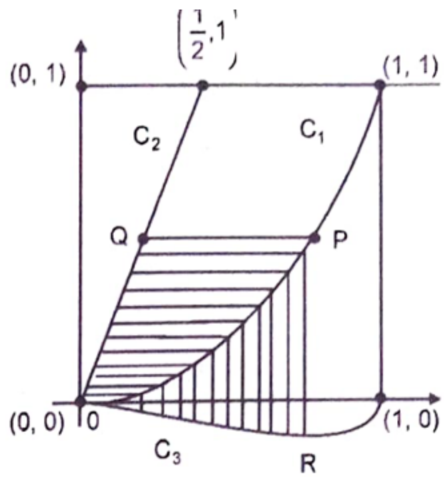
18/A/E/36-49

AI24BTECH11011 - HIMANI GOURISHETTY

1) Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x$, $0 \le x \le 1$ respectively. Let C_3 be graph of a function y=$f(x)$, $0 \le x \le 1$, $f(0)$=0. For a point **P** on C_1, let the lines through **P**, parallel to the axes, meet $C_2$and C_3 at **Q** and **R** respectively(see figure). If for every position of **P** (on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function of $f(x)$. (1998-8 Marks)



2) Integrate $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$ (1999-2 Marks)

3) Let $f(x)$ be a continuos function given by

$$f(x) = \begin{cases} 2x, |x| \le 1 \\ x^2 + ax + b, |x| > 1 \end{cases} \quad (1)$$

4) Find the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$. (1999-10marks)

5) For $x > 0$, letf(x)= $\int_e^x \frac{\ln t}{1+t}dx$.Find the function $f(x) + f\left(\frac{1}{x}\right)$ and show that $f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2}$. Here, $\ln t$=$\log t$. (2000-5Marks)

6) Let $b \neq 0$ and for $j = 0, 1, 2, \ldots\ldots, n$, S_j be the area of the region bounded by the y-axis and the curve $xe^{ay} = \sin$ by $\frac{jr}{b} \le y \le \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots\ldots, S_n$ are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$.
. (2001-5Marks)

7) Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x|$ and $y = 2$, which lies to the right of the line $x = 1$
. (2002-5 Marks)

8) If f is an even function then prove that $\int_0^{\frac{\pi}{2}} f(\cos 2x) \cos x dx = \sqrt{2} \int_0^{\frac{\pi}{4}} f(\sin 2x) \cos x dx$
. (2003-2Marks)

9) If $y(x) = \int_{\frac{\pi^2}{16}}^{x^2} \frac{\cos x \cos \sqrt{\theta}}{1+\sin^2 \sqrt{\theta}}dx$, then find $\frac{dy}{dx}$ at $x = \pi$
. (2004-2Marks)

10) Find the value of $\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \frac{\pi+4x^3}{2-\cos\left(|x|+\frac{\pi}{3}\right)}dx$
. (2004-4Marks)

11) Evaluate $\int_0^{\pi} e^{\cos x} \left((2 \sin\left(\frac{1}{2} \cos x\right) + 3 \cos\left(\frac{1}{2} \cos x\right)) \sin x\right) d$
. (2005-2Marks)

12) Find the area bounded by the curves
$x^2 = y, x^2 = -y$ and $y^2 = 4x - 3$ (2005-4Marks)

13) $f(x)$ is a differentiable function and $g(x)$ is double differentiable function such that $f(x) \le 1$ and $f'(x) = g(x)$. if $f^2(0) + g^2(0) = 9$. Prove that there exist some $c \in (-3, 3)$ such that $g(c).g'(c) < 0$.
. (2005-6Marks)

14) $$\begin{pmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{pmatrix} \begin{pmatrix} f(-1) \\ f(1) \\ f(2) \end{pmatrix} = \begin{pmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{pmatrix}$$

$f(x)$ is a quadratic function and its maximum value occurs at a point **V**. **A** is a point of intersection of y=$f(x)$ with x axis and point **B** is such that chord AB subtends a right angle at **V**. Find the area enclosed by $f(x)$ and chord AB. (2005-6Marks)

15) The value of $5050\frac{\int_0^1(1-x^50)^100}{\int 0^1(1-x^50)^101}dx$ (2006-6M)