

# 08-26-2021-shift-1-16-30

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- 1) If  $\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$ ,  $i = \sqrt{-1}$ , and  $\mathbf{Q} = \mathbf{A}^T \mathbf{B} \mathbf{A}$ , then the inverse of the matrix  $\mathbf{A} \mathbf{Q}^{2021} \mathbf{A}^T$  is equal to: (august 2021)
- $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$
  - $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$
  - $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$
  - $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$
- 2) If the sum of an infinite GP  $a, ar, ar^2, ar^3, \dots$  is 15 and the sum of the squares of its each term is 150, then the sum of  $ar^2, ar^4, ar^6, \dots$  is : (august 2021)
- $\frac{5}{2}$
  - $\frac{1}{2}$
  - $\frac{25}{2}$
  - $\frac{9}{2}$
- 3) The value of  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{t=0}^{2n-1} \frac{n^2}{n^2 + 4t^2}$  is: (august 2021)
- $\frac{1}{2} \tan^{-1}(2)$
  - $\frac{1}{2} \tan^{-1}(4)$
  - $\tan^{-1}(4)$
  - $\frac{1}{4} \tan^{-1}(4)$
- 4) Let  $ABC$  be a triangle with  $\mathbf{A}(-3, 1)$  and  $\angle ACB = \theta, 0 < \theta < \frac{\pi}{2}$ . If the equation of the median through  $\mathbf{B}$  is  $2x + y - 3 = 0$  and the equation of the angle bisector of  $\mathbf{C}$  is  $7x - 4y - 1 = 0$ , then  $\tan \theta$  is equal to: (august 2021)
- $\frac{1}{2}$
  - $\frac{3}{2}$
  - $\frac{4}{3}$
  - $\frac{3}{4}$
- 5) If the truth value of the Boolean expression  $((p \vee q) \wedge (q \rightarrow r) \wedge (\neg r)) \rightarrow (p \wedge q)$  is false, then truth values of the statements  $p, q, r$  respectively can be: (august 2021)
- T F T
  - F F T
  - T F F
  - F T F

## I. SECTION-B

- 1) Let  $z = \frac{1-i\sqrt{3}}{2}, i = \sqrt{-1}$ . Then the value of  $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \dots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$  is \_\_\_\_\_ . (august 2021)

- 2) The sum of all integral values of  $k$  ( $k \neq 0$ ) for which the equation  $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$  in  $x$  has no real roots, is \_\_\_\_\_. (august 2021)
- 3) Let the line  $L$  be the projection of the line  $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$  in the plane  $x - 2y - z = 3$ . If  $d$  is the distance of the point  $(0, 0, 6)$  from  $L$ , then  $d^2$  is equal to \_\_\_\_\_. (august 2021)
- 4) If  ${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \dots + 15 \cdot {}^{15}P_{15} = {}^qP_r = {}^qP_{r-s}$ ,  $0 \leq s \leq 1$ , the  ${}^{q+s}C_{r-s}$  is equal to \_\_\_\_\_. (august 2021)
- 5) A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is  $k(\text{meter})$ , then  $\left(\frac{4}{\pi} + 1\right)k$  is equal to \_\_\_\_\_. (august 2021)
- 6) The area of the region  $S = \{(x, y) : 3x^2 \leq 4y \leq 6x + 24\}$  is \_\_\_\_\_. (august 2021)
- 7) The locus of a point, which moves such that the sum of squares of its distances from the points  $(0, 0), (1, 0), (0, 1), (1, 1)$  is 18 units, is a circle of diameter  $d$ . Then  $d^2$  is equal to \_\_\_\_\_. (august 2021)
- 8) If  $y = y(x)$  is an implicit function of  $x$  such that  $\log_e(x + y) = 4xy$ , then  $\frac{d^2y}{dx^2}$  at  $x = 0$  is equal to \_\_\_\_\_. (august 2021)
- 9) The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is \_\_\_\_\_. (august 2021)
- 10) Let  $a, b \in \mathbb{R}, b \neq 0$ . Define a function

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x-1), & \text{for } x \leq 0 \\ \frac{\tan 2x - \sin 2x}{bx^3}, & \text{for } x > 0 \end{cases}$$

If  $f$  is continuous at  $x = 0$ , then  $10 - ab$  is equal to \_\_\_\_\_. (august 2021)