1-1.6.16

AI24BTECH11011 - Himani Gourishetty

1) Find the values of k if the points $\mathbf{A}\,(k+1, 2k)$, $\mathbf{B}\,(3k, 2k+3)$, $\mathbf{C}\,(5k-1, 5k)$ are collinear.
Solution Given,

Vertex	Coordinates
A	$\begin{pmatrix} k+1 \\ 2k \end{pmatrix}$
B	$\begin{pmatrix} 3k \\ 2k+3 \end{pmatrix}$
C	$\begin{pmatrix} 5k-1 \\ 5k \end{pmatrix}$

For Points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ to be collinear if

$$rank\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix} = 1 \tag{1}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 3k \\ 2k+3 \end{pmatrix} - \begin{pmatrix} k+1 \\ 2k \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} 2k-1 \\ 3 \end{pmatrix} \tag{3}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} 5k-1 \\ 5k \end{pmatrix} - \begin{pmatrix} k+1 \\ 2k \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 4k-2 \\ 3k \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix} = \begin{pmatrix} 2k-1 & 4k-2 \\ 3 & 3k \end{pmatrix} \tag{6}$$

$$\tag{7}$$

Now, rank of this matrix should be 1.

$$\begin{pmatrix} 2k-1 & 4k-2 \\ 3 & 3k \end{pmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 2k-1 & 4k-2 \\ 4-2k & -k+2 \end{pmatrix} \tag{8}$$

$$rank\begin{pmatrix} 2k-1 & 2\,(2k-1) \\ -2\,(-k+2) & -k+2 \end{pmatrix} = 1 \tag{9}$$

$$\tag{10}$$

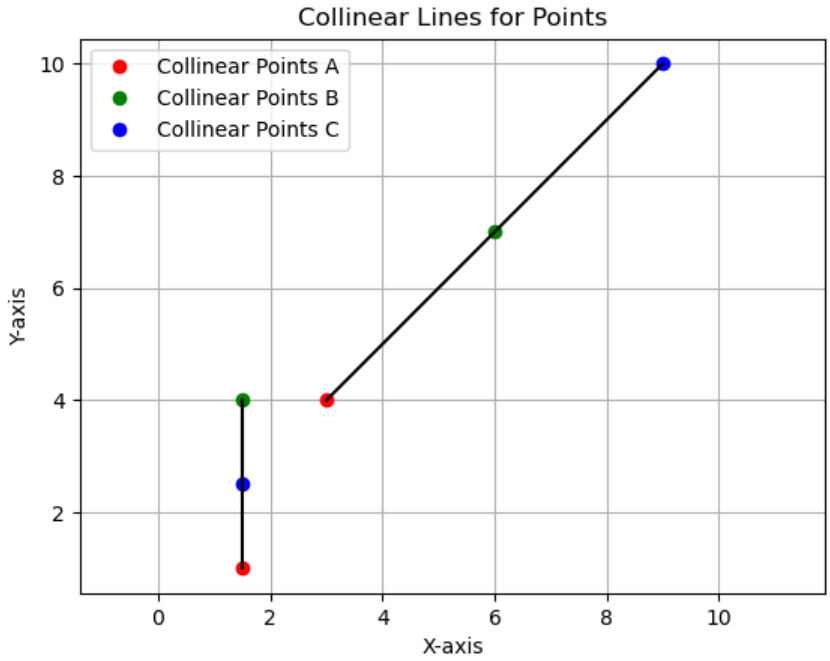
For the rank to be one, either of below should be true,

$$2k-1 = 0 \tag{11}$$

$$-k+2 = 0 \tag{12}$$

then,

$$k = 2; k = 0.5 \tag{13}$$


Collinear Lines for Points
Collinear Points A
Collinear Points B
Collinear Points C
Y-axis
X-axis
0
2
4
6
8
10