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AI24BTECH11011 - Himani Gourishetty

1) Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is
   a) 0.10
   b) 0.20
   c) 0.01
   d) 0.02
2) Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S:
   a) is a singleton.
   b) is an empty set.
   c) contains at least four elements.
   d) contains exactly two elements.
3) The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:
   a) 4.01
   b) 3.99
   c) 3.98
   d) 4.02
4) Let $\mathbf{a} = \hat{i} - 2\hat{j} + \hat{k}, \mathbf{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\mathbf{c}$ is a vector such that $\mathbf{b} \times \mathbf{c} = \mathbf{b} \times \mathbf{a}$ and $\mathbf{a}.\mathbf{b} = 0$ then $\mathbf{c}.\mathbf{b}$ is equal to:
   a) $\frac{1}{2}$
   b) $\frac{-3}{2}$
   c) $\frac{-1}{2}$
   d) -1
5) Let $f : (1, 3) \to \mathbb{R}$ be a function defined by $f(x) = \frac{x\lfloor x \rfloor}{x^2+1}$, where $\lfloor x \rfloor$ denotes the greatest integer $\leq x$. Then the range of f is:
   a) $(\frac{2}{5}, \frac{3}{5}] \cup (\frac{3}{4}, \frac{4}{5})$
   b) $(\frac{2}{5}, \frac{4}{5}]$
   c) $(\frac{3}{5}, \frac{4}{5})$
   d) $(\frac{2}{5}, \frac{1}{2}) \cup (\frac{3}{5}, \frac{4}{5}]$
6) If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$, then:
   a) $\alpha + \beta = -30$
   b) $\alpha - \beta = -132$
   c) $\alpha + \beta = 60$
   d) $\alpha - \beta = 60$
7) If a hyperbola passes through the point $(10, 16)$ and it has vertices at $(\pm 6, 0)$, then the equation of the normal at $\mathbf{P}$ is:
   a) $3x + 4y = 94$
   b) $x + 2y = 42$

c) $2x + 5y = 100$
d) $x + 3y = 58$

8) $\lim_{x\to 0} \frac{\int_0^x t\sin(10t)dt}{x}$ is equal to:
a) 0
b) $\frac{1}{10}$
c) $\frac{-1}{10}$
d) $\frac{-1}{5}$

9) If a line, $y = mx + c$ is a tangent to the circle, $(x-3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then:
a) $c^2 + 7c + 6 = 0$
b) $c^2 - 6c + 7 = 0$
c) $c^2 - 7c + 6 = 0$
d) $c^2 + 6c + 7 = 0$

10) Let $\alpha = \frac{(-1+i\sqrt{3})}{2}$. If $\alpha = (1+\alpha)\sum_{k=0}^{100} a^{2k}$ and $b = \sum_{k=0}^{100} a^{3k}$, then a and b are the roots of the quadratic equation:
a) $x^2 + 101x + 100 = 0$
b) $x^2 + 102x + 101 = 0$
c) $x^2 - 102x + 101 = 0$
d) $x^2 - 101x + 100 = 0$

11) The mirror image of the point $(1, 2, 3)$ in a plane is $\left(\frac{-7}{3}, \frac{-4}{3}, \frac{-1}{3}\right)$. Which of the following points lies on this plane?
a) $(1, -1, 1)$
b) $(-1, -1, 1)$
c) $(1, 1, 1)$
d) $(-1, -1, -1)$

12) The length of the perpendicular from the origin, on the normal to the curve,$x^2 + 2xy - 3y^2 = 0$ at the point $(2, 2)$ is
a) 2
b) $2\sqrt{2}$
c) $4\sqrt{2}$
d) $\sqrt{2}$

13) Which of the following statements is a tautology?
a) $\neg(p \wedge \neg q) \to (p \vee q)$
b) $(\neg p \vee \neg q) \to (p \wedge q)$
c) $p \wedge (\neg q) \to (p \wedge q)$
d) $\neg(p \vee \neg q) \to (p \vee q)$

14) If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then:
a) $\frac{1}{6} < I^2 < \frac{1}{2}$
b) $\frac{1}{8} < I^2 < \frac{1}{4}$
c) $\frac{1}{9} < I^2 < \frac{1}{8}$
d) $\frac{1}{16} < I^2 < \frac{1}{9}$

15) If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to:
a) 6I-A
b) A-6I
c) 4I-A

d) A-4I