

# 18/A/E/36-49

AI24BTECH11011 - HIMANI GOURISHETTY

**36** Let  $C_1$  and  $C_2$  be the graphs of the functions  $y = x^2$  and  $y = 2x, 0 \leq x \leq 1$  respectively. Let  $C_3$  be graph of a function  $y=f(x), 0 \leq x \leq 1, f(0)=0$ . For a point P on  $C_1$ , let the lines through P, parallel to the axes, meet  $C_2$  and  $C_3$  at Q and R respectively (see figure). If for every position of P (on  $C_1$ ), the areas of the shaded regions OPQ and ORP are equal, determine the function of  $f(x)$ . (1998-8 Marks)

**37** Integrate  $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}$  (1999-2 Marks)

**38** Let  $f(x)$  be a continuous function given by

$$f(x) = \begin{cases} 2x, & |x| \leq 1 \\ x^2 + ax + b, & |x| > 1 \end{cases}$$

Find the area of the region in the third quadrant bounded by the curves  $x = -2y^2$  and  $y = f(x)$  lying on the left of the line  $8x + 1 = 0$ . (1999-10 marks)

**39** For  $x > 0$ , let  $f(x) = \int_e^x \frac{\ln t}{1+t}$ . Find the function  $f(x) + f(\frac{1}{x})$  and show that  $f(e) + f(\frac{1}{e}) = \frac{1}{2}$ . Here,  $\ln = \log e$ . (2000-5 Marks)

**40** Let  $b \neq 0$  and for  $j=0,1,2,\dots,n$ ,  $S_j$  be the area of the region bounded by the y-axis and the curve  $xe^{ay} = \sin by$  by  $\frac{j\pi}{b} \leq y \leq \frac{(j+1)\pi}{b}$ . Show that  $S_0, S_1, S_2, \dots, S_n$  are in geometric progression. Also, find their sum for  $a=-1$  and  $b=\pi$ . (2001-5 Marks)

**41** Find the area of the region bounded by the curves  $y = x^2$ ,  $y = -2 - x$  and  $y = 2$ , which lies to the right of the line  $x = 1$ . (2002-5 Marks)

**42** If  $f$  is an even function then prove that  $\int_0^{\frac{\pi}{2}} f(\cos 2x) \cos x = \sqrt{2} \int_0^{\frac{\pi}{4}} f(\sin 2x) \cos x$  (2003-2 Marks)

**43** If  $y(x) = \int_{\frac{\pi}{16}}^{x^2} \frac{\cos x \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}}$ , then find  $\frac{dy}{dx}$  at  $x = \pi$  (2004-2 Marks)

**44** Find the value of  $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos(|x| + \frac{\pi}{3})}$  (2004-4 Marks)

**45** Evaluate  $\int_0^\pi e^{\cos x} (2 \sin(\frac{1}{2} \cos x) + 3 \cos(\frac{1}{2} \cos x)) \sin x$  (2005-2 Marks)

**46**

Find the area bounded by the curves

$$x^2 = y, x^2 = -y \text{ and } y^2 = 4x - 3 \quad (2005-4 \text{ Marks})$$

**47**

$f(x)$  is a differentiable function and  $g(x)$  is double differentiable function such that  $|f(x)| \leq 1$  and  $f'(x) = g(x)$ . If  $f^2(0) + g^2(0) = 9$ . Prove that there exist some  $c \in (-3, 3)$  such that  $g(c) \cdot g''(c) < 0$ . (2005-6 Marks)

**48**

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$$

$f(x)$  is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of  $y = f(x)$  with x axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by  $f(x)$  and chord AB. (2005-6 Marks)

**49** The value of  $5050 \frac{\int_0^1 (1-x^5)^{100}}{\int_0^1 (1-x^5)^{101}}$  2006-6M