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AI24BTECH11011 - Himani Gourishetty

1) The Lebesgue measure of the set $A = \left\{0 \leq x \leq 1 : x\sin\left(\frac{\pi}{2x}\right) \geq 0\right\}$ is
   a) 0
   b) 1
   c) $\ln 2$
   d) $1 - \ln\sqrt{2}$
2) Which of the following statements are **TRUE**?
   P:The set $\left\{x \in \mathbb{R} : |cox| \leq \frac{1}{2}\right\}$ is a compact.
   Q: The set $\{x \in \mathbb{R} : \tan x \text{ is not differentiable}\}$ is complete.
   R:The set $\left\{x \in \mathbb{R} : \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \text{is convergent}\right\}$ is bounded.
   S:The set $\{x \in \mathbb{R} : f(x) = \cos x \text{has a local maxima}\}$ is closed.
   a) P and Q
   b) R and S
   c) Q and S
   d) P and S
3) If a random variable X assumes only positive integral values, with the probability
$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, 4, \cdots,$$
   then $E(X)$ is
   a) $\frac{2}{9}$
   b) $\frac{2}{3}$
   c) 1
   d) $\frac{3}{2}$
4) The probability density function of the random variable X is
$$f(x) = \begin{cases} \frac{1}{\lambda}e^{\frac{-x}{\lambda}}, x > 0 \\ 0, x \leq 0, \end{cases}$$
   where $\lambda > 0$. For testing the hypothesis $H_0 : \lambda = 3$, against $H_A : \lambda = 5$, a test is given as "Reject H_0 if $X \geq 4.5$". The probability of type I error and power of this text are, respectively,
   a) 0.1353 and 0.4966
   b) 0.1827 and 0.379
   c) 0.2021 and 0.4493
   d) 0.2231 and 0.4066
5) The order of the smallest possible non trivial group consisting elements x and y such that $x^7 = y^2 = e$ and $yx = x^4y$ is
   a) 1
   b) 2
   c) 7
   d) 14
6) The number of 5-Sylow subgroup(s) in a group of order 45 is

a) 1
b) 2
c) 3
d) 4

7) The solution of the initial value problem

$$y'' + 2y' + 10y = 6\delta(t), \quad y(0) = 0, y'(0) = 0,$$

where $\delta(t)$ denotes the Dirac-delta function, is
a) $2e^{t}\sin 3t$
b) $6e^{t}\sin 3t$
c) $2e^{-t}\sin 3t$
d) $6e^{-t}\sin 3t$

8) Let $\omega = \cos\frac{2\pi}{3} + isin\frac{2\pi}{3}, \mathbf{M} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \mathbf{N} = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}$ and $G = \langle \mathbf{M}, \mathbf{N} \rangle$ be the group generated by the matrix $\mathbf{M}$ and $\mathbf{N}$ under matrix multiplication. Then
a) $\frac{G}{Z}(G) \cong C_6$
b) $\frac{G}{Z}(G) \cong S_3$
c) $\frac{G}{Z}(G) \cong C_2$
d) $\frac{G}{Z}(G) \cong C_4$

9) The flux of the vector field $\mathbf{u} = x\hat{i} + y\hat{j} + z\hat{k}$ flowing out through the surface of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, a > b > c > 0,$$

is
a) πabc
b) $2\pi abc$
c) $3\pi abc$
d) $4\pi abc$

10) The integral surface satisfying the partial differential equation $\frac{\partial z}{\partial x} + z^2\frac{\partial z}{\partial y} = 0$ and passing through the straight line $x = 1, y = z$ is
a) $(x-1)z + z^2 = y^2$
b) $x^2 + y^2 - z^2 = 1$
c) $(y-z)x + x^2 = 1$
d) $(x-1)z^2 + z = y$

11) The diffusion equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, u = u(x,t), \quad u(0,t) = 0 = u(\pi,t), \quad u(x,0) = \cos x \sin 5x$$

admits the solution
a) $\frac{e^{-36t}}{2}[\sin 6x + e^{20t}\sin 4x]$
b) $\frac{e^{-36t}}{2}[\sin 4x + e^{20t}\sin 6x]$
c) $\frac{e^{-20t}}{2}[\sin 3x + e^{15t}\sin 5x]$
d) $\frac{e^{-36t}}{2}[\sin 5x + e^{20t}\sin x]$

12) Let $f(x)$ and $xf(x)$ be a particular solutions of a differential equation

$$y'' + R(x)y' + S(x)y = 0.$$

Then the solution of the differential equation $y'' + R(x)y' + S(x)y = f(x)$ is
a) $y = \left(\frac{-x^2}{2} + \alpha x + \beta\right) f(x)$

b) $y = \left(\frac{x^2}{2} + \alpha x + \beta\right) f(x)$
c) $y = \left(-x^2 + \alpha x + \beta\right) f(x)$
d) $y = \left(x^2 + \alpha x + \beta\right) f(x)$

13) Let the Legendre equation $\left(1 - x^2\right) y'' - 2xy' + n(n+1) y = 0$ have n^{th} degree polynomial solution $y_n(x)$ such that $y_n(1) = 3$. If $\int_{-1}^{1} \left(y_n^2(x) + y_{n-1}^2(x)\right) dx = \frac{144}{15}$, then n is
a) 1
b) 2
c) 3
d) 4