Unit-II Applications of Theorems in AC circuits

Thevenin's Theorem :- Any two terminal bilateral linear dc circuit can be replaced by an equivalent ckt consisting of a voltage source and a series resistor.

In AC applications the Thevenin's theorem is changed only to include impedance in place of resistance. Since reactances of the ckt are frequency dependent the Thevenin's eqvt. ckt found for a particular network is applicable only at a specified frequency. Also voltage or current phasors in place of simply real (scalar) voltage or current for independent and dependent sources (with a control variable not in the network under study)

For dependent sources where control variable is governed by the network variable any of the two methods below are used to find the Thevenin's impedance.

Method 1 :- Remove the load impedance, V_{oc} is obtained, next short the load terminals and I_{sc} is determined

Then $Z_{int} = \dfrac{V_{oc}}{I_{sc}}$

<u>Method 2</u> :- Open ckt the load impedance, a source voltage(V) is applied. The source current (I) entering through the load terminals is determined, the original source voltage of the network being set to zero.

The Thevenin's impedance is then given by

$$Z_{int} = \frac{V}{I}$$

where $V = V_{oc}$

Same procedure is followed for finding Norton's equit ckt. The Norton's & Thevenin's equit ckts can also be found from each other by using source transformation as explained below:

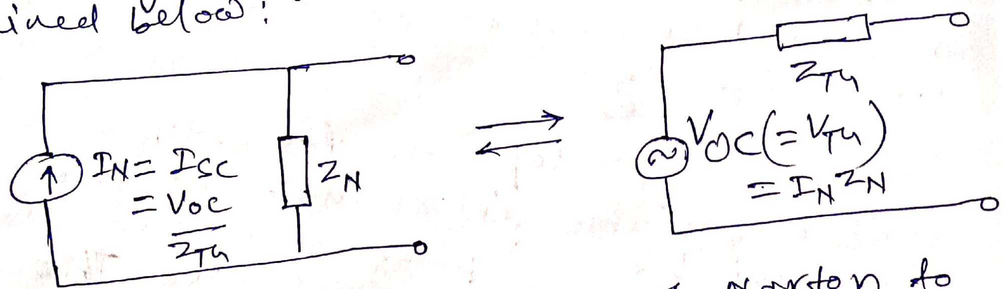
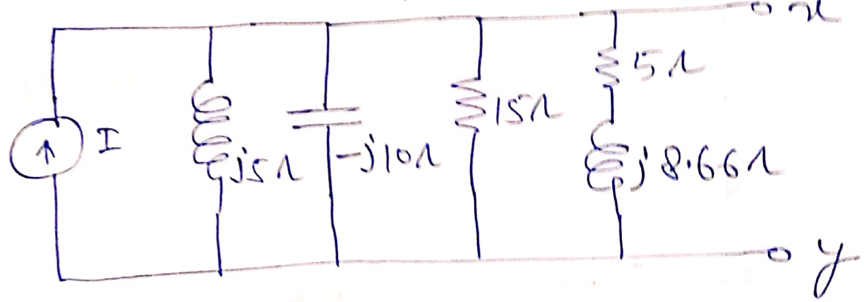


Fig: conversion of Norton to Thevenin and vice-versa.

Ex 12.19 :- If $I = 33\angle -13^\circ$ A, find the Thevenin's equivalent circuit to the left of terminals x-y in the network below.



Soln :- First find equt impedance across current source, let equt admittance to be Y_{eq}, we can write

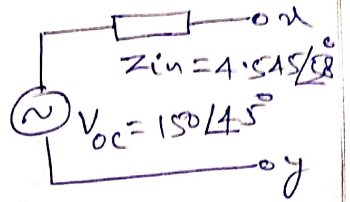
$$Y_{eq} = Y_1 + Y_2 + Y_3 + Y_4$$

where $Y_1 = \frac{1}{j5} = -j\,0.2$ mho

$Y_2 = \frac{1}{-j10} = j\,0.1$ mho

$Y_3 = \frac{1}{15} = 0.067$ mho.

& $Y_4 = \frac{1}{5 + j8.66} = \frac{1}{10\angle 60^\circ} = (0.05 - j\,0.0866)$ mho



$\therefore Y_{eq} = -j\,0.2 + j\,0.1 + 0.067 + 0.05 - j\,0.0866$

$= (0.117 - j\,0.1866)$ mho $= 0.22\angle -58^\circ$

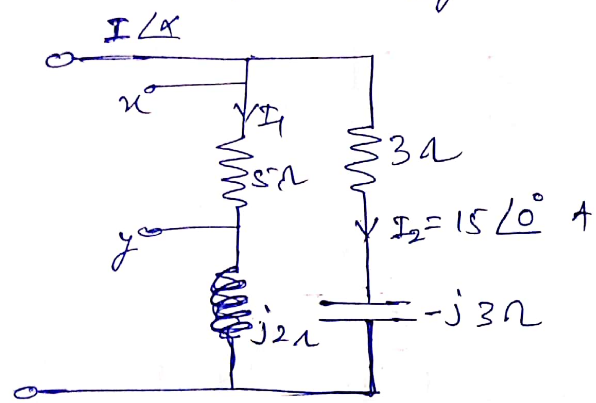
Then $V_{x\text{-}y} = V_{oc} = I \cdot \frac{1}{Y_{eq}} = \frac{33\angle -13^\circ}{0.22\angle -58^\circ}$

$= 150\angle 45^\circ$ V

To find $Z_{in}(=Z_{th})$ open the current source & by inspection $Z_{th} = 1/Y_{eq} = \frac{1}{0.22\angle -58} = 4.545\angle 58^\circ$

Ex 12.2 :- Using Thevenin's theorem, find Thevenin's equt ckt of the netw across $x-y$ in fig below:

$I \angle \alpha$



Soln :- Given $I_2 = 15\angle 0°$ A

drop across $(3-j3)\Omega$ is

$$V_{drop} = (3-j3) \times 15\angle 0° = 63.6\angle -45° \text{ V}$$

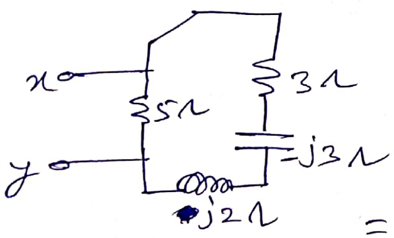
$$\therefore I_1 = \frac{V_{drop}}{5+j2} = \frac{63.6\angle -45°}{5+j2} = 11.8\angle -66.8$$

$$= (4.64 - j10.85) \text{ A}$$

$$V_{oc} (= V_{x-y}) = 5\,(4.64 - j10.85)$$

$$= 59\angle -66.8 \text{ V}$$

In order to find Zint, open ckt the current source, then find Zin looking into the terminals $x-y$ as below

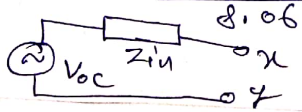


$$\therefore Z_{in} = 5\Omega \,||\, (3-j3+j2)$$

$$= \frac{5 * (3-j3+j2)}{5+3-j3+j2} = \frac{5(3-j)}{8-j.1}$$

$$= \frac{15.81\angle -18.42}{8.06\angle -7.125} = 1.96\angle -11.29°$$

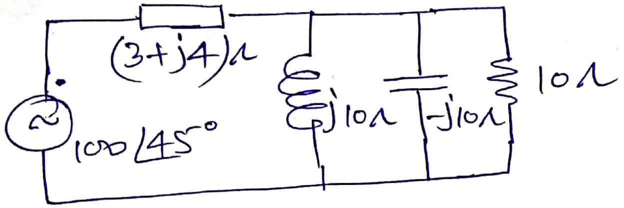
$\equiv$



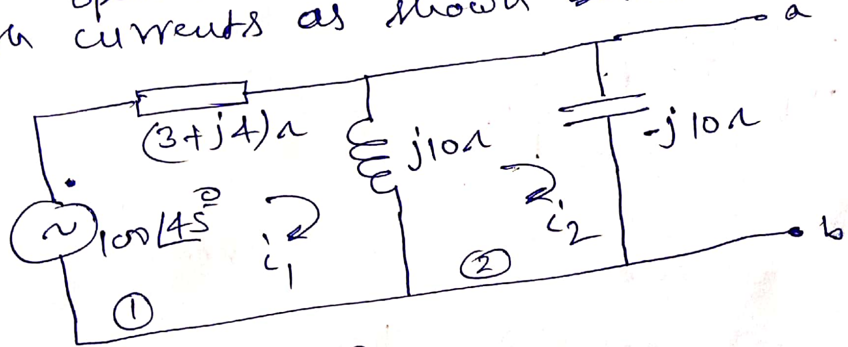
$\therefore V_{oc} = 59\angle -66.8°$

& $Z_{in} = 1.96\angle -11.29°$

12.21 :- Find the current through 10Ω resistor using Thevenin's theorem in below N/w. (25)



Solⁿ :- Remove the load resistance (10Ω), let open ckt voltage be V_{oc} and assume mesh currents as shown below



KVL in mesh 1

$100\angle 45 - i_1(3+j4) - j10(i_1 - i_2) = 0$

$100\angle 45 - i_1(3+j4) - j10 i_1 + j10 \cdot i_2 = 0$

$- i_1(3+j4) \ldots = 100\angle 45^\circ$ --- (1)

$\Rightarrow (3+j14) i_1 - j10 i_2 = 100\angle 45^\circ$ --- (1)

Similarly in 2

$-(-j10) i_2 - j10(i_2 - i_1) = 0$ ~~$\Rightarrow j10 i_2 - j10 i_2 = 0$~~ ~~$\Rightarrow i_2 = 0$~~

$j10 i_2 - j10 i_2 + j10 i_1 = 0$

$\Rightarrow i_1 = 0$ --- (2)

put in eqⁿ (1)

$-j10 i_2 = 100\angle 45^\circ$

or $i_2 = -\frac{10}{j}\angle 45 = 10j\angle 45^\circ$ check ?

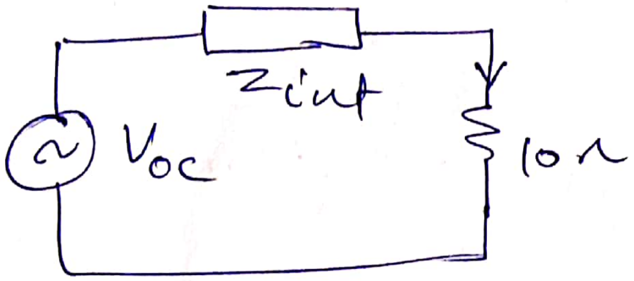
$\underline{\underline{10\angle 135^\circ A}}$

$$V_{a-b} (= V_{oc}) = -j10 \times 10 \angle 45 = j100 \angle 45$$

Internal impedance

$$Y_{int} = \frac{1}{-j10} + \frac{1}{j10} + \frac{1}{3+j4}$$

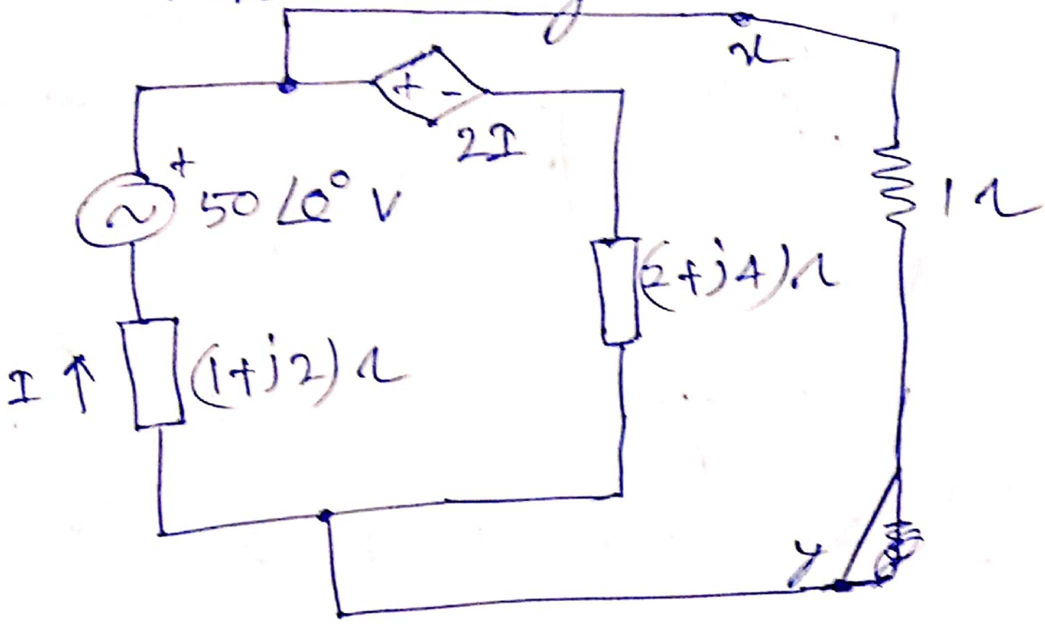
$$\therefore Z_{int} = \frac{1}{Y_{int}} = (3+j4)\,\Omega$$



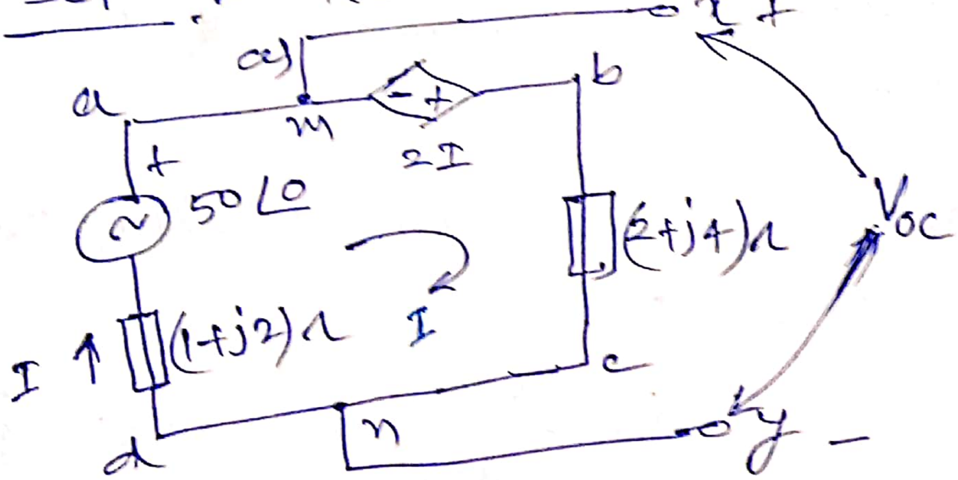
$\therefore$ current in 10Ω resistor

$$= \frac{V_{oc}}{Z_{int}+10} = \frac{100 \angle 45}{3+j4+10} = 7.35 \angle 28.3^{\circ}\,A$$

Ex 12.31 :- Find Thevenin's equit network of the ckt below across x-y and find the current through the 1Ω resistor. Verify the result using Norton's theorem.



Sol^n :- Remove 1Ω resistor, the ckt become



Applying KVL in loop abcda

$$50 + 2I - I(1+j2+2+j4) = 0$$

$$50 + 2I - I(3+j6) = 0 \Rightarrow 50 - I(1+j6) = 0$$

$$\Rightarrow I = \frac{50}{1+j6} = (1.35 - j8.11)\ A$$

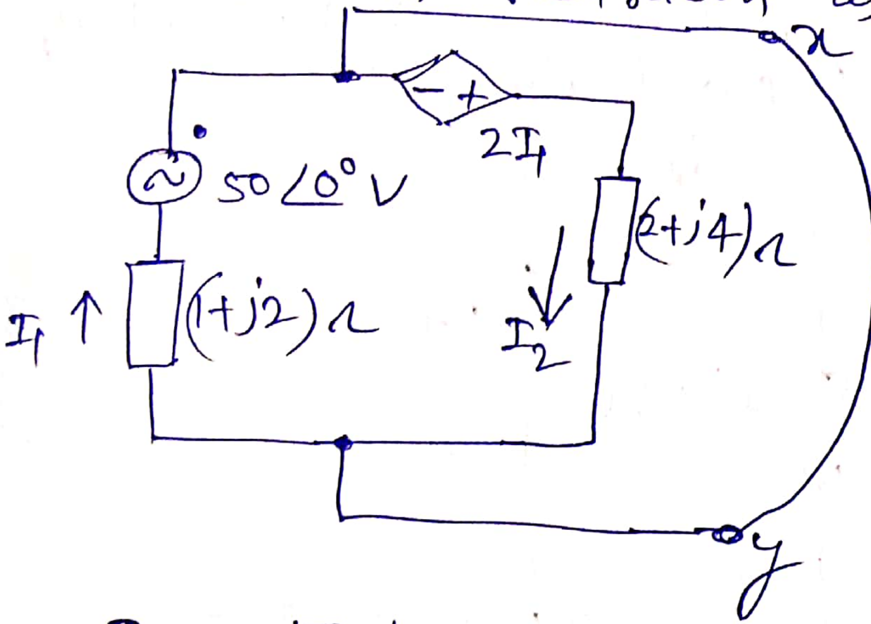
from observation of ckt

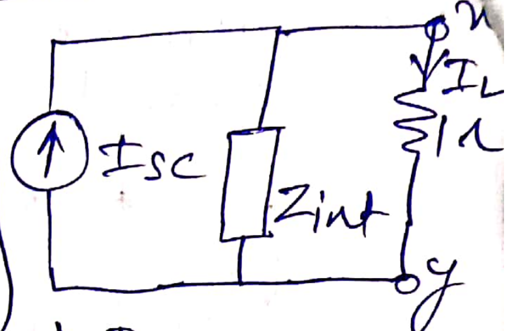
$$V_{oc} = V_{xy} = V_{mn} = -I(1+j2) + 50$$

$$= -(1.35 - j8.11)(1+j2) + 50$$

$$V_{oc} = (32.43 + j5.41)\ V = 32.88 \angle 9.5°$$

When short ckt is applied across xy, the ckt is redrawn as





$\therefore I_L = \dfrac{I_{sc} Z_{int}}{Z_{int}+1}$

$= 13.475\angle -19.66°$

verified

$$I_1 = \frac{50\angle 0 \, V}{(1+j2)\Omega} = 22.36\angle -63.43° \, A$$

and $I_2 = \dfrac{2I_1 \, V}{(2+j4)\Omega} = \dfrac{2\times 22.36\angle -63.43°}{4.472\angle 63.434°}$

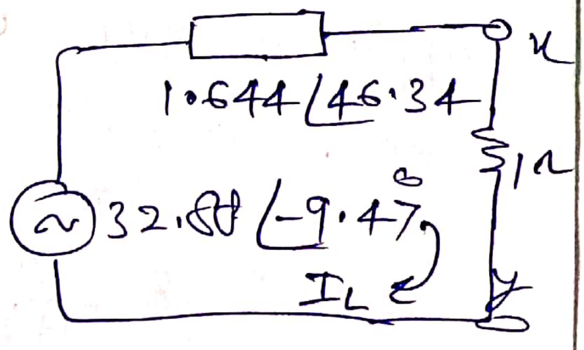
$= 10\angle -126.87° A$

$\therefore I_{sc} = I_1 - I_2 = 22.36\angle -63.43 - 10\angle -126.87°$

$= 20\angle -36.87° A$

$\therefore Z_{int} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{32.88\angle -9.47°}{20\angle -36.87°} = 1.644\angle 46.34 \Omega$

$\therefore$ Thevenin's eqvt is

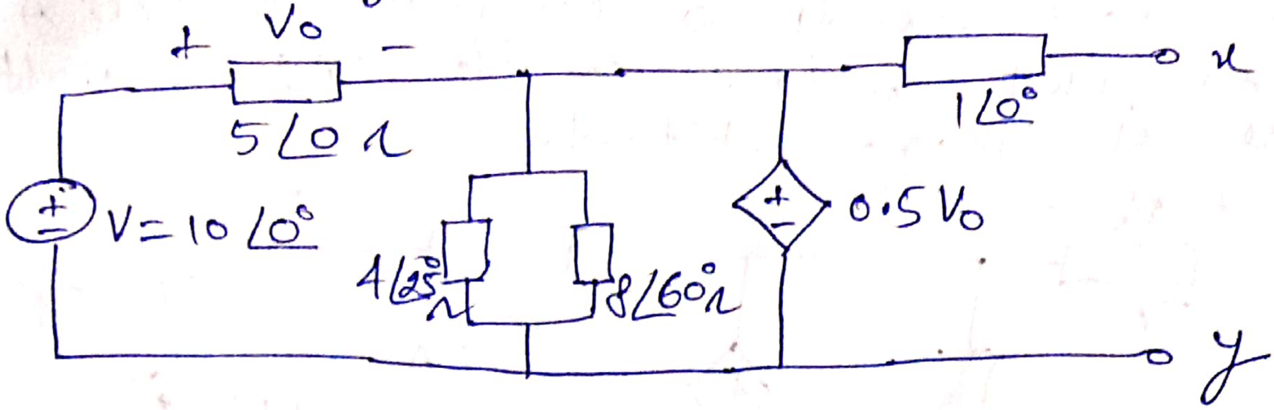


$\therefore I_L$ current through 1Ω resistor $= \dfrac{32.88\angle -9.47}{1.644\angle 46.34 + 1}$

$= 13.475\angle -19.66° A$

To validate it by Norton's theorem, Norton's eqvt is redrawn as I_{sc} & Z_{int} already calculated.

EX 12.27 :- Find Thevenin's equivalent network across x-y in below network. (27)

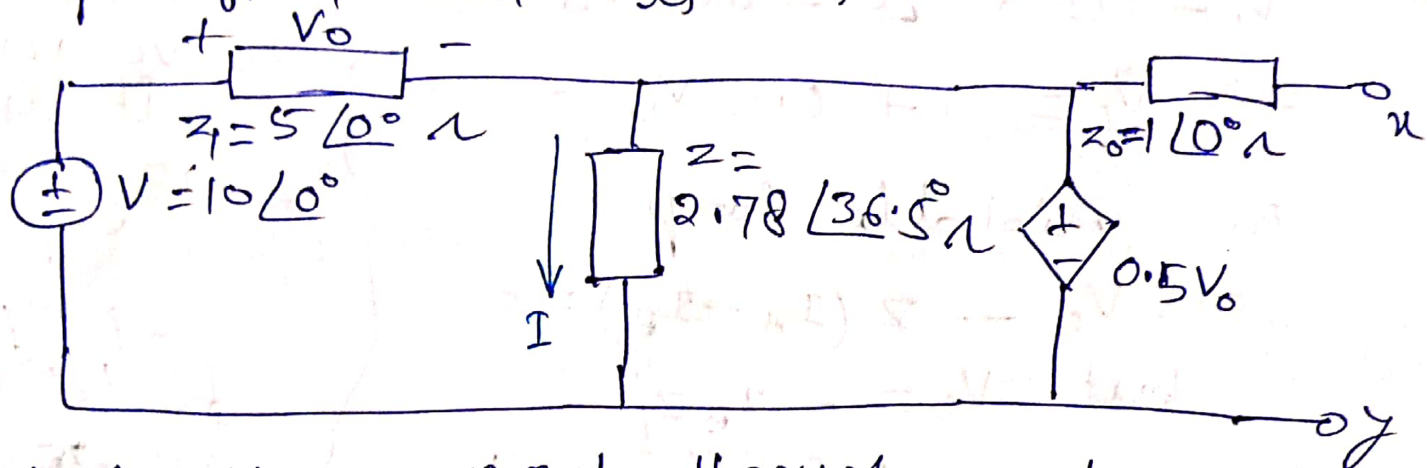


Sol^n :- Parallel combination of $4\angle 25°\,\Omega$ & $8\angle 60°\,\Omega$,

$$Z = \frac{4\angle 25° \times 8\angle 60°}{4\angle 25° + 8\angle 60°}$$

$$Z = 2.78\angle 36.5°\,\Omega$$

Simplified ckt is as below



Let the current through Z be I, we observe that $IZ = 0.5V_0 = V_{oc}$

Also in leftmost loop

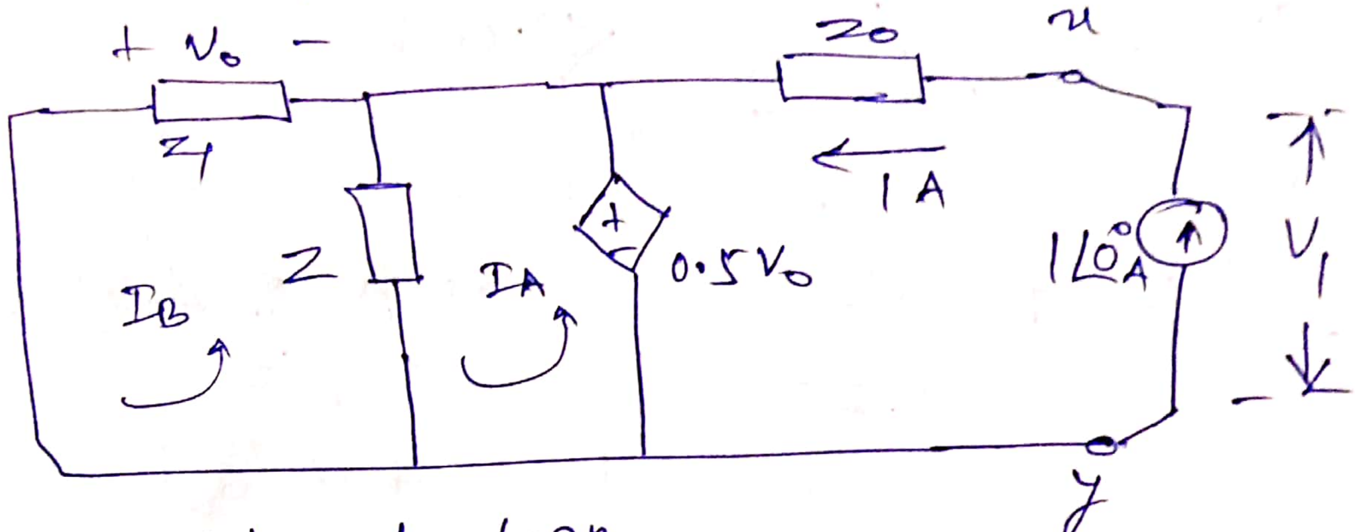
$$10\angle 0° - V_0 - 0.5V_0 = 0 \Rightarrow 10 - 1.5V_0 = 0$$

$$\Rightarrow V_0 = \frac{10\angle 0°}{1.5} = 6.67\angle 0° \text{ Volts}$$

$$\therefore V_{oc} = 0.5V_0 = 0.5 \times 6.67\angle 0° = 3.335\angle 0°$$

$$V_{oc} = 3.33\angle 0° \text{ Volts}$$

To find Z_{int} let a voltage V_1 applied across x-y sends a current of $1\angle 0^\circ$ A. as in ckt below while original voltage source is short ckted.



In rightmost loop

$$V_1 - 1\angle 0 \times 1\angle 0 - 0.5V_0 = 0$$

$$\Rightarrow V_1 = 1 + 0.5V_0 \quad \text{--- (1)}$$

In middle loop

$$0.5V_0 - Z(I_A - I_B) = 0$$

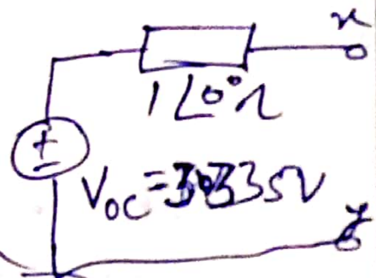
but $V_0 = -I_B Z_1$

$$\therefore -0.5(I_B Z_1) - Z(I_A - I_B) = 0$$

or $0.5 I_B Z_1 + Z(I_A - I_B) = 0$

or $I_B(0.5Z_1 - Z) + I_A Z = 0 \quad \text{--- (2)}$

Hence thevenin's equvt is drawn below b/w, x-y



Again in leftmost loop

$$-(I_B - I_A)Z - I_B Z_1 = 0$$

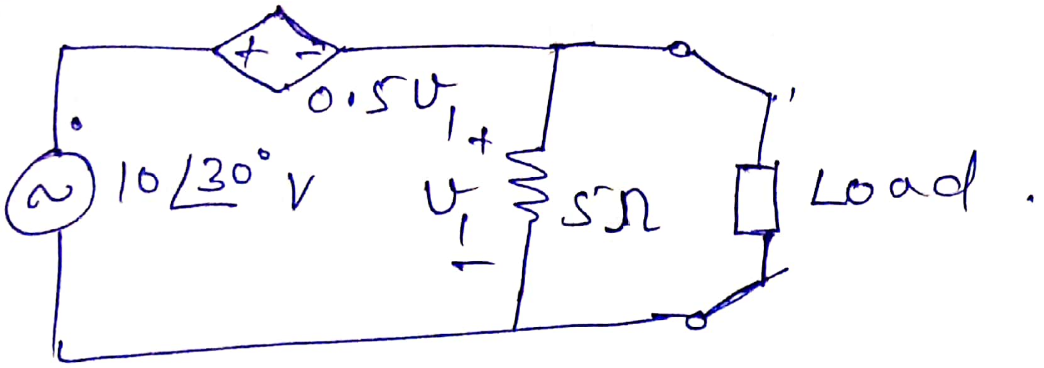
or $(I_B - I_A)Z + I_B Z_1 = 0$ or $I_B(Z_1 + Z) - I_A Z = 0 \quad \text{--- (3)}$

from (2) put value of $I_A Z$ in (3) & simplify we get $I_B = 0$ $\therefore V_0 = -I_B Z_1 = 0$. put in (1)

$V_1 = 1V$. $\therefore Z_{int} = \frac{V_1(\text{volt})}{1(\text{Amp})} = 1\Omega$

(28)

Ex 12.29 :- Find Thevenin's eq^nt ckt of the network shown below:



Sol^n :- Remove the load, the $V_{oc} = v_1$

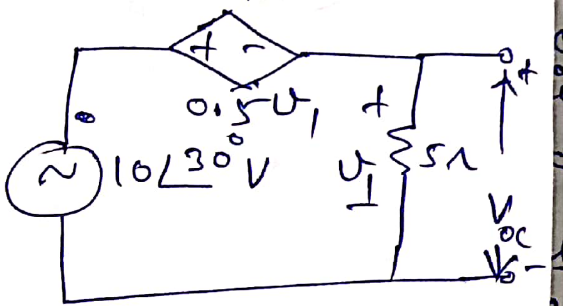
Apply KVL in left loop

$$10\angle 30 - 0.5 v_1 - v_1 = 0$$

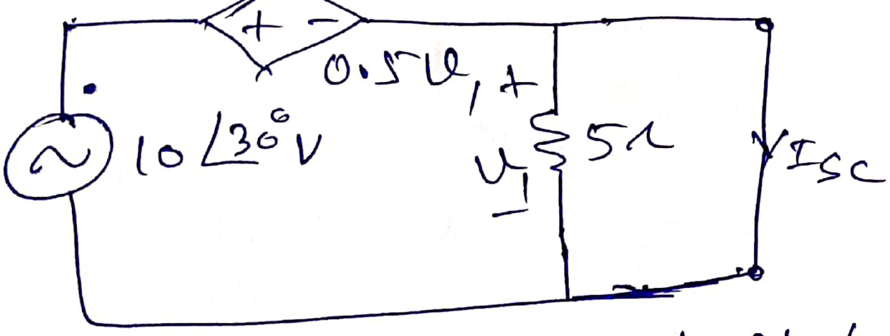
$$10\angle 30 - 1.5 v_1 = 0 \Rightarrow v_1 = \frac{10\angle 30^\circ}{1.5}$$

$v_1 = 6.67 \angle 30^\circ$ V

$\therefore V_{oc} = 6.67 \angle 30^\circ$ V



Now short circuiting the load terminals, the ckt becomes



when load is shorted whole current will flow through short circuit & $v_1 = 0$ hence

~~KVL gives~~

~~$10\angle 30^\circ - 0.5 v_1$~~

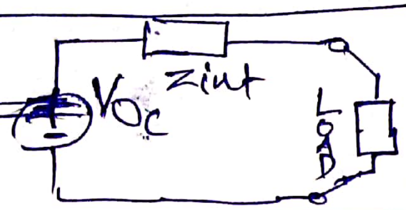
Hence

$$I_{sc} = \frac{10\angle 30 - 0}{0}$$

$\because$ no impedance in ckt

$\therefore I_{sc} = \infty$

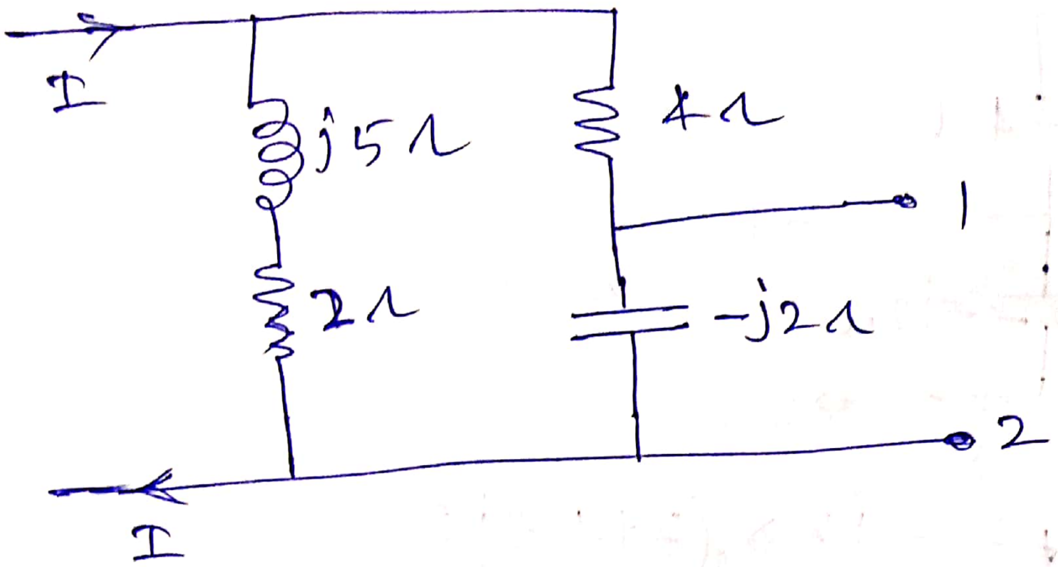
$$\therefore Z_{int} = \frac{V_{oc}}{I_{sc}} = \frac{V_{oc}}{\infty} = 0\,\Omega$$



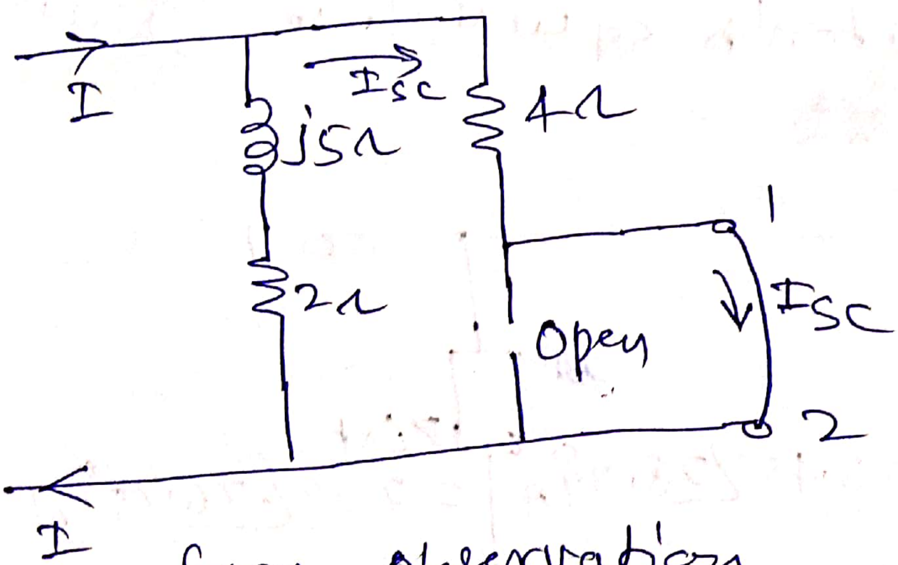
$\therefore I_L = V_{oc}/(Z_{int} + R_L)$

... above.

Ex 12.26 :- Find Norton's equivalent for the network shown below at the left of terminals 1-2, assume $I = 5\angle 0°$ A.



Sol^n :- when terminals 1-2 are shorted no current flows through -j2Ω reactance & ckt is as below

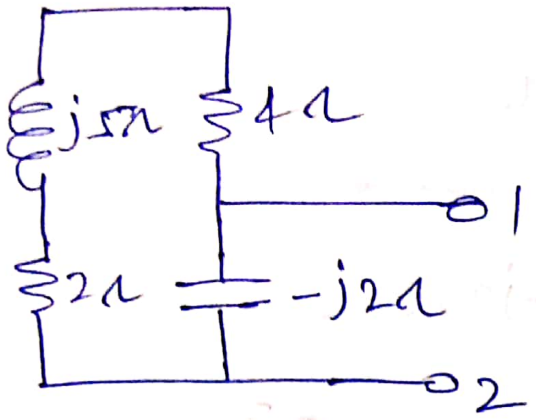


from observation

$$I_{sc} = \frac{I(2+j5)}{2+j5+4} = \frac{5\angle 0° (2+j5)}{6+j5}$$

$$= 3.45\angle 28.39° \text{ A}$$

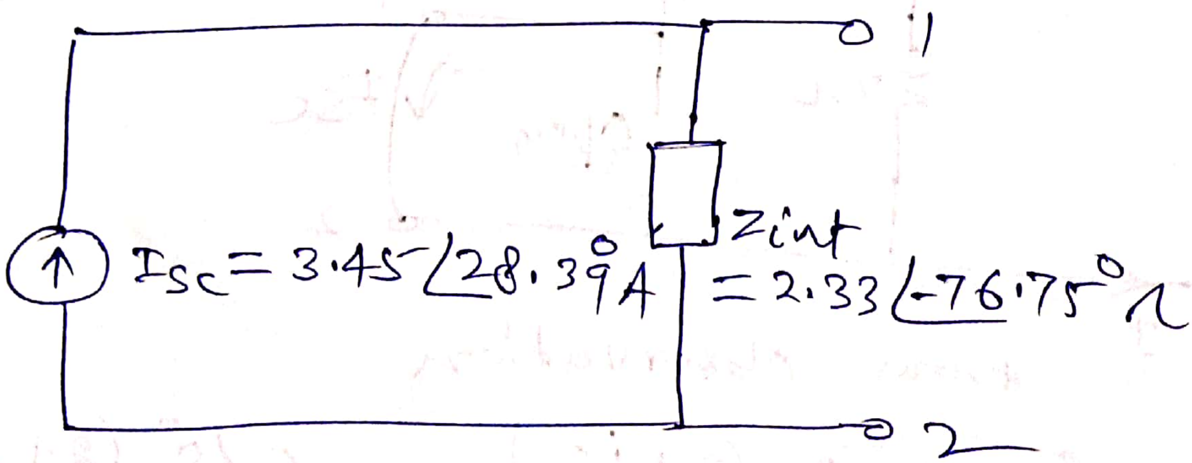
To determine the internal impedance, open ckt the current source & see from terminals 1-2 to left.



$$\therefore \; Z_{int} = \frac{-j2 \times (2+4+j5)}{-j2+2+4+j5}$$

$$= 2.33 \angle -76.75^\circ \, \Omega$$

$\therefore$ Norton's equivalent is as below:



Superposition Theorem for AC Networks

For DC networks

If a no. of voltage or current sources are acting simultaneously in a linear network, the resultant current in any branch is the algebraic sum of currents that would be produced in it, when each source acting alone replacing all other independent sources by their internal impedances.

For A C Networks :- For superposition theorem with a c networks with independent sources ckt will have to be worked out with impedances & phasors (operation with complex nos) instead of just resistors & real nos. in same way as for dc networks.

For dependent sources in which the control variable is determined by the network to which the theorem is to be applied, the dependent source can't set to be zero unless controlling variable is also zero.

For networks where both the independent and dependent sources exist, the superposition theorem is to be applied for each independent source and each dependent source not having a controlling variable in the portion of network under consideration.

In order to explain above, consider the ckt below in fig (a)

In fig ⓐ, both the sources have control variable V & I but it is to be noted that V & I does not make the source dependent on the variable of any portion of the network and each source can be treated as independent source.

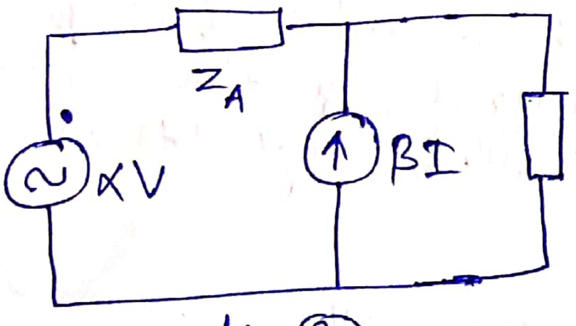


fig ⓐ

To apply superposition to calculate current through Z_B let remove current source βI & consider voltage source αV, then current through Z_B

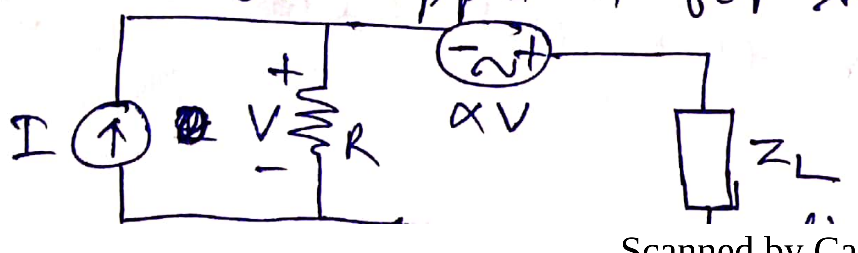
$$I' = \frac{\alpha V}{Z_A + Z_B}$$

Now consider current source βI & short voltage source αV, the current through Z_B is

$$I'' = \beta I \frac{Z_A}{Z_A + Z_B}$$

∴ Net current, by superposition $= I' + I''$

Now consider the fig ⓑ below, it may be noted that the controlling variable V is determined by the network to be analysed. If the current source is set to zero the drop across R would vanish and this will make $\alpha V = 0$, thus the n/w would then lack of any source thus superposition theorem can not be applied for such sources.



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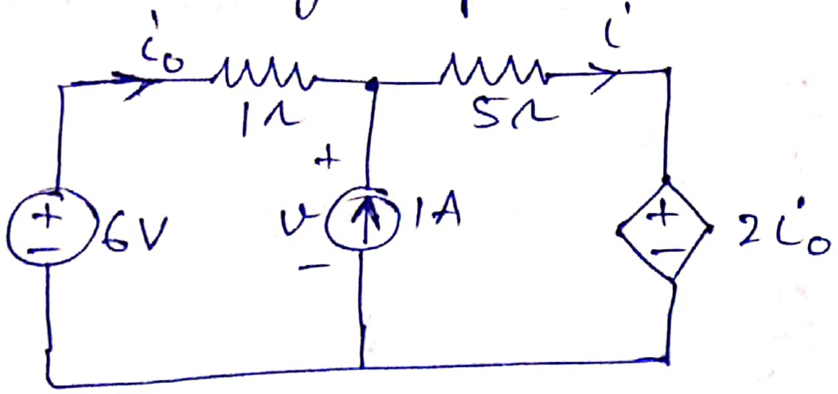
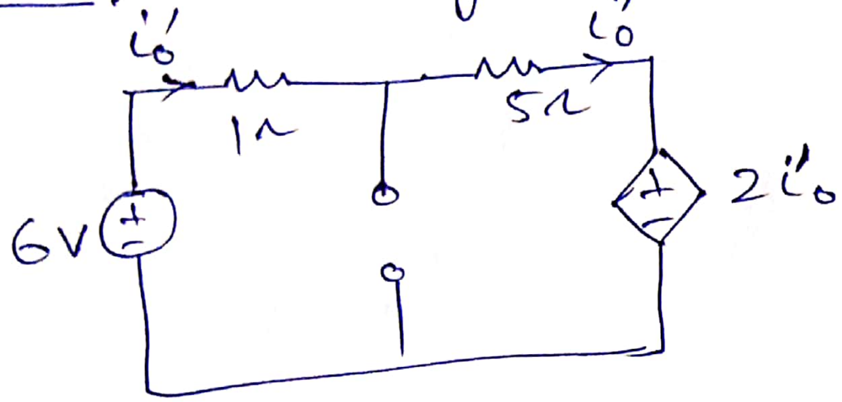
Ex 3.45 :- Find i_o and i from the ckt below using superposition theorem.



Sol^n :- Assuming 6V source to be active



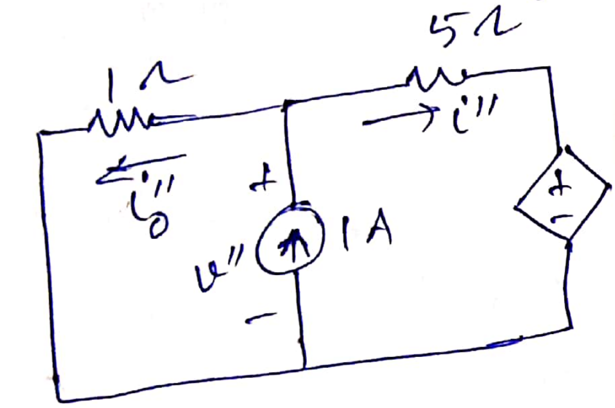
KVL gives

$$6 - 6i_o' - 2i_o' = 0$$

$$\Rightarrow 8i_o' = 6$$

$$i_o' = \frac{6}{8} = 3/4 \text{ A}$$

Now considering 1A source active



KCL gives

$$1 = i_o'' + i''$$

$$= \frac{v''}{1} + \frac{v'' - 2i_o''}{5}$$

$$= v'' + 0.2v'' - 0.4i_o''$$

$$= 1.2v'' - 0.4i_o''$$

but $i_o'' = \frac{v_o''}{1} \Rightarrow v_o'' = i_o''$

$\cancel{\therefore 1 = 1.2v'' - 0.4v'' = 0.8v''}$

$$\therefore 1 = 1.2i_o'' - 0.4i_o'' = 0.8i_o''$$

$$\Rightarrow i_o'' = \frac{1}{0.8} = 1.25 \text{ A}$$

& $i_o'' = \frac{v'' - 2i_o''}{5} = \frac{i_o'' - 2i_o''}{5} = -\frac{i_o''}{5}$

$$\therefore i'' = -\frac{1.25}{5} = -0.25 \text{ A}$$

10Ω

Hence using superposition theorem

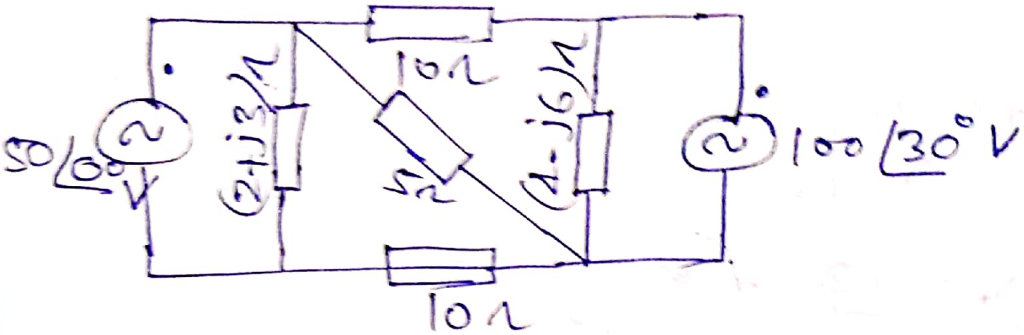
$$i_0 = i_0' - i_0''' = \frac{3}{4} - 1.25 = 0.75 - 1.25$$

$$= -0.50 \text{ A}$$

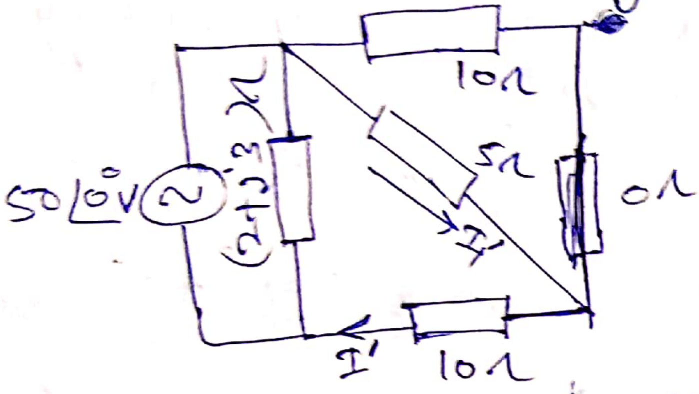
$$\& \quad i = i_0' + i'' = \frac{3}{4} - 0.25 = 0.75 - 0.2$$

$$= 0.50 \text{ A}$$

12.33 :- Find by superposition theorem, the current through 5Ω resistor in fig below.



Sol^n :- Deactivating $100\angle 30^\circ$ V source, the ckt is



since short ckt across $(4-j6)\Omega$ makes it zero.

∵ $(10\|5)+10$ is in parallel with $(2+j3)\Omega$ ∴ same voltage appear across $\{(10\|5)+10\}\Omega$.

$$I' = \frac{50\angle 0}{(10\|5)+10}$$

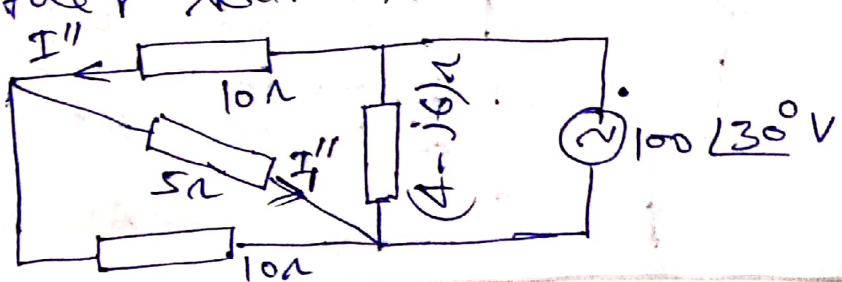
$$= 3.75\angle 0^\circ \text{ A}$$

and $$I_1' = \frac{I' \, 10}{10+5}$$

$$= 3.75\angle 0 \; \frac{\times 10}{15}$$

$$= 2.5\angle 0^\circ \text{ A}$$

Again deactivate $50\angle 0^\circ$ V source and consider the other source, ckt is as below:



above ... = ... (Rq+ j...

$\therefore \quad I'' = \dfrac{100\angle 30^\circ}{10\|5 + 10} = 7.5\angle 30^\circ A$

$\therefore \quad I_1'' = I'' \dfrac{10}{5+10}$

$= 7.5\angle 30^\circ A \times \dfrac{10}{15}$

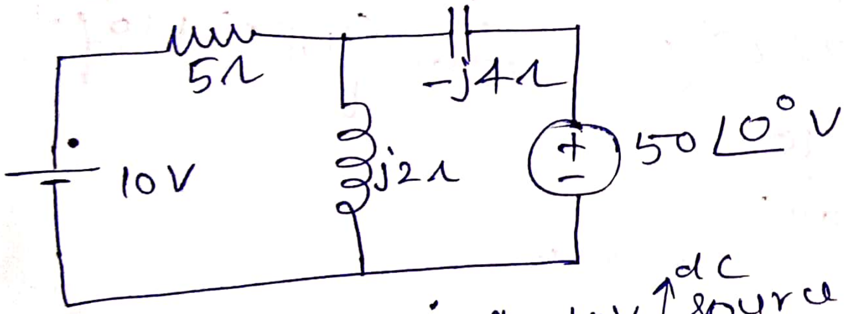
$= 5\angle 30^\circ \; A$

$\therefore$ current through 5Ω resistor, using principle of superposition is

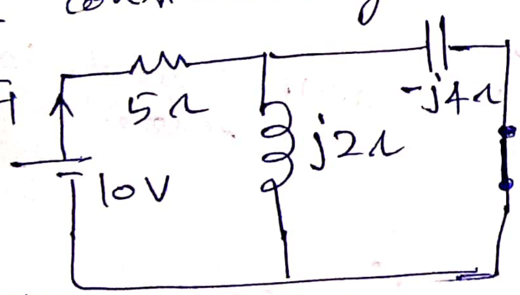
$I_{(5\Omega)} = I_1' + I_1'' = 2.5\angle 0^\circ + 5\angle 30^\circ$

$= 6.83 + j2.5 = 7.273\angle 20.1^\circ A$

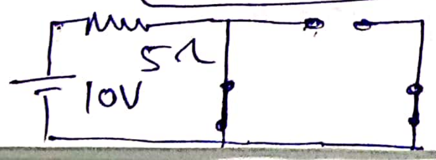
ex 12.36:- obtain the steady state current through the 10V battery in time domain in the ckt of fig below using superposition theorem.



Sol^n:- considering 10V dc source initially



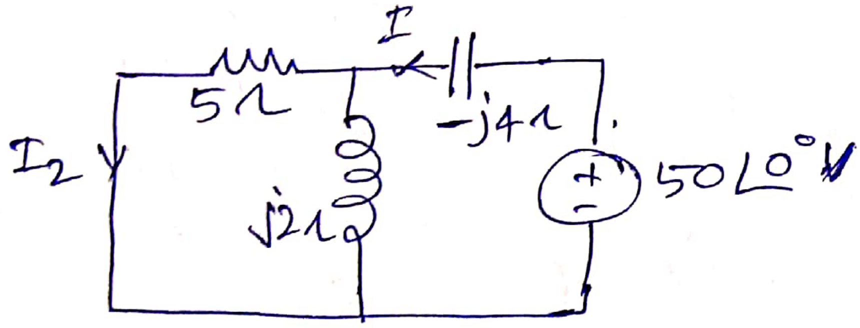
with dc source j2Ω inductive reactance behave as short ckt and -j4Ω capacitance behaves as open ckt hence

≡ 

$I_1 = \dfrac{10}{5} = 2\,A$

$= \dfrac{}{(R_g+R_L)+j(X_g}$

Now consider $50\angle 0^\circ V$ ac source only then



$$Z = -j4\Omega + \frac{5\times j2}{5+j2} = \left(\frac{8-j10}{5+j2}\right)\Omega$$

$$= \frac{12.8\angle -51.34^\circ}{5.385\angle 21.8^\circ} = 2.377\angle -73.14^\circ \Omega.$$

$$\therefore I = \frac{50\angle 0}{2.377\angle -73.14^\circ} = 21.04\angle 73.14^\circ A$$

$$\therefore I_2 = \frac{I\ j2\Omega}{5+j2} = \frac{21.04\angle 73.14^\circ\ j2}{5+j2}$$

$$= 7.81\angle 141.34^\circ A$$

In time domain instanteneous value is

$i_2 = I_m \sin(\omega t + \theta)$

$= 7.81\times\sqrt{2}\sin(\omega t + 141.34^\circ)$

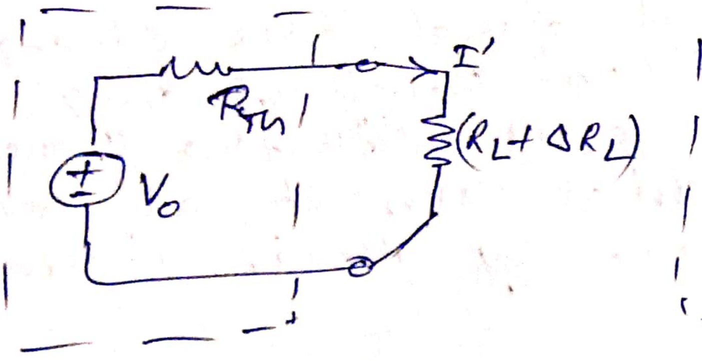
$= 11\sin(\omega t + 141.34^\circ)$

$\therefore$ Net current current through the 10V battery in time domain is

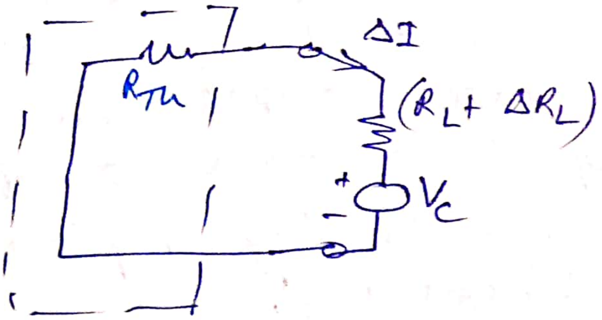
$= 2 - 11\sin(\omega t + 141.34^\circ)$ Ans

since 2 A current is due to battery i.e. dc current i.e. average value no instantaneous value.

Here $I' = \dfrac{V_0}{R_{Th} + (R_L + \Delta R_L)}$



Thevenin's equt of source



Source Network with source replaced by internal resistance

fig ⑥ : Explanation of compensation Theorem.

The change of current being denoted by ΔI, we get

$$\Delta I = I' - I$$

$$= \frac{V_0}{R_{Th} + (R_L + \Delta R_L)} - \frac{V_0}{R_{Th} + R_L}$$

$$= \frac{V_0 \{ R_{Th} + R_L - (R_{Th} + R_L + \Delta R_L) \}}{(R_{Th} + R_L + \Delta R_L)(R_{Th} + R_L)}$$

$$= -\left(\frac{V_0}{R_{Th} + R_L}\right) \frac{\Delta R_L}{(R_{Th} + R_L + \Delta R_L)}$$

$$= -\frac{I \, \Delta R_L}{R_{Th} + R_L + \Delta R_L} = \frac{-V_c}{R_{Th} + R_L + \Delta R_L}$$

where $V_c = I \Delta R_L$ is termed as compensating voltage.

Thus it has been proved that with, the change of branch resistance, branch current is changed and the change is equt to an ideal compensating voltage source in series with the branch opposing the original current, all other sources being replaced by their internal resistances.

Tellegen's Theorem :- In any linear, non-linear, passive, active time variant network, excited by a c sources, the summation of instantaneous or complex power of the sources is zero.

For a network excited by sinusoidal sources, if no. of branches be 'b',

$$\sum_{b=1}^{b} v_b i_b = 0$$

where v_b and i_b represent the instantaneous voltage and current of source at each branch

When considering complex power, if V_b and I_b be the voltage and current of each branch then as per this theorem

$$\sum_{b=1}^{b} V_b I_b^* = 0$$

where $I_b^* \rightarrow$ complex conjugate of I_b

Proof of Tellegen's theorem :- For the network shown below let the node voltages be V_1, V_2 and V_3 at nodes 1, 2 & 3 respectively. The current directions are shown arbitrarily.

Fig. 4

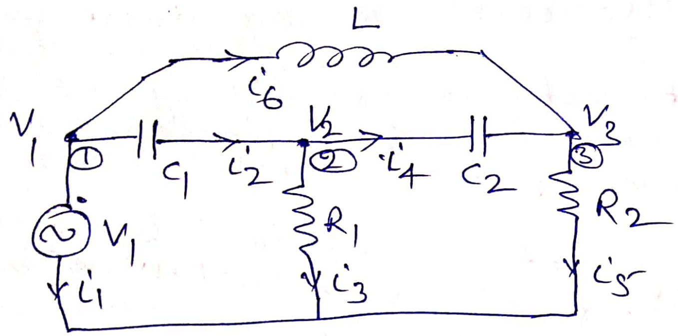


Fig: Application of Tellegen's theorem

The summation of instantaneous power in the network is given by

$$\sum_{b=1}^{6} v_b i_b = V_1 i_1 + V_{C_1} i_2 + V_{R_1} i_3 + V_{C_2} i_4 + V_{R_2} i_5 + V_L i_6$$

or $$\sum_{b=1}^{6} v_b i_b = V_1 i_1 + (V_1 - V_2) i_2 + V_2 i_3 + (V_2 - V_3) i_4 + V_3 i_5 + (V_1 - V_3) i_6$$

$$= V_1 (i_1 + i_2 + i_6) + V_2 (i_3 - i_2 + i_4) + V_3 (i_5 - i_4 - i_6)$$

Applying KCL at node 1 gives

$i_1 + i_2 + i_6 = 0$

and at node 2 $i_3 + i_4 - i_2 = 0$

and at node 3 $i_5 - i_4 - i_6 = 0$

then finally,

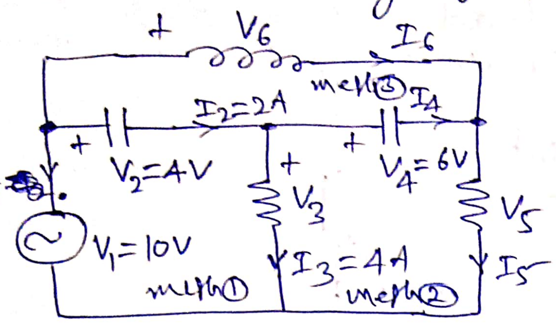
$$\sum_{b=1}^{6} v_b i_b = V_1 \times 0 + V_2 \times 0 + V_3 \times 0$$
$$= 0$$

$\therefore$ The Tellegen's theorem is proved.

(35)

(36)

Ex 12.46 :- In the network shown below, check the validity of Tellegen's theorem.



Soln :- Let us first find unknown voltages and currents. KVL in mesh (1)

$$10 - 4 - V_3 = 0 \Rightarrow V_3 = 10 - 4 = 6V$$

KVL in mesh (2)

$$6 - 6 - V_5 = 0 \Rightarrow V_5 = 0\ V$$

KVL in mesh (3)

$$4 - V_6 + 6 = 0 \Rightarrow V_6 = 10\ V$$

Next apply KCL at node (1)

$$2 + 2 + I_6 = 0 \Rightarrow I_6 = -4\ A$$

KCL at node (2)

$$4 + I_4 - 2 = 0 \Rightarrow I_4 = -2\ A$$

KCL at node (3)

$$I_5 - I_4 - I_6 = 0 \Rightarrow I_5 + 2 + 4 = 0 \Rightarrow I_5 = -6A$$

Now
$$\sum_{b=1}^{6} v_b i_b = V_1 I_1 + V_2 I_2 + V_3 I_3 + V_4 I_4 + V_5 I_5 + V_6 I_6$$
$$= 10 \times 2 + 4 \times 2 + 6 \times 4 + 6 \times (-2) + 0 \times (-6) + 10 \times (-4)$$
$$= 20 + 8 + 24 - 12 + 0 - 40$$
$$= 52 - 52 = 0$$

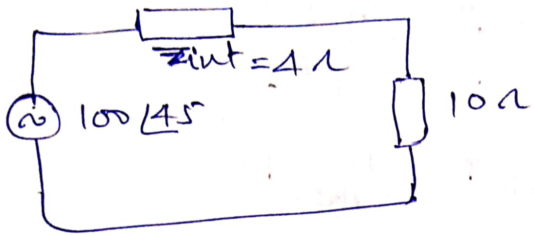
$\therefore$ Proved

(39)

Transfer Theorem:-

... energy

ex:- use compensation theorem to determine change in current through 10Ω resistive load when the load resistance is increased by 1Ω in the ckt below:



Soln:- As per compensation theorem the change in current

$$\Delta I = \frac{V_c}{R_{Th} + R_L + \Delta R_L} = \frac{I \Delta R_L}{R_{Th} + R_L + \Delta R_L}$$

[where $V_c = I \Delta R_L$]

$$I = \frac{V_{oc}}{Z_{int} + R_L} = \frac{100}{4+10} = \frac{100}{14} = 7.14 \text{ A}$$

$\therefore$ change in current

$$= \frac{7.14 \times 1}{4+10+1} = \frac{7.14}{15} = 0.476 \text{ A}$$

Maximum Power Transfer Theorem:-

In a linear network having energy sources and impedances, the maximum amount of power is transferred from source to load impedance if the load impedance is the complex conjugate of the total impedance of the network.

Proof :- consider ckt below having load impedance Z_L and source impedance Z_g, connected in series with the source V_g. Let I be the current through load.

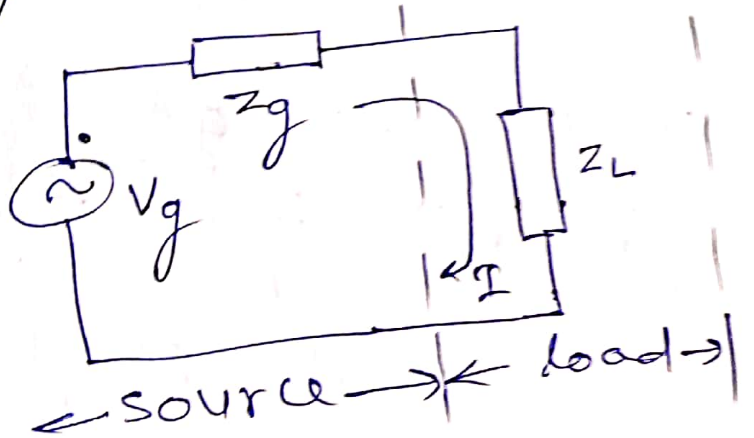


Fig: ckt for Maximum Power Transfer Theorem.

from above ckt

$$I = \frac{V_g}{Z_g + Z_L} = \frac{V_g}{(R_g + jX_g) + (R_L + jX_L)}$$

$$= \frac{V_g}{(R_g + R_L) + j(X_g + X_L)}$$

$\therefore$ Real Power, $P_L = I^2 R_L$

$$P_L = \frac{V_g^2}{(R_g+R_L)^2 + (X_g+X_L)^2} \cdot R_L \quad \text{--- (1)}$$

for maximum power flow from source to load

$$\frac{dP_L}{dX_L} = 0$$

$$\Rightarrow \frac{dP_L}{dX_L} = \frac{d}{dX_L}\left(\frac{V_g^2}{(R_g+R_L)^2 + (X_g+X_L)^2} \cdot R_L\right)$$

$$\frac{dP_L}{dX_L} = -\frac{V_g^2 R_L \cdot 2(X_g+X_L)}{\left[(R_g+R_L)^2 + (X_g+X_L)^2\right]^2} \quad \text{--- (2)}$$

Put $\frac{dP_L}{dX_L} = 0$ and simplifying

we get $X_g = -X_L$

Now put $X_g = -X_L$ in eqn (1)

$$P_L = \frac{V_g^2 R_L}{(R_g+R_L)^2} = \frac{V_g^2}{4R_g}\left[1 - \left(\frac{R_g - R_L}{R_g+R_L}\right)^2\right]$$

$$\boxed{P_L = \frac{V_g^2}{4R_g}\left[1 - \left(\frac{R_g - R_L}{R_g+R_L}\right)^2\right]}$$

gx

w

It may be proved that P_L attains maximum value when $R_L = R_g$. Thus maximum power transfer is possible only when

$$R_g + jX_g = R_L - jX_L$$

i.e. $$\boxed{Z_g = Z_L^*}$$

$Z_L^* \rightarrow$ complex conjugate of Z_L.

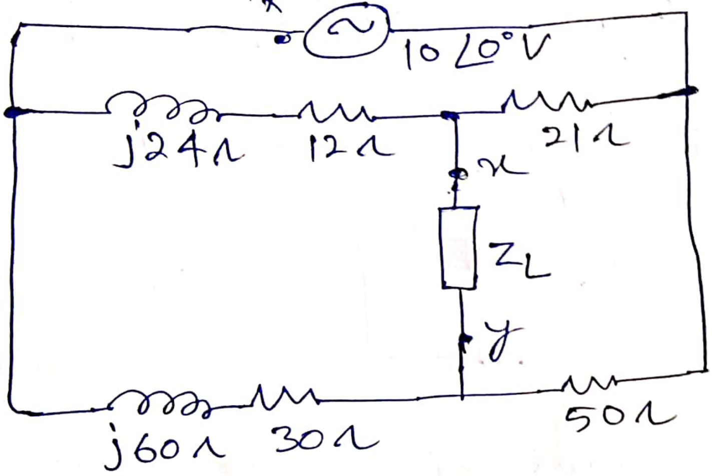
and amount of maximum power transfer becomes

$$\boxed{P_{L\,max} = \frac{V_g^2}{4R_L}}$$

In solving, the problems, the internal impedance of the network across the load is to be determined as in Thevenin's theorem. Next open ckt voltage across load terminals is determined thus the amount of maximum power transfer becomes

$$\boxed{P_{L,max} = \frac{V_{oc}^2}{4R_L}}$$

ex 12.44 :- In the network of fig below the value of Z_L, so that the transfer from source to load is maxi. Also find P_{max}.



Soln :- First remove the load Z_L, the internal impedance of ckt looking into terminals x-y is

$$Z_{int} = Z_{x-y} = \frac{21(12+j24)}{21+12+j24} + \frac{50(30+j60)}{50+30+j60}$$

$$Z_{int} = (42.26 + j21.35)\Omega$$

for max power transfer

$$Z_L = Z_{int}^* = (42.26 - j21.35)\Omega$$

Next when source is present and load terminals open, let open ckt voltage is V_{oc} and potential of terminals x & y are V_x & V_y respectively

$$V_x = 10\angle 0 \cdot \frac{12+j24}{12+j24+21}$$ [Voltage division rule

$$= 6.577 \angle 27.43^\circ \text{ V}$$

Similarly, $V_y = 10\angle 0 \; \frac{30+j60}{30+j60+50}$

$$= 6.71 \angle 26.56^\circ \text{ V}$$

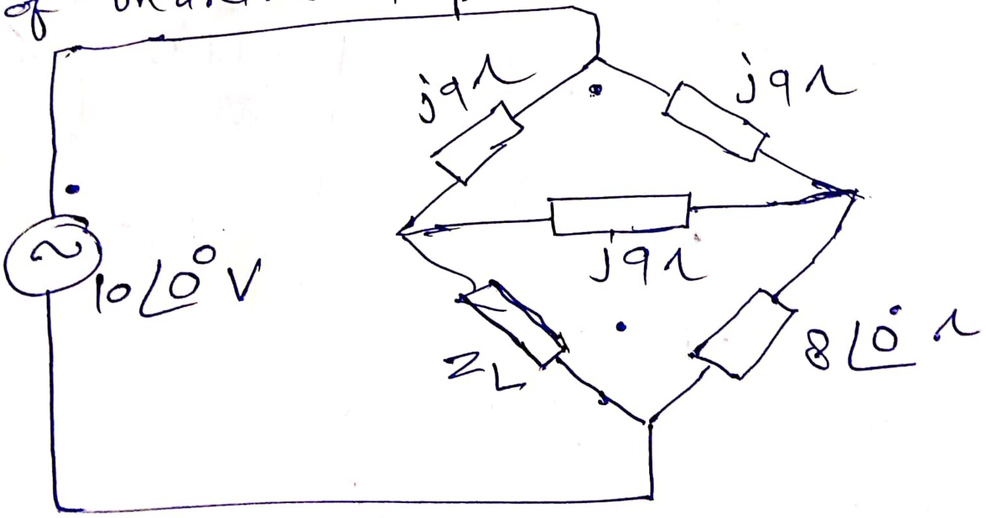
$\therefore \; V_{oc} = V_x - V_y$

$$= 6.577 \angle 27.43^\circ - 6.71 \angle 26.56^\circ$$

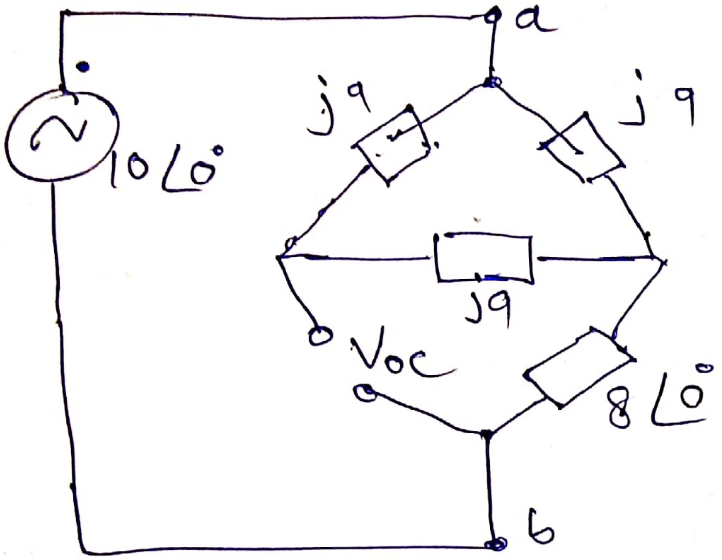
$$= 0.1657 \angle 170^\circ \text{ V}$$

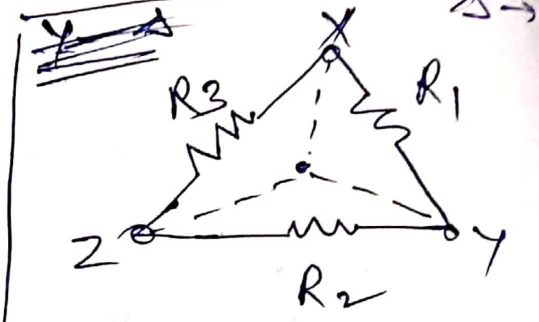
$$\therefore \; P_{max} = \frac{V_{oc}^2}{4R_L} = \frac{0.1657}{4 \times 42.26} = 0.1624 \text{ mW}$$

Ex 12.45 :- Find the value of Z_L to have maximum power transfer from the $10\angle 0^\circ$ voltage source. Also determine the amount of maximum power.



Sol^n :- Let us first remove load Z_L and find V_{oc}

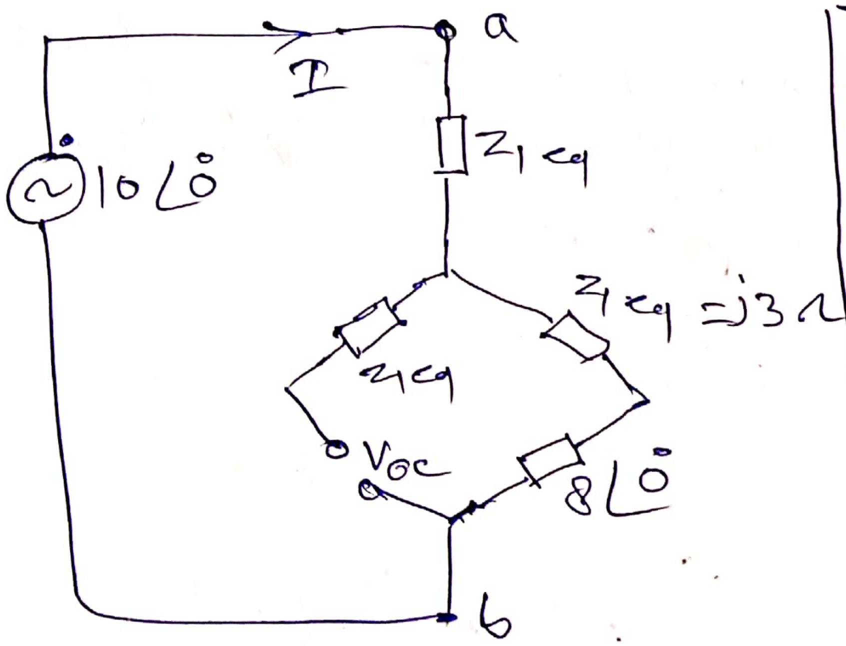


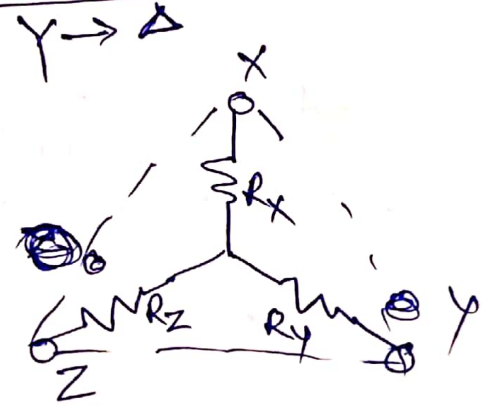


$$R_X = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

convert $j9\,\Omega$ impendances ~~in~~ Δ to equivalent Y

$$Z_{1eq} = \frac{j9 \cdot j9}{j9 + j9 + j9} = \frac{j9 \cdot \cancel{j9}}{3 \cdot \cancel{j9}} = j\frac{9}{3} = j3\,\Omega$$



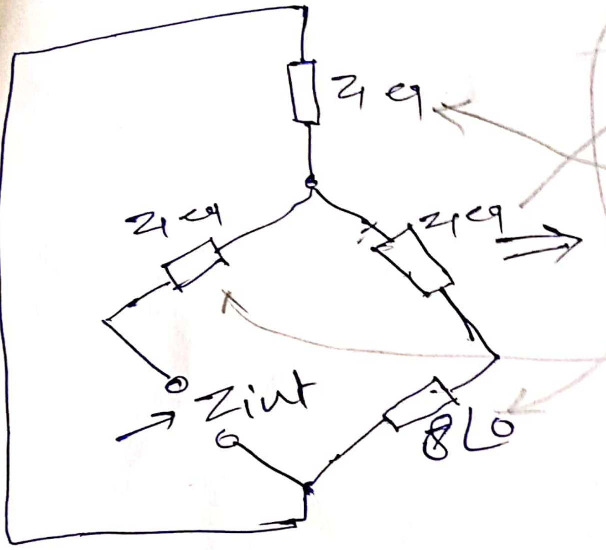


$$R_1 = R_X + R_Y + \frac{R_X R_Y}{R_Z}$$

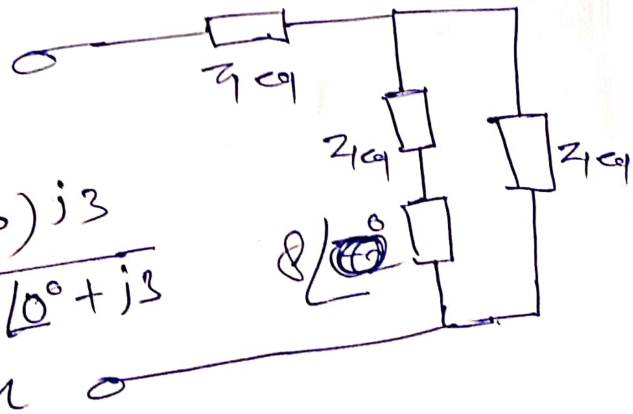
$$I = \frac{10\angle 0^\circ}{j3 + j3 + 8\angle 0^\circ} = \frac{10\angle 0^\circ}{8 + j6}$$

$$\therefore V_{oc} = I\,(Z_{eq} + 8\angle 0^\circ)$$
$$= \frac{10\angle 0}{8+j6}\,(j3 + 8)$$
$$V_{oc} = 8.54\angle -16.31^\circ \text{ V}$$

To find out Z_{int}, open the load & replace voltage source by short ckt



Z_{eq} & $8\angle 0$ in series are in parallel with Z_{eq} and this combn is parallel with Z_{eq}



$$\therefore Z_{int} = j3 + \frac{(j3 + 8\angle 0)\,j3}{j3 + 8\angle 0^\circ + j3}$$
$$= (0.72 + j5.46)\,\Omega$$

$\therefore$ for max power transfer

Z_L = complex conjugate of Z_{int}

$$= (0.72 - j5.46)\,\Omega$$

Also $P_{Lmax} = \dfrac{V_{oc}^2}{4R_L} = \dfrac{(8.54)^2}{4 \times 0.72} = 25.32\text{ W}$

Reciprocity Theorem :- For a linear
ning generators and network contai-
ratio of Voltage V introduced in impedances, the
loop to the current I produced in one
any other loop is same as the ratio
of Voltage and current obtained if the
position of voltage source V and the curr-
ent ~~source~~ measured is interchanged.

To explain Reciprocity theorem consider
ckt below:

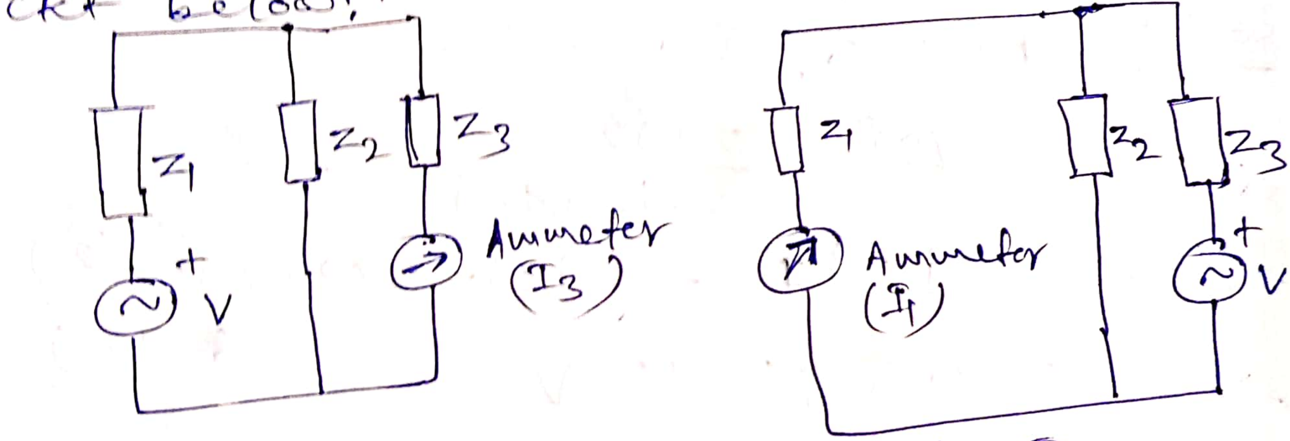


fig (a) fig (b)

Fig: Application of Reciprocity theorem

Let voltage V in fig (a) produces a
current I_3 in the branch having impe-
dance Z_3. In fig (b) we have just inter-
exchanged the position of voltage source
and ammeter, as per reciprocity theorem

$$\boxed{\frac{V}{I_3} = \frac{V}{I_1}} \Rightarrow I_1 = I_3$$

Ex 12.40 :- In fig ⓐ and fig ⓑ obtain V_0 and establish reciprocity theorem.

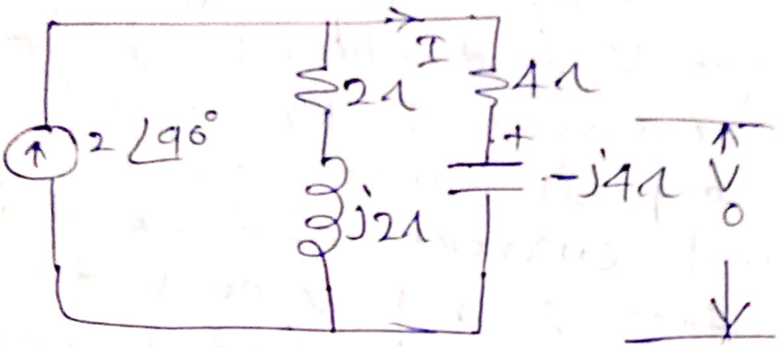


fig ⓐ

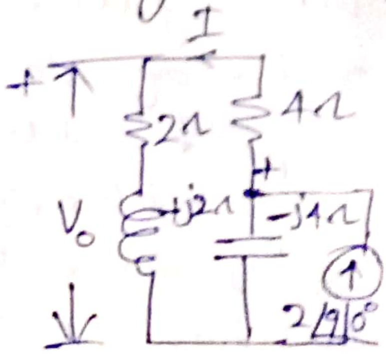


fig ⓑ

Soln :- from fig ⓐ

$$I = 2\angle 90^\circ \times \frac{2+j2}{2+j2+4-j4}$$

$$\therefore V_0 = (-j4)I = (-j4)\times 2\angle 90^\circ \times \frac{(2+j2)}{(6-j2)}$$

$$= 3.58 \angle 63.43^\circ \ V$$

from fig ⓑ

$$V_0 = I(2+j2) = \frac{2\angle 90^\circ \times -j4 \times (2+j2)}{-j4+4+2+j2}$$

$$= 3.58 \angle 63.43^\circ \ V$$

Thus we see that in both the figures V_0 is same with the interchange of sources, this proves the reciprocity theorem.

Tutorial/Assignment-2

Course: B.Tech **Section: EC-1,2,3& EI**

Session: 2019-20 **Sem: III**

Subject: Network Analysis and Synthesis **Sub. Code: KEC-303**

1. In the circuit of Fig.1, find the current through R_L connected across x-y terminals by utilising Thevenin's theorem. Verify the results by Norton's theorem also.

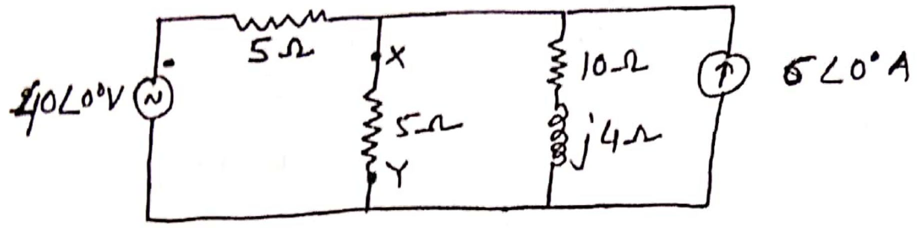


Fig. 1

2. In the network of Fig.2, find the current through the 10 ohm resistance using Norton's theorem. Also find the power loss in the 10 ohm resistance.

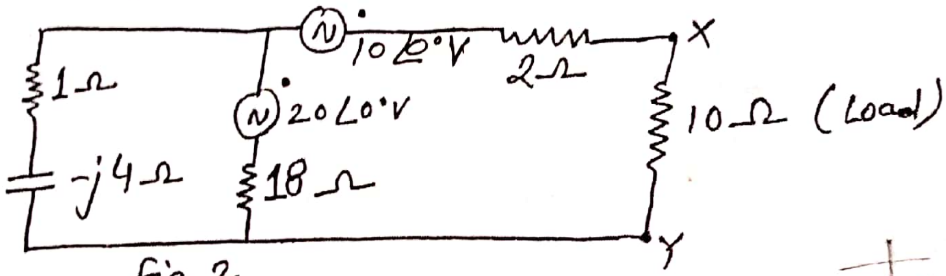


Fig. 2

3. Find the Norton's equivalent circuit across x-y for the network shown in Fig.3.

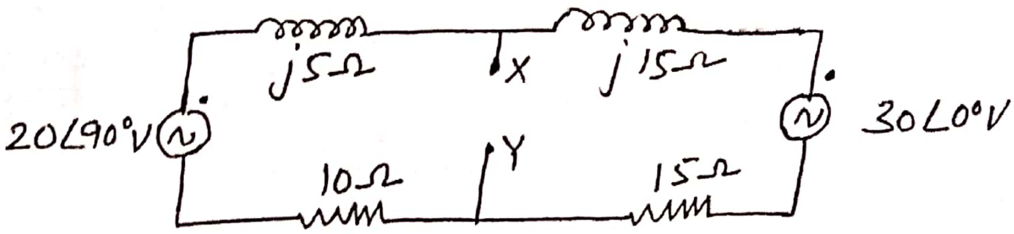


Fig.3

4. Find Thevenin's equivalent circuit across terminal a-b for the left part of the network of the circuit shown in the Fig.4

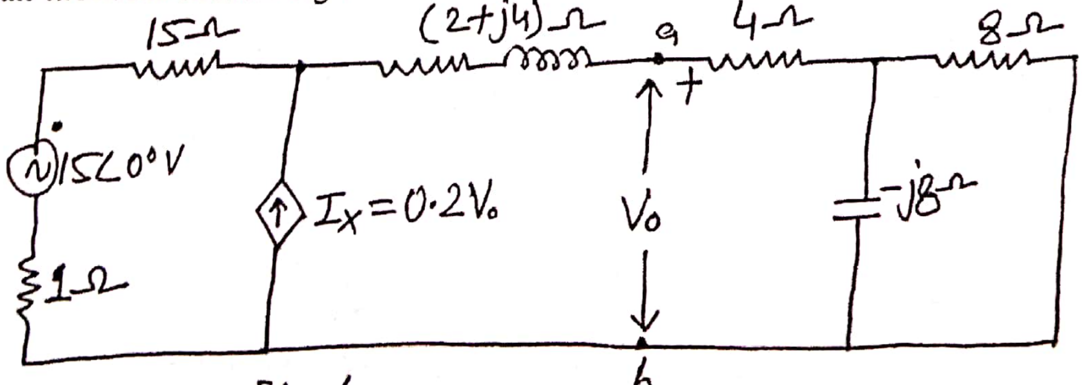


Fig. 4

Scanned by CamScanner

5. Obtain equivalent circuit across x-y terminals in Fig.5 and find the value of Z_L to have maximum power transfer.

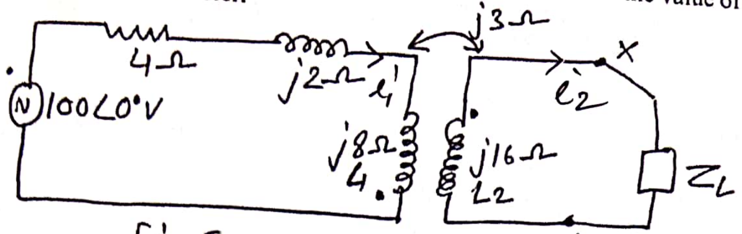


Fig. 5

6. Find the current through the capacitor of (-j5) ohm reactance in Fig.6 using superposition theorem.

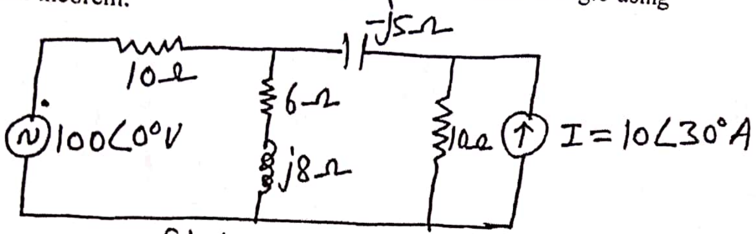


Fig. 6

7. In the circuit shown in the Fig.7 determine the values of R and C so that the maximum power is absorbed by R. Determine the maximum power transferred to the load.

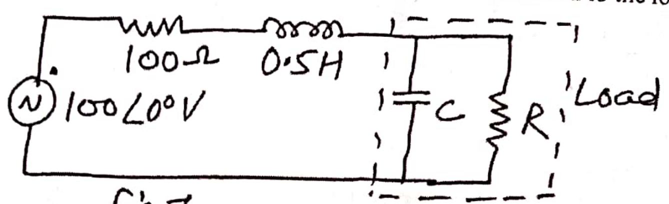
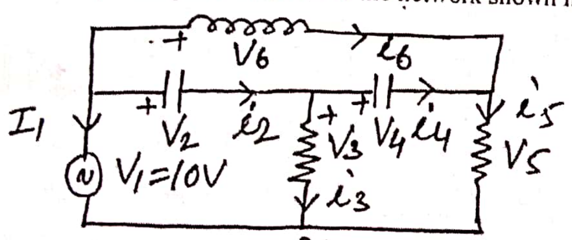


Fig. 7

8. Verify Tellegen's theorem in the network shown in Fig.8



$V_1 = 10V$ $\quad I_1 = I_2 = 2A$

$V_2 = 4V$ $\quad I_3 = 4A$

$V_4 = 6V$

Fig. 8.

9. Find Thevenin's equivalent circuit across a-b in the circuit of Fig.9 and find the current through 10 ohm resistance.

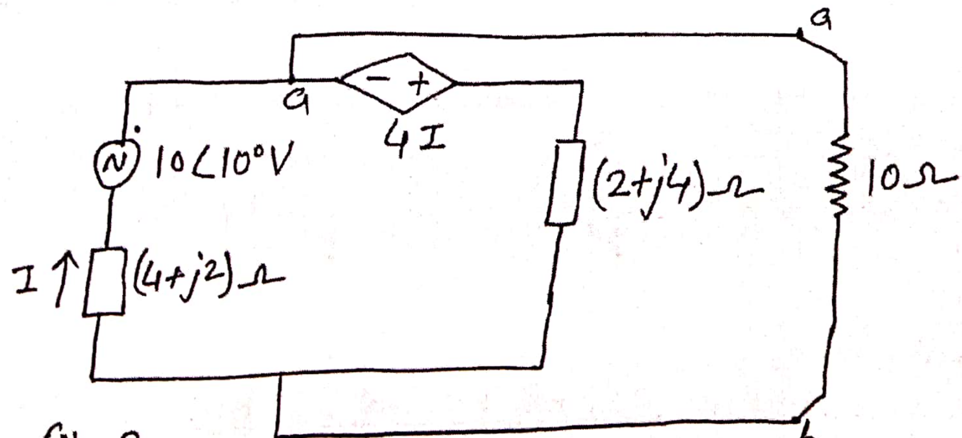


Fig. 9

10. Using principle of superposition find the current in (-j4) ohm impedance in the circuit of Fig.10.

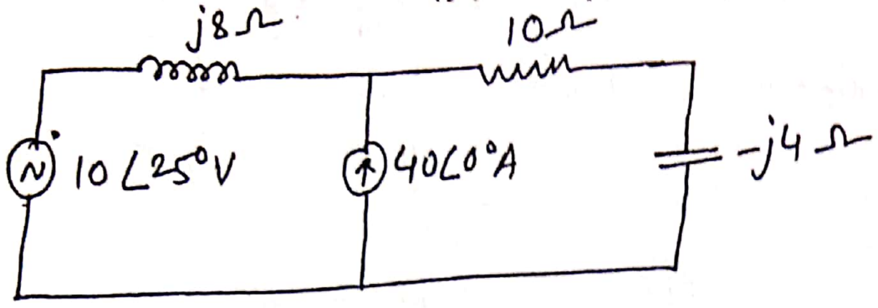


Fig. 10.

11. In the circuit shown in the Fig.11 two voltage sources act on the load impedance connected to terminal a-b. If the load impedance is variable, what load Z_L will draw the maximum power. Also calculate the maximum power.

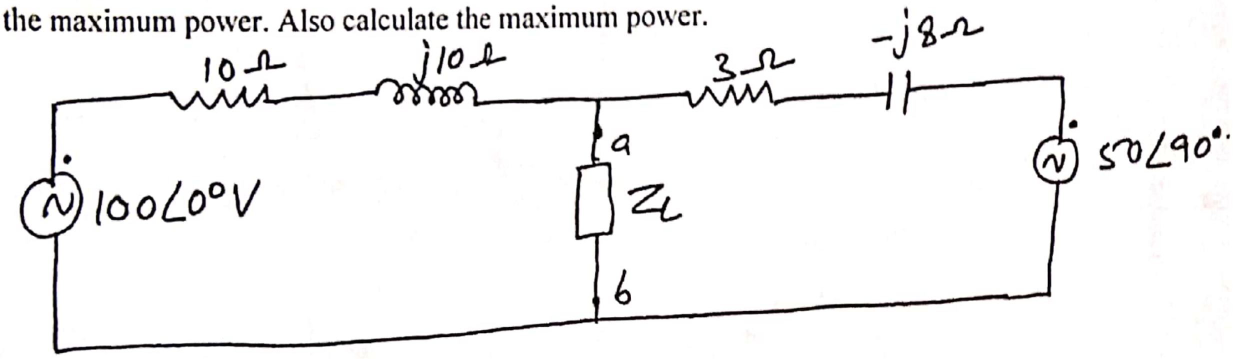


Fig. 11

12. Verify the Reciprocity theorem in the network shown in the Fig.12.

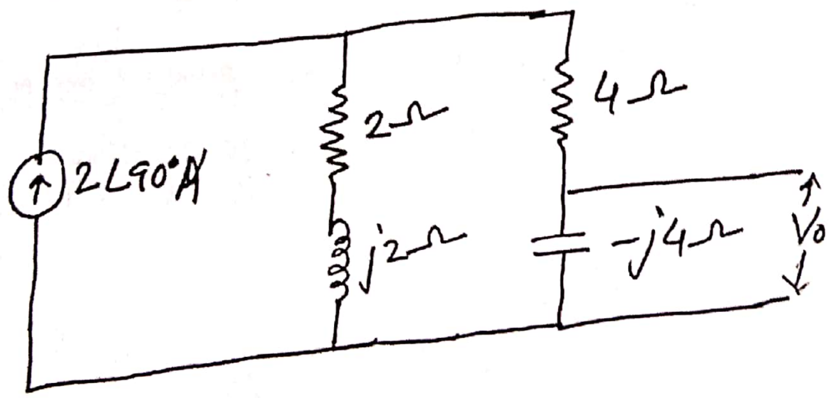


Fig. 12

13. State and Prove the maximum Power Transfer theorem.