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1) If $0 < x < \frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is
   a) $4\sqrt{(1-x^2)}\left(1-2x^2\right)$
   b) $4x\sqrt{(1-x^2)}\left(1-2x^2\right)$
   c) $2x\sqrt{(1-x^2)}\left(1-4x^2\right)$
   d) $4\sqrt{(1-x^2)}\left(1-4x^2\right)$
2) Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is
   a) $(\neg p) \wedge q$
   b) $p \wedge (\neg q)$
   c) $(\neg p) \vee (\neg q)$
   d) $(\neg p) \wedge (\neg q)$
3) Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then $54P(X \leq 2)$ is equal to
   a) $\frac{73}{27}$
   b) $\frac{146}{27}$
   c) $\frac{146}{81}$
   d) $\frac{126}{81}$
4) The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)} dx$ is equal to
   a) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\frac{x}{2}+\frac{\pi}{6}}\right| + C$
   b) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\frac{x}{2}+\frac{\pi}{3}}\right| + C$
   c) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\frac{x}{2}+\frac{\pi}{12}}\right| + C$
   d) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\frac{x}{2}-\frac{\pi}{6}}\right| + C$
5) The area bounded by the curves $y = \left|x^2-1\right|$ and $y = 1$ is
   a) $\frac{2}{3}\left(\sqrt{2}+1\right)$
   b) $\frac{4}{3}\left(\sqrt{2}-1\right)$
   c) $2\left(\sqrt{2}-1\right)$
   d) $\frac{8}{3}\left(\sqrt{2}-1\right)$

I. SECTION-B

1) Let $A = \{1,2,3,4,5,6,7\}$ and $B = \{3,6,7,9\}$. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is _____ .
2) The largest value of a, for which the perpendicular distance of the plane containing the lines $r = \left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+a\hat{j}-\hat{k}\right)$ and $r = \left(\hat{i}+\hat{j}\right)+\mu\left(-\hat{i}+\hat{j}-a\hat{k}\right)$ from the point $(2,1,4)$ is $\sqrt{3}$, is _____ .
3) Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1,2,3,4,5 and 6 without repetition of digits. Then the total number of such numbers is _____ .

4) If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then $m+n$ is equal to
5) If the sum of solutions of the system of equations $2sin^2\theta - \cos\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $\lfloor 0, 2\pi \rfloor$ is $k\pi$, then k is equal to _____ .
6) The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to _____ .
7) The plane passing through the line : $L : lx - y + 3(1-l)z = 1, x + 2y - z = 2$ and perpendicular to the plane $3x + 2y + z = 6$ is $3x - 8y + 7z = 4$. If θ is the acute angle between the line L and the y-axis, then $415\cos^2\theta$ is equal to _____ .
8) Suppose $y = y(x)$ be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x and y-intercept of the tangent to the curve at $x = 0$, then the value of $a - 4b$ is equal to _____ .
9) Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.
10) The number of matrices $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in \{-1, 0, 1, 2, 3, 4, \cdots, 10\}$, such that $\mathbf{A} = \mathbf{A}^{-1}$, is _____ .