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1) A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is
   a) 64
   b) 66
   c) 56
   d) 54
2) For $\alpha, \beta \in \mathbb{R}$ and a natural number n,let $A_r = \begin{pmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n-1)}{2} \end{pmatrix}$. Then $2A_{10} - A_8$ is
   a) $4\alpha + 2\beta$
   b) 0
   c) $2\alpha + 4\beta$
   d) $2n$
3) Let the relations $\mathbb{R}_1$ and $\mathbb{R}_2$ on the set $X = \{1, 2, 3, \cdots, 20\}$ be given by $\mathbb{R}_1 = \{(x, y) : 2x - 3y = 2\}$ and $\mathbb{R}_2 = \{(x, y) : -5x + 4y = 0\}$.If M nd N be the minimum number of elements required to be added in $\mathbb{R}_1$ and $\mathbb{R}_2$ respectively, in order to make the relations symmetric, then $M + N$ equals
   a) 10
   b) 8
   c) 12
   d) 16
4) Let $A = \{n \in [100, 700] \cap N :$ n is neither a multiple of 3 nor a multiple of 4$\}$. Then the number of elements in A is
   a) 290
   b) 300
   c) 280
   d) 310
5) A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then $m + n^2$ is equal to
   a) 408
   b) 396
   c) 312
   d) 414

I. SECTION-B

1) For $n \in \mathbb{N}$, if $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{1} n = \frac{\pi}{4}$. then n is equal to _____ .
2) Let $\alpha\beta\gamma = 45; \alpha, \beta, \gamma \in \mathbb{R}$. If $x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$ for some $x, y, z \in \mathbb{R}, xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to _____ .

3) Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3T_{r-1} + 6^r, r = 2, 3, \cdots, n$.If sum of the first n terms of the series is $\frac{1}{5}\left(n^2 - 12n + 39\right)(4 \cdot 6^n - 5 \cdot 3^n + 1)$, then n is equal to _____ .
4) Let $\mathbf{P}$ be the point $(10, -2, -1)$ and $\mathbf{Q}$ be the foot of perpendicular drawn from the point $\mathbf{R}(1, 7, 6)$ on the line passing through the points $(2, -5, 11)$ and $(-6, 7, -5)$.Then the length of the line segment PQ is equal to _____ .
5) If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135,30 and $\frac{10}{3}$ respectively. Then $6\left(n^3 + x^2 + y\right)$ is equal to _____ .
6) Let L_1, L_2 are the lines passing through the point $\mathbf{P}(0, 1)$ and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let $\mathbf{Q}$ and $\mathbf{R}$ be the points on the lines L_1, L_2 such that ΔPQR is an isosceles triangle with base QR.If the slopes of the lines QR are m_1, m_2, then $16\left(m_1^2 + m_2^2\right)$ is equal to _____ .
7) Given the vectors:$\mathbf{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\mathbf{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and a vector $\mathbf{c}$ such that:$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) + \mathbf{b} \times \mathbf{c} = \hat{i} + 8\hat{j} + 13\hat{k}$ with the condition: $\mathbf{a} \cdot \mathbf{c} = 13$ Then, $((24 - \mathbf{b} \cdot \mathbf{c})$ is equal to _____ .
8) Let conic C pass through the point $(4, -2)$ and $\mathbf{P}(x, y), x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point $\mathbf{P}$ be half the slope of the line joining the points $\mathbf{P}$ and $(3, -5)$. If the focal distance of the point $(7, 1)$ on C is d, then 12d equals _____ .
9) Let $r_k = \frac{\int_0^1 \left(1-x^7\right)^k dx}{\int_0^1 \left(1-x^7\right)^{k+1} dx}, k \in \mathbb{N}$. Then the value of $\sum_{k=1}^{10} \frac{1}{7(r_k - 1)}$ is equal to _____ .
10) Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $\left(4 + x_1^2\right)\left(4 + x_2^2\right)\left(4 + x_3^2\right)\left(4 + x_4^2\right) =$ $\frac{125}{16}m$. Then the value of m is _____ .