02-24-2021-shift-2-1-15

AI24BTECH11011 - Himani Gourishetty

1) Let $a, b \in \mathbb{R}$. If the mirror image of the point $\mathbf{P}(a, 6, 9)$ with respect to the line $\frac{(x-3)}{7} = \frac{(y-2)}{5} = \frac{(z-1)}{-9}$ is $(20, b, -a, -9)$, then $|a+b|$ is equal to: (February 2021)
   a) 86
   b) 88
   c) 84
   d) 90
2) Let f be a twice differentiable function defined on $\mathbb{R}$ such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in \mathbb{R}$. If $|f(x) f'(x) f'(x) f''(x)| = 0$, for all $x \in \mathbb{R}$, then the value of $f(1)$ lies in the interval: (February 2021)
   a) $(9, 12)$
   b) $(6, 9)$
   c) $(3, 6)$
   d) $(0, 3)$
3) A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is : (February 2021)
   a) $\frac{1}{2\sqrt{2}}$
   b) $\frac{1}{\sqrt{7}}$
   c) $\sqrt{7} - 1$
   d) $2\sqrt{2} - 1$
4) The probability that two randomly selected subsets of the set $\{1, 2, 3, 4, 5\}$ have exactly two elements in their intersection, is: (February 2021)
   a) $\frac{65}{2^7}$
   b) $\frac{135}{2^9}$
   c) $\frac{65}{2^8}$
   d) $\frac{35}{2^7}$
5) The vector equation of the plane passing through the intersection of the planes $\mathbf{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\mathbf{r}\left(\hat{i} - 2\hat{j}\right) = -2$ and the points$(1, 0, 2)$ is: (February 2021)
   a) $\mathbf{r}.\left(\hat{i} - 7\hat{j} + 3\hat{k}\right) = \frac{7}{3}$
   b) $\mathbf{r}.\left(\hat{i} + 7\hat{j} + 3\hat{k}\right) = 7$
   c) $\mathbf{r}.\left(3\hat{i} + 7\hat{j} + 3\hat{k}\right) = 7$
   d) $\mathbf{r}.\left(\hat{i} + 7\hat{j} + 3\hat{k}\right) = \frac{7}{3}$
6) If $\mathbf{P}$ is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of $\mathbf{P}$ are : (February 2021)
   a) $(-2, 8)$
   b) $(1, 5)$
   c) $(3, 13)$
   d) $(2, 8)$
7) Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a, c), (2, b)$ and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is: (February 2021)

a) $\frac{71}{256}$
b) $\frac{-69}{256}$
c) $\frac{69}{256}$
d) $\frac{-71}{256}$

8) The value of the integral, $\int_1^3 \lfloor x^2 - 2x - 2 \rfloor dx$ where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x, is : (February 2021)
   a) -4
   b) -5
   c) $-\sqrt{2} - \sqrt{3} - 1$
   d) $-\sqrt{2} - \sqrt{3} + 1$
9) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4 \end{cases}$$

   Let $A = \{x \in \mathbb{R} : f \text{ is increasing}\}$.Then A is equal to : (February 2021)
   a) $(-5, 4) \cup (4, \infty)$
   b) $(-5, \infty)$
   c) $(-\infty, -5) \cup (4, \infty)$
   d) $(-\infty, -5) \cup (-4, \infty)$
10) If the curve $y = ax^2 + bx + c, x \in \mathbb{R}$ passes through the point $(1, 2)$ and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are: (February 2021)
   a) $a = 1, b = 1, c = 0$
   b) $a = -1, b = 1, c = 1$
   c) $a = 1, b = 0, c = 1$
   d) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$
11) The negation of the statement $\neg p \wedge (p \vee q)$ is: (February 2021)
   a) $\neg p \wedge q$
   b) $p \wedge \neg q$
   c) $\neg p \vee q$
   d) $p \vee \neg q$
12) For the system of linear equations:$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbb{R}$
   Consider the following statements:
   (A) The system has a unique solution if $k \neq 2, k \neq -2$.
   (B) The system has a unique solution if $k = -2$.
   (C) The system has a unique solution if $k = 2$.
   (D) The system has no solution if $k = 2$.
   (E) The system has an infinite number of solutions if $k \neq -2$.

   (February 2021)
   a) (B) and (E) only
   b) (C) and (D) only
   c) (A) and (D) only
   d) (A) and (E) only
13) For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$? (February 2021)
   a) $x^2 + 9y^2 = 9$
   b) $2x^2 - 18y^2 = 9$

c) $y^2 = \frac{x}{6\sqrt{3}}$
d) $x^2 + y^2 = 7$

14) The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of $432\frac{km}{hour}$, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is: (February 2021)

a) $1200\sqrt{3}m$
b) $1800\sqrt{3}m$
c) $3600\sqrt{3}m$
d) $2400\sqrt{3}m$

15) For the statements p and q, consider the following compound statements: (February 2021)

(a) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
(b) $((p \vee q) \wedge \neg p) \rightarrow q$

(February 2021)

a) (a) is a tautology but not (b)
b) (a) and (b) both are not tautologies
c) (a) and (b) both are tautologies
d) (a) is a tautology but not (b)