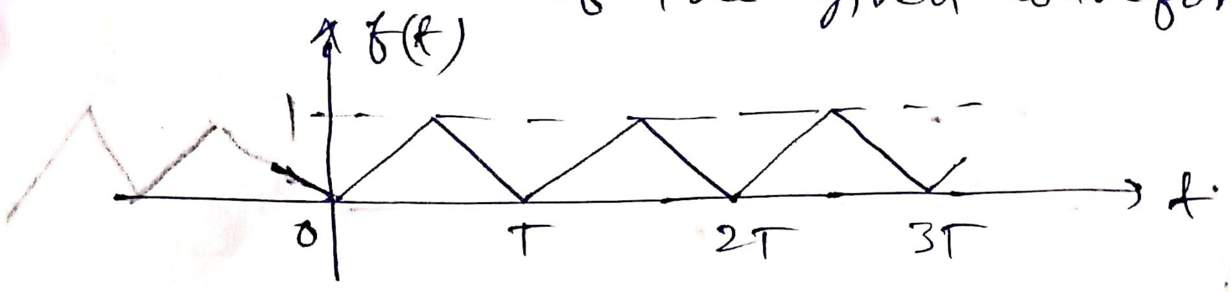


Half wave symm Functions

$$f\left(t-\frac{T}{2}\right) = -f(t)$$

$-\frac{T}{2}$ to 0 invert 0 to $\frac{T}{2}$

$\times$ 15.3 :- Find the amplitude of the 5th harmonic of the given waveform (10)



Sol^n :- since $f(-t) = f(t)$ by observation the function is even hence only cosine terms will be present

~~We k, a~~ we know, $a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\, dt$

$\cancel{\therefore\ a_5 = \frac{2}{T}\int_0^{T}}$

$f(t) =$

$n = 5, \quad \omega_0 = 2\pi/T$

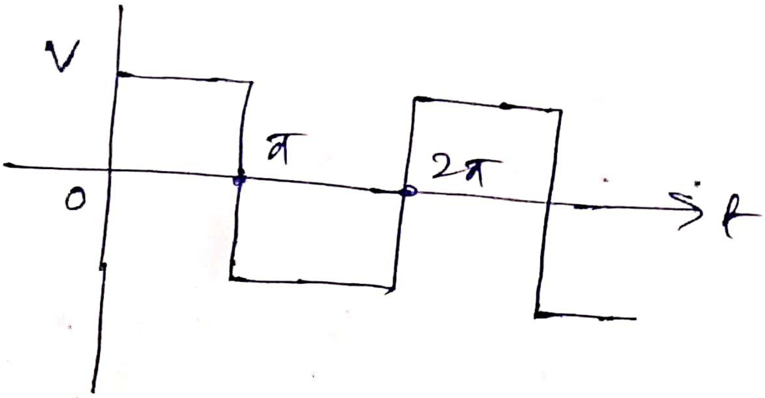
$\therefore\ a_n = \frac{4}{T}\int_0^{T/2} \frac{2t}{T}\cos\left(5.\frac{2\pi}{T}.t\right) dt$

$= \frac{4\times 2}{T\times T}\int_0^{T/2} t\cos\frac{10\pi}{T}t\ dt$

$= \frac{8}{T^2}\int_0^{T/2} t.\cos\frac{10\pi t}{T}\ dt$

$= -\frac{4}{25\pi^2}$

Ex 15.4 :- A square waveform is shown below, obtain the Fourier series.



Soln :- When waveform extended left to origin we can observe that $f(-t) = -f(t)$ hence function has odd symmetry and hence

$a_0 = a_n = 0$

$$\boxed{b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_0 t)\,dt}$$

Here $T = 2\pi$ and $f(t) = V \quad 0<t<\pi$

$= -V \quad \pi<t<2\pi$

or

$$b_n = \frac{2}{T}\int_0^{T} f(t)\sin(n\omega_0 t)\,dt$$

$$= \frac{2}{2\pi}\left\{\int_0^{\pi} V\sin(n\omega_0 t)\,dt + \int_{\pi}^{2\pi} -V\sin n\omega_0 t\,dt\right\}$$

$$= \frac{V}{\pi}\left\{\left(-\frac{\cos n\omega_0 t}{n}\right)\Big|_0^{\pi} + \left(\frac{\cos n\omega t}{n}\right)\Big|_{\pi}^{2\pi}\right\}$$

$$= \frac{V}{n\pi}\left\{-\cos n\pi + \cos 0 + \cos 2n\pi - \cos n\pi\right\}$$

$$= \frac{V}{n\pi}\left\{-\cos n\pi + 1 + 1 - \cos n\pi\right\}$$

$$= \frac{V}{n\pi}(2 - 2\cos n\pi) = \frac{2V}{n\pi}(1 - \cos n\pi)$$

$$= \frac{V}{n\pi}$$

$b_n = 2V/n\pi \,(1 - \cos n\pi)$ (11)

for $n = 1$ $\quad b_1 = 4V/\pi$

$n = 2$ $\quad b_2 = 0$

$n = 3$ $\quad b_3 = 4V/3\pi$

$n = 4$ $\quad b_4 = 0$

$n = 5$ $\quad b_5 = 4V/5\pi$.

and so on. Since $\cos n\pi = -1$ for odd & $\cos n\pi = +1$ for even value of n.

Hence the Fourier series of given wave form is

$$f(t) = \frac{4V}{\pi} \sin \omega t + \frac{4V}{3\pi} \sin 3\omega t + \frac{4V}{5\pi} \sin 5\omega t + \frac{4V}{7\pi} \sin 7\omega t + \cdots$$

$\int x \cos x \, dx$

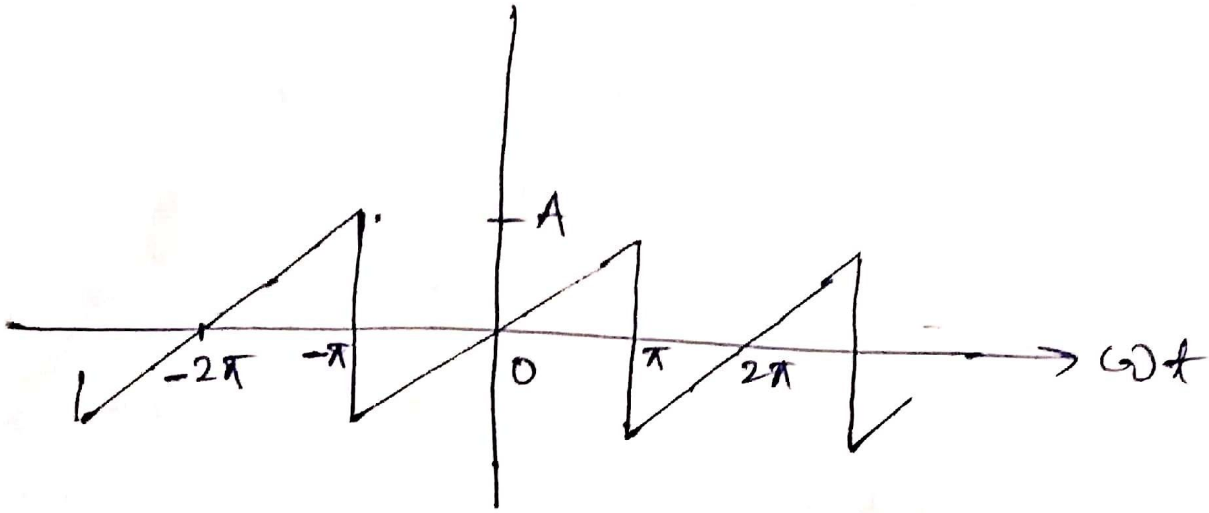
let $u = x$ $\quad$ let $\frac{dv}{dx} = \cos x$

$\frac{du}{dx} = 1$ $\quad$ $v = \sin x$

$$\therefore \int x \cos x \, dx = x \sin x - \int \sin x . 1 \, dx$$

$$= x \sin x + \cos x + c$$

Q. :- Find the Fourier expansion of the given waveform



Solⁿ :- By observation, the function has odd symmetry since $f(-t) = -f(t)$

$\therefore a_0 = a_n = 0$

also $T = 2\pi$

It may be seen that if we consider the period $0 - 2\pi$, the integration has to be carried out in two parts i.e. from 0 to π & π to 2π since function ~~that~~ is different for these two spans. On the other hand from $-\pi$ to π in one way the full period is covered and function has one form, and given by

$$f(t) = \frac{A}{\pi}(\omega t)$$

$$\therefore \quad b_n = \frac{2}{T}\int_0^T f(t)\,\sin(n\omega t)\,d(\omega t)$$ (12)

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{A}{\pi}(\omega t)\,\sin(n\omega t)\,d(\omega t)$$

$$= \frac{A}{\pi^2}\left[\frac{\sin(n\omega t)}{n^2} - \frac{\omega t}{n}\cos n\omega t\right]_{-\pi}^{\pi}$$

$$b_n = -\frac{2A}{n\pi}\cos n\pi$$

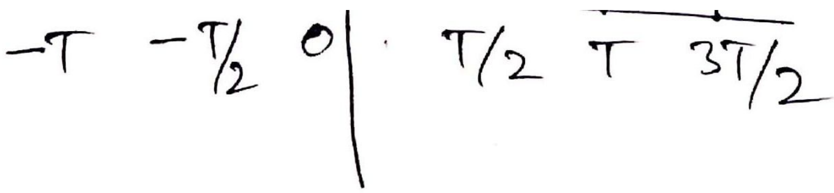
for $n=1$ $\quad b_1 = \frac{2A}{\pi}$, for $n=2$, $b_2 = \frac{-2A}{2\pi}$

for $n=3$ $\quad b_3 = \frac{2A}{3\pi}$ $\quad$ for $n=4$, $b_4 = \frac{-2A}{4\pi}$

$\therefore$ The Fourier series is

$$f(t) = \frac{2A}{\pi}\sin\omega t - \frac{2A}{2\pi}\sin 2\omega t + \frac{2A}{3\pi}\sin 3\omega t - \frac{2A}{4\pi}\sin 4\omega t + \cdots$$

$$= \frac{2A}{\pi}\left[\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \frac{1}{4}\sin 4\omega t + \cdots\right]$$



$f(t) = A \quad \text{for } 0 < t < T/2$

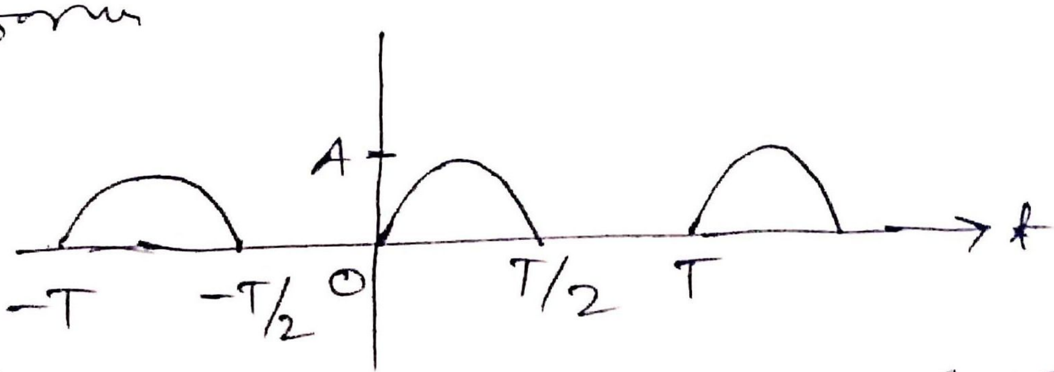
$\quad\;\; = 0 \quad \text{for } T/2 < t < T$

$a_0 = A/2$, $a_n = \frac{A}{n\pi} \sin n\pi = 0$ for any value of n

& $b_n = -\frac{A}{n\pi} [\cos n\pi - 1]$

$\therefore \; f(t) = \frac{A}{2} + \frac{2A}{\pi} \left\{ \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \cdots \right\}$

ex!- obtain Fourier series for the waveform



here $f(t) = A \sin\left(\frac{2\pi}{T} t\right) \quad \text{for } 0 \le t \le T/2$

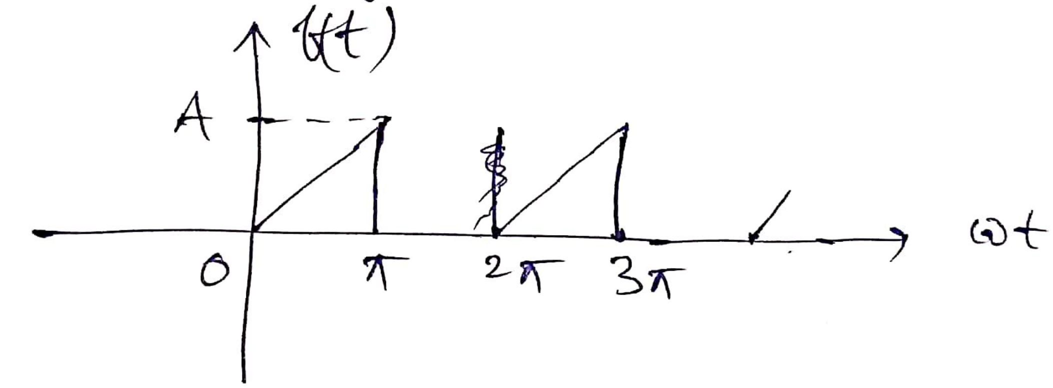
$\quad\;\; = 0 \quad \text{for } T/2 \le t \le T$

find a_0, a_n & b_n

$f(t) = A/\pi + A/2 \sin \omega t - 2A/3\pi \cos 2\omega t - \frac{2A}{15\pi} \cos 4\omega t + \cdots$

Frequency spectrum / Discrete spectra :- The plot of amplitude of harmonics versus frequency is known as discrete spectra or frequency spectrum or amplitude spectrum. The plot of phase versus frequency is known as phase spectrum. Here frequency is discrete the spectra are called line spectra and it illustrate the frequency content of the signal.

Ex 15.8 :- Find the line spectrum of the following wave using Fourier series.



Soln :- Here $T = 2\pi$ & $f(t) = \frac{A}{\pi}(\omega t)$ for $0 < \omega t < \pi$

$= 0 \quad \pi < \omega t < 2\pi$

$$\therefore a_0 = \frac{1}{T}\int_0^T f(t)\, d(\omega t)$$

$$= \frac{1}{2\pi}\left\{\int_0^{\pi} \frac{A}{\pi}(\omega t)\, d(\omega t) + \int_{\pi}^{2\pi} 0 \times d\omega t\right\}$$

$$= \frac{A}{2\pi^2} \left.\frac{(\omega t)^2}{2}\right|_0^{\pi} = \frac{A}{4\pi^2} \times \pi^2 = \frac{A}{4}$$

Fourier Trans...

operati...

$$a_n = \frac{2}{2\pi}\left\{\int_0^{\pi} \frac{A}{\pi}(\omega t)\cos n\omega t \, d\omega t\right\}$$

$$= \frac{A}{\pi^2}\left\{\int_0^{\pi} (\omega t)\cos n\omega t \, d\omega t\right\}$$

$$= \frac{A}{n\pi^2}\left[n\omega t \sin n\omega t - \int_0^{\pi} \sin n\omega t \cdot 1 \, d\omega t \right]$$

$$= \frac{A}{n\pi^2}\left[n\omega t \sin n\omega t + \frac{1}{n}\cos n\omega t \Big|_0^{\pi} \right]$$

$$= \frac{A}{n\pi^2}\left[n\omega t \sin n\omega t + \frac{1}{n}\left\{\cos n\pi - 1\right\}\right.$$

The Fourier series is given by (14)

$$f(t) = \frac{A}{4} - \frac{2A}{\pi^2}\cos\omega_0 t - \frac{2A}{9\pi^2}\cos 3\omega_0 t - \frac{2A}{25\pi^2}\cos 5\omega_0 t$$

$$- \ldots\ldots + \frac{V}{\pi}\sin\omega_0 t - \frac{V}{2\pi}\sin 2\omega_0 t + \frac{V}{3\pi}\sin 3\omega_0 t - \ldots\ldots$$

Here $C_0 = |a_0| = \frac{A}{4}$

for n = odd numbers the series contains both sine and cosine terms. Thus for n = odd

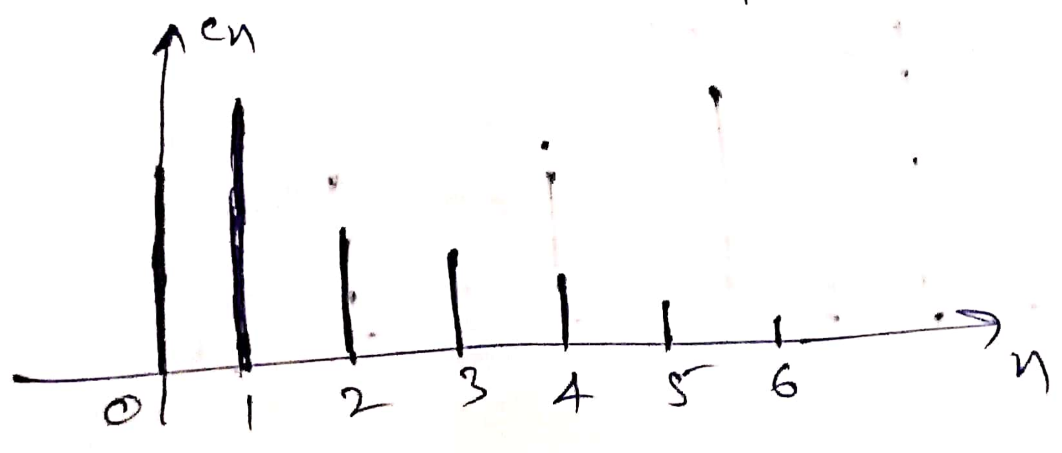
$$c_n = \sqrt{a_n^2 + b_n^2}$$

$$\Rightarrow C_1 = \sqrt{\left(\frac{2A}{\pi}\right)^2 + \left(\frac{V}{\pi}\right)^2}$$

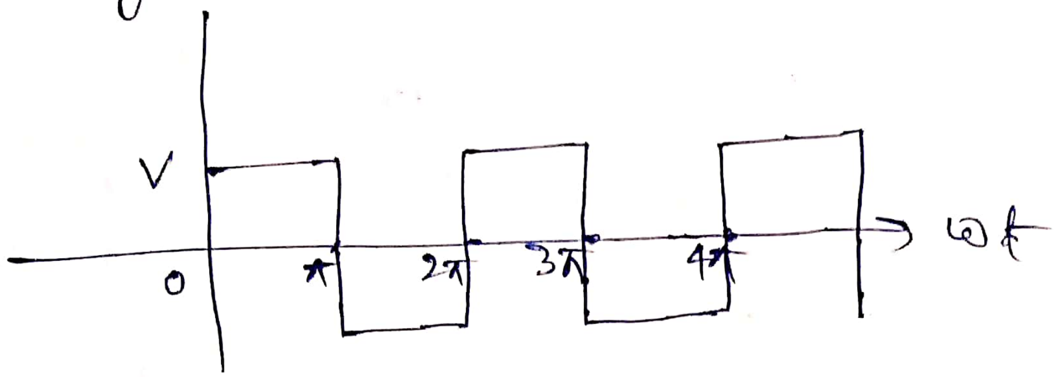
$$C_3 = \sqrt{\left(\frac{2A}{9\pi^2}\right)^2 + \left(\frac{V}{3\pi}\right)^2}$$

and for n = even there is no cosine term for n even hence the line spectrum amplitude is given by directly $b_n = \frac{V}{n\pi}$

Hence the line spectrum is shown below



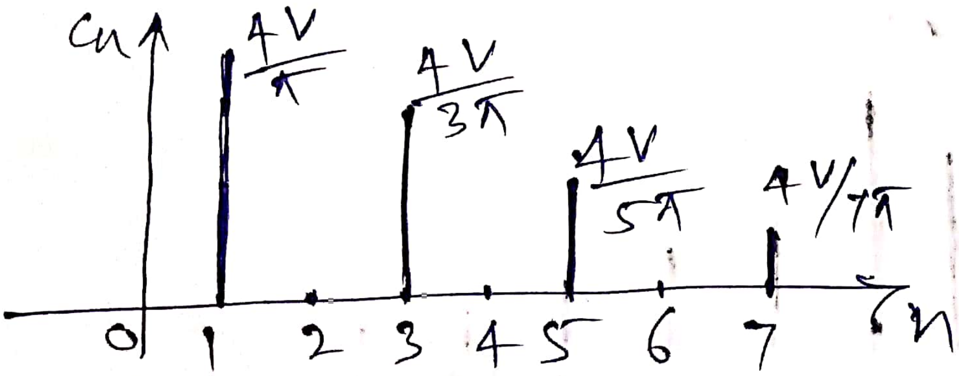
Ex :- Draw line spectrum for the following waveform



Soln :- Fourier series for above waveform is

$$f(t) = \frac{4V}{\pi}\sin\omega t + \frac{4V}{3\pi}\sin 3\omega t + \frac{4V}{5\pi}\sin 5\omega t + \ldots\ldots$$

Here constant term (a_0) and cosine term (a_n) are zero ~~...~~ and b_n has values for only odd values of n. Hence the series contains only odd harmonics given by $\frac{4V}{n\pi}$ which is given below



Average Value of Periodic complex wave :- (15)

Let us assume a periodic complex wave (non sinusoidal)

$$e = E_0 + E_{max_1} \sin(x+\phi_1) + E_{max_2} \sin(x+\phi_2) + \cdots + E_{max_n} \sin(x+\phi_n)$$

represented in fourier series where $x=\omega t$ and e is the instantaneous voltage & E_0 constant value.

we know average value of periodic function is

$$f(t)_{av} = \frac{1}{T}\int_0^T f(t)\,dt,$$

i.e. $$E_{av} = \frac{1}{T}\int_0^T e\,dx$$

We also know that integration of any sine function between 0 to T is zero

$$\therefore E_{av} = E_0 + 0 + 0 + 0 + \cdots$$

$$= E_0$$

Hence for a periodic complex wave, the average value is given by const term. This is true for ~~voltage~~, current and any other periodic complex wave.

RMS value / Effective value of periodic complex wave :- The rms value of periodic function is

$$x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

in our case periodic non sinusoidal complex voltage wave is represented by

$$e = E_0 + E_{max1}\sin(\alpha+\phi_1) + E_{max2}\sin(\alpha+\phi_2) + \cdots + E_{maxn}\sin(\alpha+\phi_n)$$

$$\therefore E_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[E_0^2 + E_{max1}^2\sin^2(\alpha+\phi_1) + \cdots + E_{maxn}^2\sin^2(\alpha+\phi_n)\right]d\alpha}$$

Let us represent

$E_{maxn}^2 \sin^2(n\alpha+\phi_n)$ as

$$P(\alpha) = E_{mn}\sin(n\alpha+\phi) \times E'_{mn}\sin(n\alpha+\theta)$$

$$= E_{mn}E'_{mn}\sin(n\alpha+\phi)\sin(n\alpha+\theta)$$

$$= \frac{E_{mn}E'_{mn}}{2}\left\{\cos(\phi-\theta) - \cos(2n\alpha+\phi+\theta)\right\}$$

$$\therefore \frac{1}{2\pi}\int_0^{2\pi} P(\alpha)\,d\alpha = \frac{E_{mn}E'_{mn}}{2}\cos(\phi-\theta)$$

$\because$ α is only variable

& $\int_0^{2\pi}\cos(2n\alpha+\phi+\theta) = 0$

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with $E_{mn} = E'_{mn}$ & $\phi = \theta$ (16)

$$\frac{1}{2\pi}\int_0^{2\pi} p(\alpha)\, d\alpha = \frac{E_{mn}^2}{2}$$

$$\therefore E_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[E_0^2 + E_{max1}^2 \sin^2(\alpha+\phi_1) + \cdots + E_{max\,n}^2 \sin^2(n\alpha+\phi_n)\right] d\alpha}$$

~~But~~ $\cancel{\dfrac{E_{max\,n}^2}{2}}$

But we have just proved

$$\frac{1}{2\pi}\int_0^{2\pi} E_{max}^2 \sin^2(n\alpha+\phi_n)\, d\alpha = \frac{E_{max\,n}^2}{2}$$

for $n = 1, 2, 3, \cdots n$

$$\therefore E_{rms} = \sqrt{E_0^2 + \frac{E_{max1}^2}{2} + \frac{E_{max2}^2}{2} + \cdots + \frac{E_{max\,n}^2}{2}}$$

$$= \sqrt{E_0^2 + \left(\frac{E_{max1}}{\sqrt{2}}\right)^2 + \left(\frac{E_{max2}}{\sqrt{2}}\right)^2 + \cdots + \left(\frac{E_{max\,n}}{\sqrt{2}}\right)^2}$$

$$E_{rms} = \sqrt{E_0^2 + E_1^2 + E_2^2 + \cdots + E_n^2}$$

where $E_1, E_2, E_3, \cdots E_n$ are the rms values of the harmonic components of the wave. This true for voltage, current or any other periodic wave.

we have seen that a non-sinusoidal complex wave (say a voltage wave) can be expressed as

$$e = E_0 + E_{max1} \sin(\alpha + \phi_1) + E_{max2} \sin(2\alpha + \phi_2)$$
$$E_{max3} \sin(3\alpha + \phi_3) + \cdots + E_{maxn} \sin(n\alpha + \phi_n) \quad \text{--- (1)}$$

where $e \rightarrow$ instantaneous voltage

$E_0 \rightarrow$ average voltage/constant value

$n \rightarrow$ integer

$\alpha = \omega t$

$E_{max1}, E_{max2}, E_{max3} \cdots$ peak amplitudes of various harmonics

$\phi_1, \phi_2, \cdots \phi_n \rightarrow$ angles measured from origin to the point of crossing from negative to positive value of component wave.

Similarly non-sinusoidal current wave expressed as

$$i = I_0 + I_{max1} \sin(\alpha + \phi_1 + \psi_1) + I_{max2} \sin(2\alpha + \phi_2 + \psi_2)$$
$$+ I_{max3} \sin(3\alpha + \phi_3 + \psi_3) + \cdots + I_{maxn} \sin(n\alpha + \phi_n + \psi_n) \quad \text{--- (2)}$$

where $\psi_n \rightarrow$ represents the phase angle between the n^{th} harmonic voltage and corresponding current wave.

The average power is given by (17)

$$P = \frac{1}{2\pi}\int_0^{2\pi} e \cdot i \, d\alpha \quad \text{--- (3)}$$

Equations (1) & (2) may be written as

$$e = E_0 + \sum_{n=1}^{n} E_{max\,n} \sin(n\alpha + \phi_n) \quad \text{--- (4)}$$

$$\& \quad i = I_0 + \sum_{n=1}^{n} I_{max\,n} \sin(n\alpha + \phi_n + \psi_n) \quad \text{--- (5)}$$

$\therefore$ from eqn (3)

$$P = \frac{1}{2\pi}\int_0^{2\pi} \Big[\Big\{E_0 + \sum_{n=1}^{n} E_{max\,n} \sin(n\alpha + \phi_n)\Big\} \times \Big\{I_0 + \sum_{n=1}^{n} I_{max\,n} \sin(n\alpha + \phi_n + \psi_n)\Big\}\Big] d\alpha$$

$$P = \frac{1}{2\pi}\int_0^{2\pi} \Big[E_0 I_0 + E_0 \sum_{n=1}^{n} I_{max\,n} \sin(n\alpha + \phi_n + \psi_n) + I_0 \sum_{n=1}^{n} E_{max\,n} \sin(n\alpha + \phi_n) + \sum_{n=1}^{n} E_{max\,n} \sin(n\alpha + \phi_n) \sum_{n=1}^{n} I_{max\,n} \sin(n\alpha + \phi_n + \psi_n)\Big] d\alpha$$

$$P = E_0 I_0 + \frac{E_{max1} I_{max1}}{2} \cos\psi_1 + \frac{E_{max2} I_{max2}}{2} \cos\psi_2 + \cdots + \frac{E_{max\,n} I_{max\,n}}{2} \cos\psi_n$$

$$P = E_0 I_0 + E_1 I_1 \cos\psi_1 + E_2 I_2 \cos\psi_2 + \cdots + E_n I_n \cos\psi_n$$

Steady state Response of a network to non-sinusoidal periodic input :-

When a non-sinusoidal waveform of voltage is applied to a linear network, its Fourier analysis gives harmonic voltages. Utilizing principle of superposition, the corresponding harmonic terms of currents can be obtained. The equivalent impedance of the network at each harmonic frequency nω is used to compute the current at that harmonic.

Each term of the Fourier series of voltage is then assumed to represent as seperate source. The sum of these individual responses provides the total response of the current.

Fourier Transform :- The Fourier transform is an operation that converts a function of time into a function of frequency ω. (18)

We know that the exponential Fourier series is given by

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t} \quad \text{---} (1)$$

where $c_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega t} dt$

$$= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega t} dt \quad \text{--} (2)$$

& $\omega = \frac{2\pi}{T}$ = fundamental frequency

Now let $T \to \infty$, then

(i) $\omega = \frac{2\pi}{T}$ becomes extremely small

(ii) the limit is then represented by a differential

$\omega \to d\omega$

(iii) $\frac{1}{T} = \frac{\omega}{2\pi} \to \frac{d\omega}{2\pi}$

(iv) the frequency of any harmonic $n\omega$ must now correspond to the general frequency variable which describe the continuous spectrum. In other words n must approach infinity if ω approaches zero, so that the product is finite.

$n\omega \rightarrow \omega$

If we multiply each side of eqn ② by T and then undertake limiting process, then

$$c_n T = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \quad \text{--- ③}$$

The right side of the above expression is a function of $j\omega$ and we represent it as

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \quad \text{--- ④}$$

In order to apply the limiting process to eqn ①, we multiply and divide ~~the~~ summation by T

$$f(t) = \sum_{n=-\infty}^{\infty} c_n T\, e^{jn\omega t} \times \frac{1}{T}$$

In the limit, the summation becomes an integral and

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega \quad \text{--- ⑤}$$

eqns ④ & ⑤ are together called the Fourier transform pair. The function $F(j\omega)$ is the Fourier transform or Fourier integral of function $f(t)$ and $f(t)$ is called inverse Fourier transform of $F(j\omega)$

Note :- A non periodic signal may be assumed as a limiting case of a periodic signal where period of signal approaches infinity. We can use this approach to obtain freq representation of non periodic signal.

$n \neq 0$, c_n ... coefficient

Conditions of existence of Fourier transform :- A function $f(t)$ is said to be Fourier transferable, if it satisfies the following Dirichlet conditions : (19)

(i) $f(t)$ is absolutely integrable i.e.

$$\int_{-\infty}^{\infty} |f(t)| \, dt < \infty$$

(ii) $f(t)$ has finite no. of maxima & minima

(iii) $f(t)$ is single valued and has a finite no. of discontinuities.

Fourier Transform of some useful functions :-

i) Fourier transform of single sided exponential function :- The single sided exponential function is defined as

$$f(t) = e^{-at} \quad ; \quad 0 < t < \infty$$
$$= 0 \quad ; \quad t < 0$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) \, e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{\infty} e^{-at} \, e^{-j\omega t} \, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \, dt \qquad \left[\because f(t) = 0 \text{ for } t < 0 \right.$$

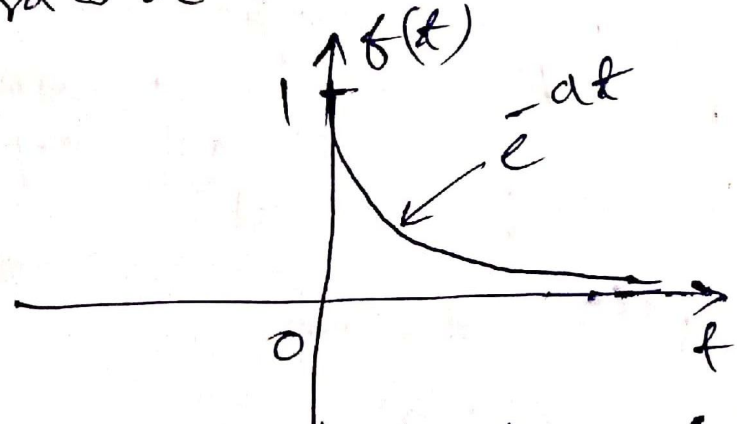
$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty} = \frac{-1}{(a+j\omega)} \left\{ e^{-\infty} - e^{0} \right\}$$

$$= -\frac{1}{(a+j\omega)} \left\{ \frac{1}{e^{\infty}} - 1 \right\} = \frac{-1}{a+j\omega} \left\{ 0 - 1 \right\}$$

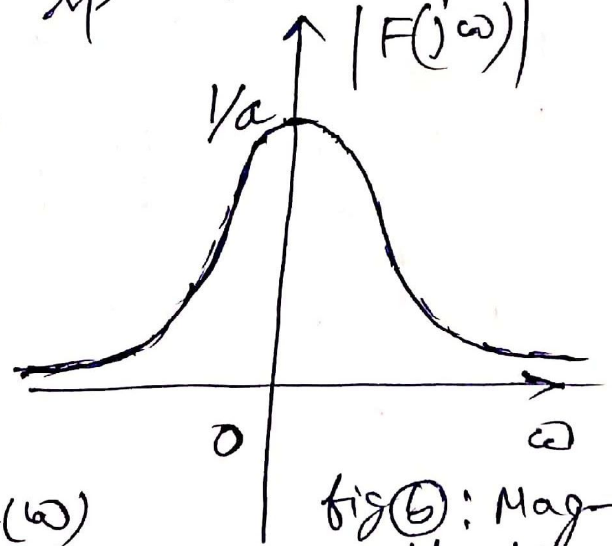
$$\cancel{F(j\omega) = \frac{1}{a+j\omega}} \Rightarrow F(j\omega) = \frac{1}{(a+j\omega)} = \frac{a-j\omega}{a^2+\omega^2}$$

$$\therefore |F(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \quad \& \quad \theta(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

In the diagram below the single-sided exponential signal, the magnitude plot & phase spectra are drawn.



fig(a): single sided exp function



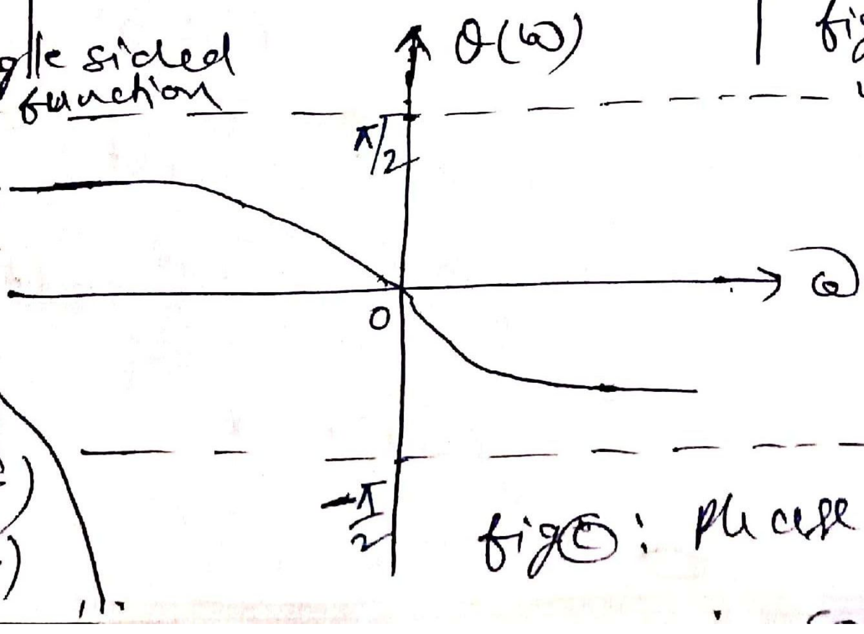
fig(b): Magnitude spectrum

To find phase angle of

$$F(j\omega) = \frac{a-j\omega}{a^2+\omega^2}$$

$$\therefore \theta(\omega) = \tan^{-1}\left(\frac{-\omega}{a}\right)$$

$$= -\tan^{-1}\left(\frac{\omega}{a}\right)$$



fig(c): Phase spectrum

Fourier transform of double sided exponential function :- The double sided exponential function is defined as

$$f(t) = e^{-at} \quad ; \quad 0 < t < \infty$$

$$= e^{at} \quad ; \quad -\infty < t < 0$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

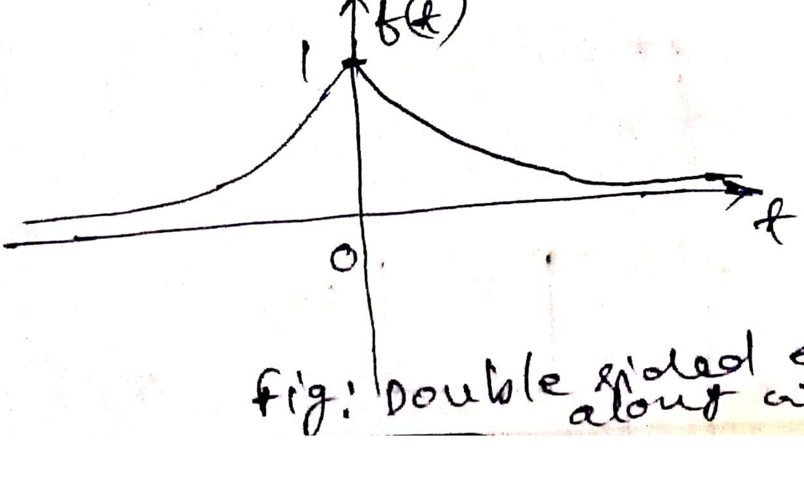
$$= \int_{-\infty}^{0} \cancel{f(t)}\, e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left. \frac{e^{(a-j\omega)t}}{a-j\omega} \right|_{-\infty}^{0} + \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$F(j\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$

$$\therefore \; |F(j\omega)| = \frac{2a}{\omega^2+a^2} \quad \& \quad \theta(\omega) = 0$$



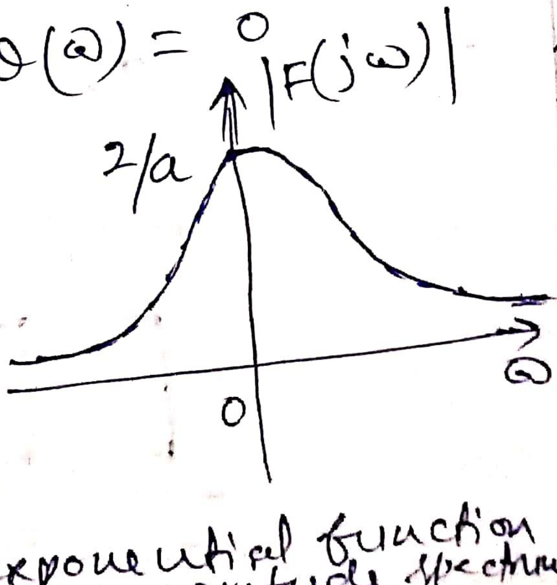


fig: Double sided exponential function along with magnitude spectrum

(iii) <u>Fourier transform of pulse or gate</u> signal or <u>function</u>: The pulse or gate function is defined as

$$f(t) = A \; ; \quad -\frac{T}{2} < t < \frac{T}{2}$$

$$= 0 \; ; \quad |t| > T/2$$

$$\therefore F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = \int_{-T/2}^{T/2} A\, e^{-j\omega t}\, dt$$

$$= \left| A \frac{e^{-j\omega t}}{-j\omega} \right|_{-T/2}^{T/2} = \frac{-A}{j\omega}\left(e^{\frac{-j\omega T}{2}} - e^{\frac{j\omega T}{2}} \right)$$

$$\therefore F(j\omega) = \frac{2A}{j\omega}\left(\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2} \right)$$

$$= \frac{2A}{\omega} \sin(\omega T/2) = AT \frac{\sin(\omega T/2)}{(\omega T/2)}$$

The gate signal, magnitude spectrum & phase spectrum are shown below:

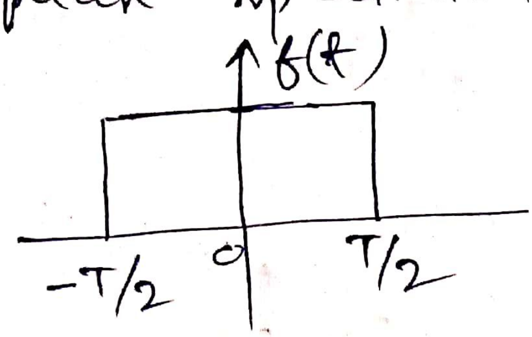


fig (a): Gate Signal

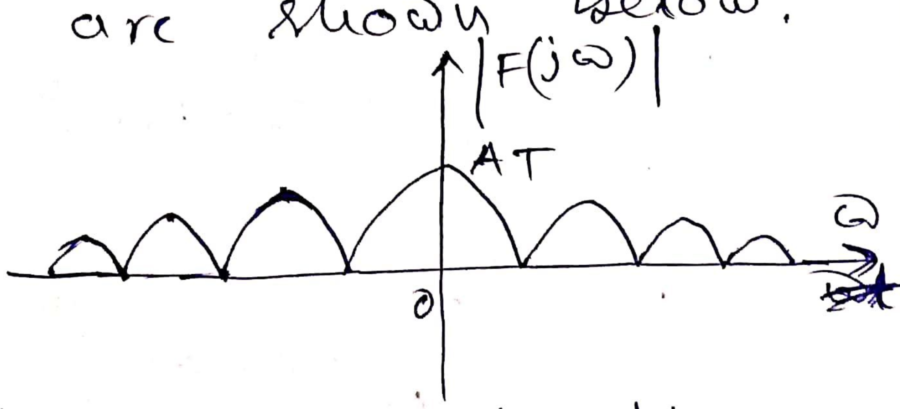


fig (b): Magnitude spectrum

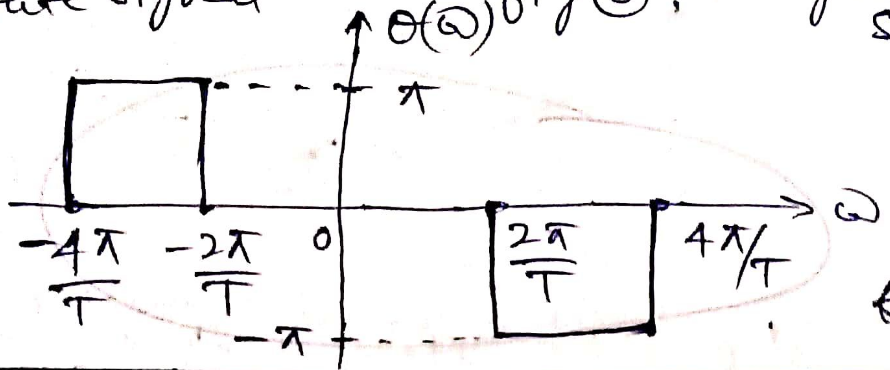


fig (c) Phase spectrum

(IV) Fourier Transform of Impulse Funcⁿ (21)

The impulse function is defined as

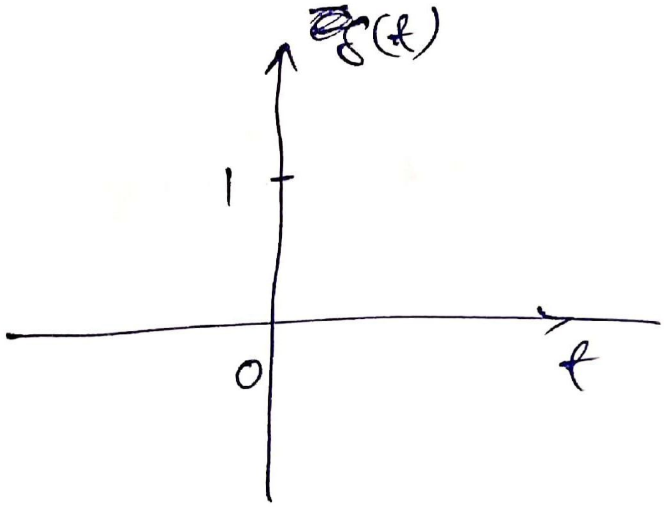
$\delta(t) = 0 \quad ; \quad t \neq 0$

$\quad\;\; = 1 \quad ; \quad t = 0$

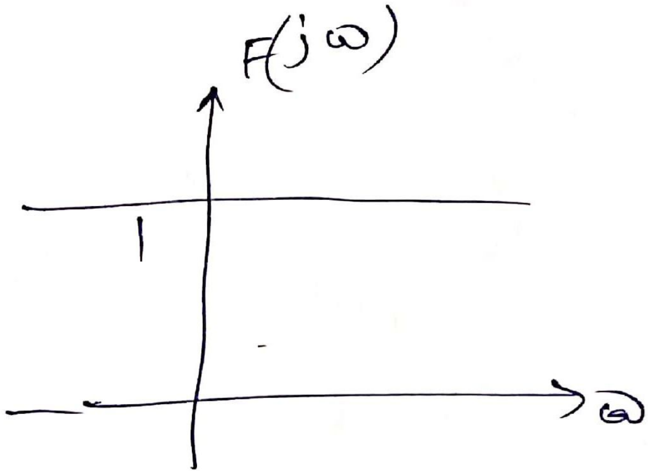
$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad$ for $t = 0$

$\therefore F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$

$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega \times 0}\, dt = 1.1 = 1$



fig(a): Impulse Signal



fig(b) Magnitude Spectrum

(V) Fourier Transform of Constant Funcⁿ

$f(t) = A \quad ; \quad -\infty \; t < \infty$

This funcⁿ is not absolutely integrable since limits are not finite

i.e. $\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$

it does not satisfies the condition

Since Fourier Transform can't be found directly, it can be evaluated ~~by~~ as a limiting value of the F.T. of the gate function as below: ①

$$f(t) = \lim_{T\to\infty} [G(t)]$$

$$\therefore F(j\omega) = FT[f(t)] = FT\left[\lim_{T\to\infty} [G(t)]\right]$$

$$= \lim_{T\to\infty} FT[G(t)]$$

$$= \lim_{T\to\infty} \left[\frac{2A}{\omega} \sin\left(\frac{\omega T}{2}\right)\right]$$

$$= \lim_{T\to\infty} AT \left(\frac{\sin \omega T/2}{\omega T/2}\right)$$

$$F(j\omega) = 2\pi A\,\delta(\omega)$$

$$\therefore |F(j\omega)| = 2\pi A$$

$$\boxed{\begin{aligned} \delta(\omega) &= \lim_{T\to\infty} \frac{\sin \omega T/2}{\omega T/2} \\ &= 1 \\ &\text{as } T\to\infty \\ &\omega \to 0 \\ \therefore \delta(\omega) &= \lim_{T\to\infty} \frac{\sin 0}{0} \end{aligned}}$$

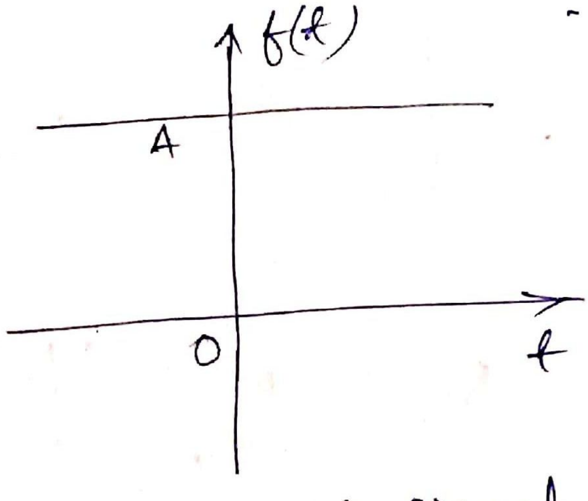


fig ⓐ: const signal

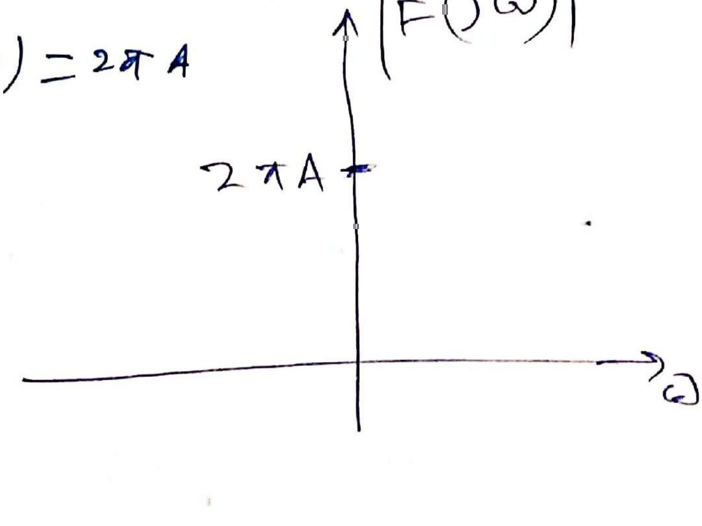


fig ⓑ: Magnitude spectrum

Fourier Transform of Signum function :- (22)

The Signum function is defined as

$$f(t) = 1 \quad ; \quad t > 0$$
$$\quad = -1 \quad ; \quad t < 0$$

or $\quad sgn(t) = u(t) - u(-t)$

$$\left[\because u(t) = 1 \; ; \; t \geq 0 \atop \quad\quad = 0 \; ; \; t < 0 \right]$$

Since the signum funⁿ is not absolutely integrable, its Fourier transform is evaluated as limiting value of FT of an exponential function.

$$f(t) = \lim_{a \to 0} \begin{cases} e^{-at} & ; \; t > 0 \\ -e^{at} & ; \; t < 0 \end{cases}$$

$$\therefore F(j\omega) = \lim_{a \to 0} \left[-\int_{-\infty}^{0} e^{at} e^{-j\omega t} \, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} \, dt \right]$$

$$= \lim_{a \to 0} \left(\frac{-1}{a - j\omega} + \frac{1}{a + j\omega} \right) = \lim_{a \to 0} \left[\frac{-2j\omega}{a^2 + \omega^2} \right]$$

$$= -\frac{2j}{\omega} = \frac{2}{j\omega}$$

$$\therefore |F(j\omega)| = \frac{2}{\omega} \quad \& \quad \theta(\omega) = -\tan^{-1}\left(\frac{2/\omega}{0}\right)$$

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0$$

$u(t)$ is not absolutely integrable.

It can be seen as

~~sig~~ $\text{sgn}(t) = 2\,u(t) - 1$

or $u(t) = \frac{1}{2}[\text{sgn}(t) + 1]$

$$FT[u(t)] = \frac{1}{2} FT[\text{sgn}(t)] + \frac{1}{2} FT[1]$$
$$= \frac{1}{2}\left(\frac{-2j}{\omega}\right) + \frac{1}{2}[2\pi\,\delta(\omega)]$$
$$= \frac{1}{j\omega} + \pi\,\delta(\omega)$$

$$\therefore F(j\omega) = FT[u(t)] = \frac{1}{j\omega} + \pi\,\delta(\omega)$$

$$\therefore |F(j\omega)| = \frac{1}{\omega} + \pi\,\delta(\omega) \quad \& \quad \theta(\omega) = -\tan^{-1}\left(\frac{1/\omega}{0}\right)$$

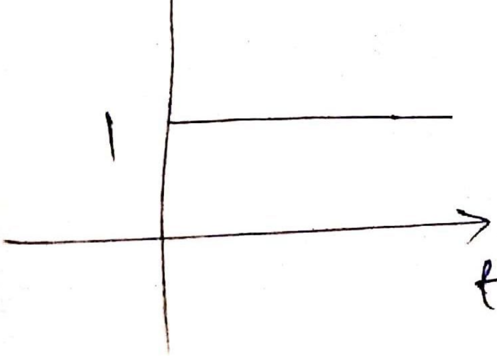


fig (a): unit step function

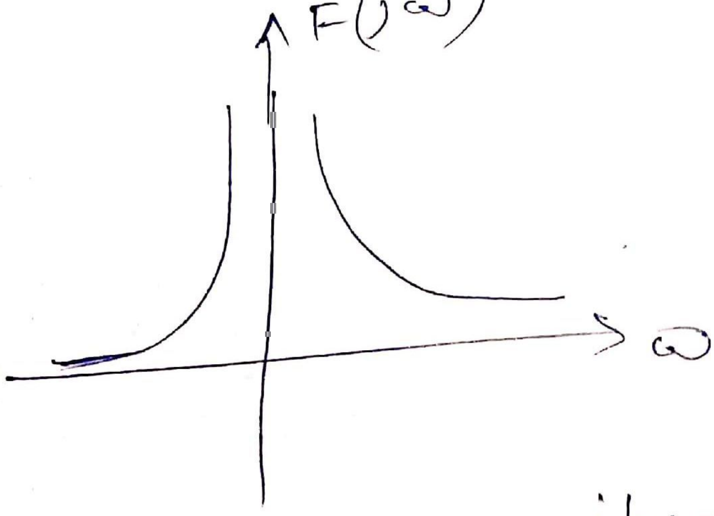


fig (b) Magnitude spectrum

Properties of Fourier Transforms: (23)

i) Linearity :- The FT of a function is a linear operation. This implies that if

$$f_1(t) \leftrightarrow F_1(j\omega)$$

$$f_2(t) \leftrightarrow F_2(j\omega)$$

Then for any constants K_1 & K_2

$$K_1 f_1(t) \pm K_2 f_2(t) = K_1 F_1(j\omega) \pm K_2 F_2(j\omega)$$

ii) Symmetry : If $f(t) \leftrightarrow F(j\omega)$ then

$$F(t) \leftrightarrow 2\pi f(-\omega)$$

iii) Time Scaling : If $f(t) \leftrightarrow F(j\omega)$ then for a real constant K, $f(Kt) \leftrightarrow \frac{1}{|K|} F\left(\frac{j\omega}{K}\right)$

(iv) Time shifting / translation in time:

If $f(t) \leftrightarrow F(j\omega)$ then $\cancel{f(t-T) \leftrightarrow F(j\omega) e^{-j}}$

$$f(t-T) \leftrightarrow F(j\omega)\, e^{-j\omega T}$$

(V) Frequency shifting / Translation in frequency

If $f(t) \leftrightarrow F(j\omega)$ then $\cancel{f(t) e^{j\omega t} \leftrightarrow F[j(\omega-\omega)]}$

$$f(t)\, e^{j\omega_0 t} \leftrightarrow F[j(\omega - \omega_0)]$$

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Ex:- Find the Fourier Transform of the following: (i) $\cos\omega_0 t$ (ii) $\sin\omega_0 t$ (iii) $f(t)\cos\omega_0 t$ (iv) $f(t)\sin\omega_0 t$.

Sol^n:- $\cos\omega_0 t = \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$

Now $1 \longleftrightarrow 2\pi\delta(\omega)$

$\therefore 1.e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega-\omega_0)$ (freq. shifting)

$\because 1.\cos\omega_0 t = \cos\omega_0 t$ & $1.e^{j\omega_0 t} = e^{j\omega_0 t}$

$\therefore \cos\omega_0 t \longleftrightarrow \frac{1}{2}\left[2\pi\delta(\omega-\omega_0) + 2\pi\delta(\omega+\omega_0)\right]$

$\longleftrightarrow \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$

(ii) $\sin\omega_0 t = \frac{1}{2j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right]$

Now $1 \longleftrightarrow 2\pi\delta(\omega) \Rightarrow 1.e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega-\omega_0)$

$\therefore \sin\omega_0 t \longleftrightarrow \frac{1}{2j}\left[2\pi\delta(\omega-\omega_0) - 2\pi\delta(\omega+\omega_0)\right]$

$\longleftrightarrow j\pi\left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right]$

(iii) $f(t)\cos\omega_0 t = \frac{1}{2}\left[f(t)e^{j\omega_0 t} + f(t)e^{-j\omega_0 t}\right]$

According to freq shifting property

$f(t)\cos\omega_0 t \longleftrightarrow \frac{1}{2}\left[FT\left[j(\omega-\omega_0)\right] + FT\left[j(\omega+\omega_0)\right]\right]$

(iv) $f(t)\sin\omega_0 t = \frac{1}{2j}\left[f(t)e^{j\omega_0 t} - f(t)e^{-j\omega_0 t}\right]$

According to freq property

$f(t)\sin\omega_0 t \longleftrightarrow \frac{1}{2j}\left[FT\left[j(\omega-\omega_0)\right] - FT\left[j(\omega+\omega_0)\right]\right]$

Three phase unbalanced circuits (loads) :- (24)

When the loads in all the three phases of either star or delta connection are not identical to each other, they are called unbalanced loading. The phase current in delta and phase or line current in star differ in unbalanced loading giving rise to flow of neutral current.

At star point

$$I_R + I_Y + I_B = I_N \quad \text{for unbalanced loading}$$

& $$I_R + I_Y + I_B = 0 \quad \text{for balanced loading}$$

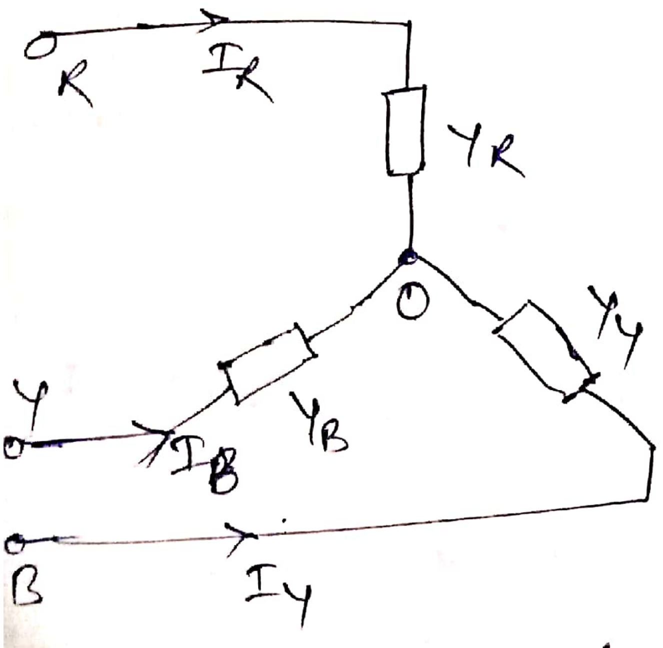


fig (a) : unbalanced loading

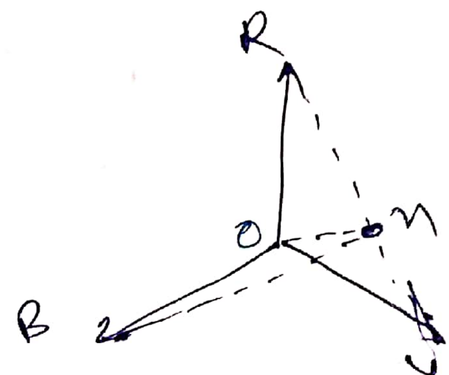


fig (b) Neutral Voltage

At ~~Star~~ star point O

$$I_R + I_Y + I_B = V_{RO} Y_R + V_{YO} Y_Y + V_{BO} Y_B$$
$$= (V_{RN} - V_{ON}) Y_R + (V_{YN} - V_{ON}) Y_Y + (V_{BN} - V_{ON}) Y_B$$
$$= V_{RN} Y_R + V_{YN} Y_Y + V_{BN} Y_B - V_{ON}(Y_R + Y_Y + Y_B)$$

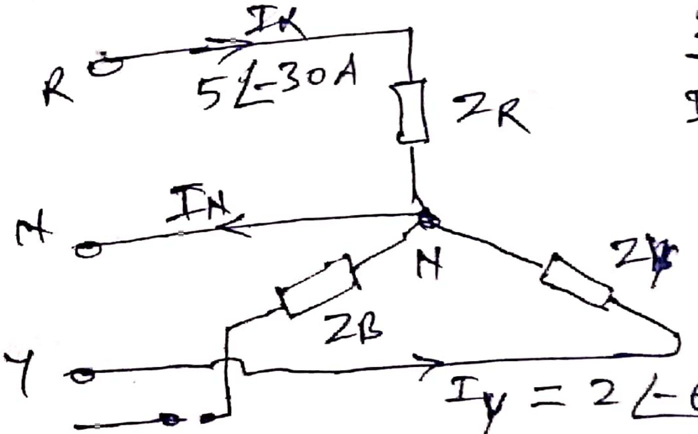
However, KCL gives $I_R + I_Y + I_B = 0$

$$\Rightarrow V_{RN} Y_R + V_{YN} Y_Y + V_{BN} Y_B - V_{ON}(Y_R + Y_Y + Y_B) = 0$$

$$\Rightarrow V_{ON} = \frac{V_{RN} Y_R + V_{YN} Y_Y + V_{BN} Y_B}{Y_R + Y_Y + Y_B}$$

V_{ON} represents the neutral shift.

CKT.30 :- In fig below, phase B is open, input currents of phase R & Y being given, find the neutral current, draw phasor diagram



Sol^n :-

$$I_N = I_R + I_Y + I_B$$
$$= 5\angle -30 + 2\angle -60 + 0$$
$$= 5.33 - j4.22$$
$$= 6.81\angle -38.45^\circ \text{ A}$$