01-08-2020-shift-2-1-15

AI24BTECH11011 - Himani Gourishetty

1) Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is . (january 2020)
   a) 0.10
   b) 0.20
   c) 0.01
   d) 0.02
2) Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S: (january 2020)
   a) is a singleton.
   b) is an empty set.
   c) contains at least four elements.
   d) contains exactly two elements.
3) The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is: (january 2020)
   a) 4.01
   b) 3.99
   c) 3.98
   d) 4.02
4) Let $\mathbf{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}, \mathbf{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ be two vectors. If $\mathbf{c}$ is a vector such that $\mathbf{b} \times \mathbf{c} = \mathbf{b} \times \mathbf{a}$ and $\mathbf{a}.\mathbf{b} = 0$ then $\mathbf{c}.\mathbf{b}$ is equal to: (january 2020)
   a) $\frac{1}{2}$
   b) $\frac{-3}{2}$
   c) $\frac{-1}{2}$
   d) -1
5) Let $f : (1, 3) \to \mathbb{R}$ be a function defined by $f(x) = \frac{x\lfloor x \rfloor}{x^2+1}$, where $\lfloor x \rfloor$ denotes the greatest integer $\leq x$. Then the range of f is: (january 2020)
   a) $(\frac{2}{5}, \frac{3}{5}] \cup (\frac{3}{4}, \frac{4}{5})$
   b) $(\frac{2}{5}, \frac{4}{5}]$
   c) $(\frac{3}{5}, \frac{4}{5})$
   d) $(\frac{2}{5}, \frac{1}{2}) \cup (\frac{3}{5}, \frac{4}{5}]$
6) If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$, then: (january 2020)
   a) $\alpha + \beta = -30$
   b) $\alpha - \beta = -132$
   c) $\alpha + \beta = 60$
   d) $\alpha - \beta = 60$
7) If a hyperbola passes through the point $(10, 16)$ and it has vertices at $(\pm 6, 0)$, then the equation of the normal at $\mathbf{P}$ is: (january 2020)

a) $3x + 4y = 94$
b) $x + 2y = 42$
c) $2x + 5y = 100$
d) $x + 3y = 58$

8) $\lim_{x \to 0} \frac{\int_0^x t\sin(10t)dt}{x}$ is equal to: (january 2020)
a) 0
b) $\frac{1}{10}$
c) $\frac{-1}{10}$
d) $\frac{-1}{5}$

9) If a line, $y = mx + c$ is a tangent to the circle, $(x-3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then: (january 2020)
a) $c^2 + 7c + 6 = 0$
b) $c^2 - 6c + 7 = 0$
c) $c^2 - 7c + 6 = 0$
d) $c^2 + 6c + 7 = 0$

10) Let $\alpha = \frac{(-1+i\sqrt{3})}{2}$. If $\alpha = (1+\alpha)\sum_{k=0}^{100} a^{2k}$ and $b = \sum_{k=0}^{100} a^{3k}$, then a and b are the roots of the quadratic equation: (january 2020)
a) $x^2 + 101x + 100 = 0$
b) $x^2 + 102x + 101 = 0$
c) $x^2 - 102x + 101 = 0$
d) $x^2 - 101x + 100 = 0$

11) The mirror image of the point $(1, 2, 3)$ in a plane is $\left(\frac{-7}{3}, \frac{-4}{3}, \frac{-1}{3}\right)$. Which of the following points lies on this plane? (january 2020)
a) $(1, -1, 1)$
b) $(-1, -1, 1)$
c) $(1, 1, 1)$
d) $(-1, -1, -1)$

12) The length of the perpendicular from the origin, on the normal to the curve,$x^2 + 2xy - 3y^2 = 0$ at the point $(2, 2)$ is (january 2020)
a) 2
b) $2\sqrt{2}$
c) $4\sqrt{2}$
d) $\sqrt{2}$

13) Which of the following statements is a tautology? (january 2020)
a) $\neg(p \wedge \neg q) \to (p \vee q)$
b) $(\neg p \vee \neg q) \to (p \wedge q)$
c) $p \wedge (\neg q) \to (p \wedge q)$
d) $\neg(p \vee \neg q) \to (p \vee q)$

14) If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then: (january 2020)
a) $\frac{1}{6} < I^2 < \frac{1}{2}$
b) $\frac{1}{8} < I^2 < \frac{1}{4}$
c) $\frac{1}{9} < I^2 < \frac{1}{8}$
d) $\frac{1}{16} < I^2 < \frac{1}{9}$

15) If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to: (january 2020)
a) 6I-A

b) A-6I
c) 4I-A
d) A-4I