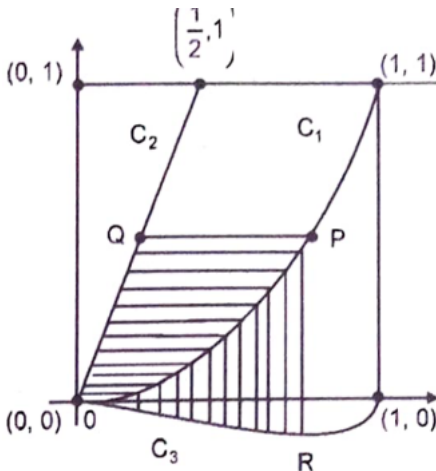


18/A/E/36-49

AI24BTECH11011 - HIMANI GOURISHETTY

36 Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x$, $0 \leq x \leq 1$ respectively. Let C_3 be graph of a function $y = f(x)$, $0 \leq x \leq 1$, $f(0) = 0$. For a point P on C_1 , let the lines through P , parallel to the axes, meet C_2 and C_3 at Q and R respectively (see figure). If for every position of P (on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function of $f(x)$. (1998-8 Marks)



37 Integrate $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}$ (1999-2 Marks)

38 Let $f(x)$ be a continuous function given by

$$f(x) = \begin{cases} 2x, & |x| \leq 1 \\ x^2 + ax + b, & |x| > 1 \end{cases}$$

Find the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$. (1999-10 marks)

39 For $x > 0$, let $f(x) = \int_e^x \frac{\ln t}{1+t}$. Find the function $f(x) + f(\frac{1}{x})$ and show that $f(e) + f(\frac{1}{e}) = \frac{1}{2}$. Here, $\ln = \log e$. (2000-5 Marks)

40 Let $b \neq 0$ and for $j = 0, 1, 2, \dots, n$, S_j be the area of the region bounded by the y-axis and the curve $xe^{ay} = \sin by$ by $\frac{jr}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \dots, S_n$ are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$. (2001-5 Marks)

41 Find the area of the region bounded by the curves $y = x^2$, $y = -2 - x$ and $y = 2$, which lies to the right of the line $x = 1$. (2002-5 Marks)

42 If f is an even function then prove that $\int_0^{\frac{\pi}{2}} f(\cos 2x) \cos x = \sqrt{2} \int_0^{\frac{\pi}{4}} f(\sin 2x) \cos x$. (2003-2 Marks)

43 If $y(x) = \int_{\frac{\pi}{16}}^{\frac{x^2}{16}} \frac{\cos x \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}}$, then find $\frac{dy}{dx}$ at $x = \pi$. (2004-2 Marks)

44 Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos(|x| + \frac{\pi}{3})}$. (2004-4 Marks)

45 Evaluate $\int_0^\pi e^{\cos x} (2 \sin(\frac{1}{2} \cos x) + 3 \cos(\frac{1}{2} \cos x)) \sin x$. (2005-2 Marks)

46 Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$. (2005-4 Marks)

47 $f(x)$ is a differentiable function and $g(x)$ is double differentiable function such that $|f(x)| \leq 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 9$. Prove that there exist some $c \in (-3, 3)$ such that $g(c) \cdot g''(c) < 0$. (2005-6 Marks)

48

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$$

$f(x)$ is a quadratic function and its maximum value occurs at a point V . A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V . Find the area enclosed by $f(x)$ and chord AB . (2005-6 Marks)

49 The value of $5050 \int_0^1 \frac{(1-x^5)^{100}}{0^1(1-x^5)^{101}}$ 2006-6M