Mini Project: Systems Thinking

Consider the following system dynamics of a 2-link manipulator:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau}, \tag{1}$$

$$\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix}, \mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix},$$

$$M_{11} = (m_1 + m_2)l_1^2 + m_2 l_2 (l_2 + 2l_1 \cos(q_2)),$$

$$M_{12} = m_2 l_2 (l_2 + l_1 \cos(q_2)), M_{22} = m_2 l_2^2,$$

$$\mathbf{C} = \begin{bmatrix} -m_2 l_1 l_2 \sin(q_2)\dot{q}_2 & -m_2 l_1 l_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ 0 & m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} m_1 l_1 g \cos(q_1) + m_2 g (l_2 \cos(q_1 + q_2) + l_1 \cos(q_1)) \\ m_2 g l_2 \cos(q_1 + q_2) \end{bmatrix}$$

where (m_1, l_1, q_1) and (m_2, l_2, q_2) denote the mass, length and joint angle positions of link 1 and 2 respectively.

The following parametric values are selected: $m_1 = 10$kg, $m_2 = 5$kg, $l_1 = 0.2$m, $l_2 = 0.1$m, $g = 9.81$m/s^2. The joint angles are initially at positions $[q_1(0)\ q_2(0)] = [0.1\ 0.1]$rad.

The objective is to bring the the joint angles from the initial position to $[q_1\ q_2] = [0\ 0]$.

Q. Via MATLAB simulations (choose P, I, and D gains of your choice) show differences in responses (i.e., plot q_1 vs. t and q_2 vs. t) when (i) PD (ii) PI and (iii) PID controllers are applied separately.