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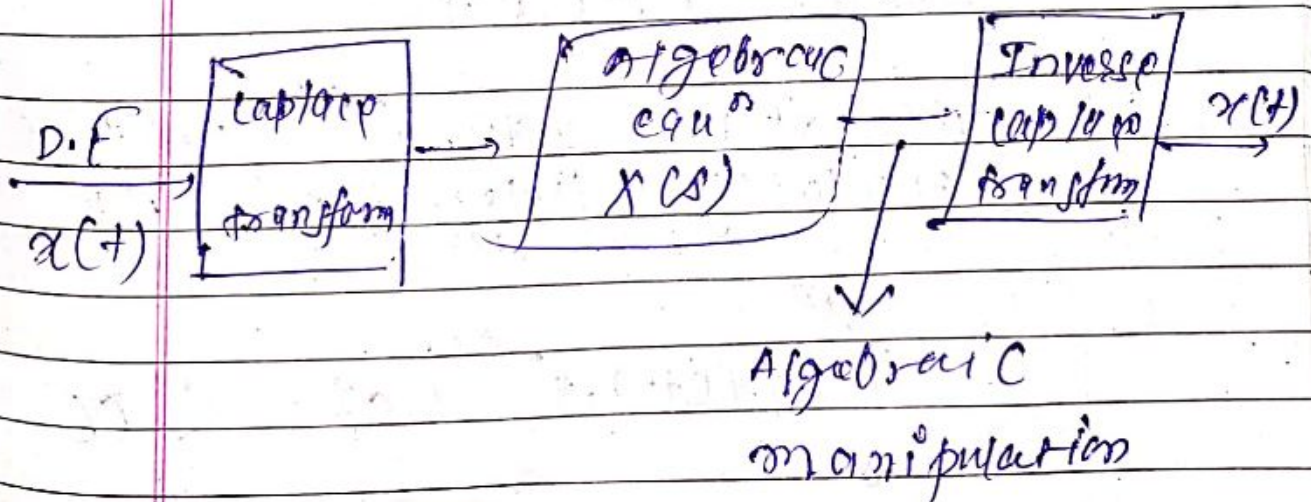
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UNIT 8-4

- 1) Laplace transform and properties
- 2) Partial fraction
- 3) singularity $8x^n$
- 4) waveform synthesis
- 5) Analysis of RC, RL, RLC circuit with & without initial condⁿ using Laplace transformation.
- 6) Evaluation of initial condition.

⇒ Laplace transform and properties (For Dis-Cont)

- 1) Determinⁿ of CF
- 2) Determinⁿ of P.I
- 3) Determinⁿ of arbitrary const



$$\mathcal{L}[x(t)] = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \cdot dt = X(s)$$

where

$s = \text{complex number}$

Bilateral L.T

$$s = \sigma + j\omega$$

Two-sided L.T

$$x(t) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \cdot dt \rightarrow \text{unilateral or one-sided L.T}$$

Note:

→ Necessary and sufficient condition

1) $x(t)$ must be linear or piecewise linear in finite time interval

2) the term $x(t) \cdot e^{-st}$ must be absolutely integrable. i.e.

$$\int_{-\infty}^{\infty} |x(t) \cdot e^{-st}| \cdot dt < \infty$$



The reason for which Laplace transform converges is known as reason of convergence and it is decided by Range of

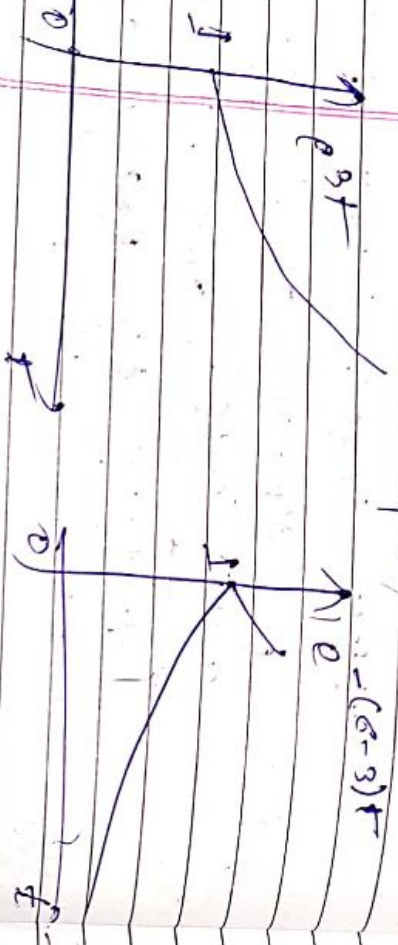
$$x(t) = e^{3t} \cdot u(t)$$

$$\mathcal{L}[e^{3t} \cdot u(t)] = \int_{-\infty}^{\infty} e^{3t} \cdot u(t) \cdot e^{-st} \cdot dt$$

$$= \int_0^{\infty} e^{3t} \cdot e^{-st} \cdot dt = \int_0^{\infty} e^{-(s-3)t} \cdot dt$$

will converge if $s > 3$

$$s > 3 \Rightarrow \mathcal{L}[e^{3t} \cdot u(t)] > 3$$



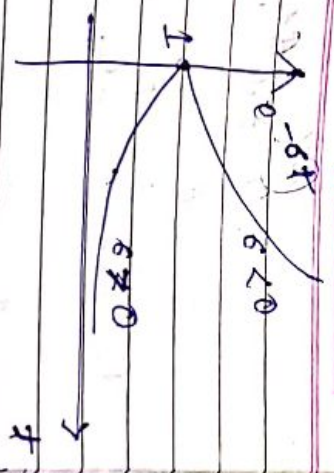
Find the Laplace transform of $u(t)$

① $\mathcal{L}[u(t)] = \int_{-\infty}^{\infty} u(t) \cdot e^{-st} dt$

$$= \int_0^{\infty} 1 \cdot e^{-st} dt = \left(-\frac{1}{s} \right) \Big|_0^{\infty} = \frac{1}{s}$$

$s > 0 \rightarrow \text{converge}$
 $s < 0 \rightarrow \text{diverge}$
 $\text{Re}(s) > 0$

$s = (\sigma + j\omega)t$



Que ② Find the Laplace transform of $\delta(t)$ (unit impulse)

$\mathcal{L}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-st} dt$

$$= e^{-st} \Big|_{-\infty}^{\infty} = 1$$

Que ③ Ramp t^n

$\delta(t) = t \cdot u(t)$

$$= \int_0^{\infty} t \cdot e^{-st} dt$$

$$\mathcal{L}[t^n] = \int_{-\infty}^{\infty} t^n \cdot u(t) \cdot e^{-st} dt$$

$\frac{1}{s}$ = integrator
 s = differentiator

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$$= \int_0^{\infty} f(t) \cdot e^{-st} \cdot dt$$

$$R(s) = \frac{1}{s^2} = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s} [V(s)]$$

$$V(s) = s \cdot R(s)$$

- ④ exponential form e^{at} and e^{-at}
- ⑤ complex expo form $e^{-j\omega t}$, $e^{j\omega t}$
- ⑥ sine & cosine form $\cos \omega t$ $\frac{\omega}{s^2 + \omega^2}$
- ⑦ damped sinusoidal form

⑧ $e^{-at} \sin \omega t$, $e^{-at} \cos \omega t$ $\frac{s\omega}{s^2 + (s+a)^2}$

⑧ hyperbolic sine & cosine form

⑨ Parabolic

$$f^2 = \frac{2}{s^3}$$

$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

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Ques Find the response $i(t)$ when a unit step is applied by unit step form

