Signals And Systems KEC-403
Mandatory Assignment

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## Section -A

### Question 1:

Given $h(n) = \left(\frac{1}{2}\right)^n u(n)$ —①

and $h(n) - A\, h(n-1) = \delta(n)$ —②

From ① and ②,

$$\left(\frac{1}{2}\right)^n u(n) - A\left(\frac{1}{2}\right)^{n-1} \cdot u(n-1) = \delta(n)$$

For $n=1$, $\frac{1}{2}\cdot 1 - A \cdot 1 \cdot 1 = 0$

$\Rightarrow \boxed{A = \frac{1}{2}}$ Ans

### Question -2

To prove : $x(t) * \delta(t) = x(t)$

We know by definition of convolution of two signals,

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

but $\delta(t-\tau) = 1 \; ; \tau = t$
$\qquad\qquad\quad 0 \; ; \tau \neq t$

$\therefore \boxed{x(t) * \delta(t) = x(t)}$ Hence Proved

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### Question 3:

For a system to be causal, the impulse respons h(t) of the system must use only the present and past values of the input to determine the output.

This requirement is a necessary and sufficient condition for a system to be causal.

# Question 4.

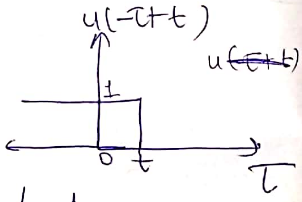
Given $x(t) = u(t)$
$h(t) = u(t)$

$\therefore\ y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau) \cdot u(t-\tau)\, d\tau$$

$$= \int_{0}^{\infty} u(t-\tau)\, d\tau$$




$$= \int_{0}^{t} d\tau$$

$$= \tau\Big|_0^t = t \text{ Ans}$$

$\therefore\ \boxed{y(t) = t}$

# Question 5.

Limitations of Fourier Transform –

i) The signal should have finite number of discontinuities.

ii) The signal ~~to~~ should have a finite averag value over the time period T.

iii) It must have finite number of maximas and minimas in period T.

This conditions are called Drichelet's Condition.

# Question 6.

Given $x(n) = a^n$ for $0<a<1$

$$X(k) = \sum_{n=0}^{N-1} x(n) \exp\left(-j\frac{2\pi k}{N} n\right)$$

$$= 1 + a e^{-j\frac{2\pi k}{N}} + a^2 e^{-j\frac{2\pi k \cdot 2}{N}} + \dots + a^{N-1} \cdot e^{-j\frac{2\pi k}{N} \cdot (N-1)}$$

$\Rightarrow$ This is a GP of $a = 1$
$r = a e^{-j\frac{2\pi k}{N}}$

$$X(k) = \frac{a(1-r^n)}{1-r}$$

$$= \frac{1 - a e^{-j\frac{2\pi k}{\cancel{N}}(\cancel{N})}}{1 - a e^{-j\frac{2\pi k}{N}}}$$

$$= \frac{1 - a e^{-j 2\pi k}}{1 - a e^{\frac{-j2\pi k}{N}}}$$ Ans

## Question 7.

z-transform, $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

and DTFT, $X(\omega) = \sum_{-\infty}^{\infty} x(n) e^{-j\omega n}$

$\therefore$ We can find DTFT from z-transform by

$$\boxed{X(z)\Big|_{z=e^{j\omega}} = X(\omega)}$$

## Question 8.

By the property of ROC of Z-transform,

If $x(n)$ is a infinite duration anti-causal sequence, ROC is interior of the circle with radius a ie

$|z| < a$

## Question 9.

Natural Response : It is the system's response to initial conditions with all external forces set to zero.

Forced Response : It is the system's response to an external stimulus with zero initial conditions.

## Question 10.

$x(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1, 2\}$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

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$$= x(-2)\, z^{-(-2)} + x(-1)\, z + x(0)\, z^{0} + x(1)\, z^{-1} + x(2)\, z^{-2} + x(3)\, z^{-3} + x(4)\, z^{-4}$$

$$= 2z^{2} + 4z + 5 + 7z + z^{-3} + 2z^{-4}$$

Ans

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## Section-B

### Question 11.

Given $h(t) = e^{-2t} \sin 3t \, u(t)$

The system is said to be BIBO stable if

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

where $h(t)$ = Impulse Response of system

$$\therefore \int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |e^{-2t} \sin 3t \, u(t)|\, dt$$

$$= \int_{0}^{\infty} e^{-2t} \sin 3t \, dt$$

$$\left[ \int e^{ax} \sin bx \, dx = \frac{1}{a^2+b^2} e^{ax} (a \sin bx - b \cos bx) \right]$$

$$= \left[ \frac{1}{[(-2)^2+3^2]} e^{-2t} (-2 \sin 3t - 3 \cos 2t) \right]_{0}^{\infty}$$

$$= \frac{1}{13} [\, 0 - 1 \cdot (0 - 3) \,]$$

$$\frac{1}{4+9} = \frac{1}{13}$$

$$= \frac{3}{13}$$ ie finite value

therefore system is stable.

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### Question 12.

$x(n) = \{1, \underset{\uparrow}{2}, 3\}$ and $h(n) = \{1, \underset{\uparrow}{2}, 3, 3\}$

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$