# 02-24-2021-shift-2-1-15

AI24BTECH11011 - Himani Gourishetty

1) Let $a, b \in \mathbb{R}$. If the mirror image of the point $\mathbf{P}(a, 6, 9)$ with respect to the line $\frac{(x-3)}{7} = \frac{(y-2)}{5} = \frac{(z-1)}{-9}$ is $(20, b, -a, -9)$, then $|a+b|$ is equal to:
   a) 86
   b) 88
   c) 84
   d) 90
2) Let f be a twice differentiable function defined on $\mathbb{R}$ such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in \mathbb{R}$. If $|f(x)\, f'(x)\, f'(x)\, f''(x)| = 0$, for all $x \in \mathbb{R}$, then the value of $f(1)$ lies in the interval:
   a) $(9, 12)$
   b) $(6, 9)$
   c) $(3, 6)$
   d) $(0, 3)$
3) A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is :
   a) $\frac{1}{2\sqrt{2}}$
   b) $\frac{1}{\sqrt{7}}$
   c) $\sqrt{7}-1$
   d) $2\sqrt{2}-1$
4) The probability that two randomly selected subsets of the set $\{1, 2, 3, 4, 5\}$ have exactly two elements in their intersection, is:
   a) $\frac{65}{2^7}$
   b) $\frac{135}{2^9}$
   c) $\frac{65}{2^8}$
   d) $\frac{35}{2^7}$
5) The vector equation of the plane passing through the intersection of the planes $\mathbf{r}.\left(\hat{i}+\hat{j}+\hat{k}\right) = 1$ and $\mathbf{r}\left(\hat{i}-2\hat{j}\right) = -2$ and the points$(1, 0, 2)$ is:
   a) $\mathbf{r}.\left(\hat{i}-7\hat{j}+3\hat{k}\right) = \frac{7}{3}$
   b) $\mathbf{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right) = 7$
   c) $\mathbf{r}.\left(3\hat{i}+7\hat{j}+3\hat{k}\right) = 7$
   d) $\mathbf{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right) = \frac{7}{3}$
6) If $\mathbf{P}$ is a point on the parabola $y = x^2+4$ which is closest to the straight line $y = 4x-1$, then the co-ordinates of $\mathbf{P}$ are :
   a) $(-2, 8)$
   b) $(1, 5)$
   c) $(3, 13)$
   d) $(2, 8)$
7) Let $a, b, c$ be in arithmetic progression. Let the centroid of the triangle with vertices $(a, c), (2, b)$ and $(a, b)$ be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If $\alpha, \beta$ are the roots of the equation $ax^2+bx+1 = 0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is:

a) $\frac{71}{256}$
b) $\frac{-69}{256}$
c) $\frac{69}{256}$
d) $\frac{-71}{256}$

8) The value of the integral, $\int_1^3 \lfloor x^2 - 2x - 2 \rfloor dx$ where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to $x$, is :
a) -4
b) -5
c) $-\sqrt{2} - \sqrt{3} - 1$
d) $-\sqrt{2} - \sqrt{3} + 1$

9) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4 \end{cases}$$

Let $A = \{x \in \mathbb{R} : f \text{ is increasing}\}$.Then $A$ is equal to :
a) $(-5, 4) \cup (4, \infty)$
b) $(-5, \infty)$
c) $(-\infty, -5) \cup (4, \infty)$
d) $(-\infty, -5) \cup (-4, \infty)$

10) If the curve $y = ax^2 + bx + c, x \in \mathbb{R}$ passes through the point $(1, 2)$ and the tangent line to this curve at origin is $y = x$, then the possible values of $a, b, c$ are:
a) $a = 1, b = 1, c = 0$
b) $a = -1, b = 1, c = 1$
c) $a = 1, b = 0, c = 1$
d) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$

11) The negation of the statement $\neg p \wedge (p \vee q)$ is:
a) $\neg p \wedge q$
b) $p \wedge \neg q$
c) $\neg p \vee q$
d) $p \vee \neg q$

12) For the system of linear equations:$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbb{R}$
Consider the following statements:
(A) The system has a unique solution if $k \neq 2, k \neq -2$.
(B) The system has a unique solution if $k = -2$.
(C) The system has a unique solution if $k = 2$.
(D) The system has no solution if $k = 2$.
(E) The system has an infinite number of solutions if $k \neq -2$.
a) (B) and (E) only
b) (C) and (D) only
c) (A) and (D) only
d) (A) and (E) only

13) For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?
a) $x^2 + 9y^2 = 9$
b) $2x^2 - 18y^2 = 9$
c) $y^2 = \frac{x}{6\sqrt{3}}$
d) $x^2 + y^2 = 7$

14) The angle of elevation of a jet plane from a point $A$ on the ground is 60°. After a flight of 20 seconds at the speed of $432\frac{km}{hour}$, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is:
   a) $1200\sqrt{3}m$
   b) $1800\sqrt{3}m$
   c) $3600\sqrt{3}m$
   d) $2400\sqrt{3}m$
15) For the statements p and q, consider the following compound statements:
   (a) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
   (b) $((p \vee q) \wedge \neg p) \rightarrow q$
   a) $(a)$ is a tautology but not $(b)$
   b) $(a)$ and $(b)$ both are not tautologies
   c) $(a)$ and $(b)$ both are tautologies
   d) $(a)$ is a tautology but not $(b)$