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AI24BTECH11011 - Himani Gourishetty

1) If $\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}, i = \sqrt{-1}$, and $\mathbf{Q} = \mathbf{A^T B A}$, then the inverse of the matrix $\mathbf{A Q^{2021} A^T}$ is equal to:
   a) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$
   b) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$
   c) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$
   d) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$
2) If the sum of an infinite GP $a, ar, ar^2, ar^3, \cdots$ is 15 and the sum of the squares of its each term is 150, then the sum of $ar^2, ar^4, ar^6, \cdots$ is :
   a) $\frac{5}{2}$
   b) $\frac{1}{2}$
   c) $\frac{25}{2}$
   d) $\frac{9}{2}$
3) The value of $\lim_{n\to\infty} \frac{1}{n} \sum_{t=0}^{2n-1} \frac{n^2}{n^2+4r^2}$ is:
   a) $\frac{1}{2} \tan^{-1}(2)$
   b) $\frac{1}{2} \tan^{-1}(4)$
   c) $\tan^{-1}(4)$
   d) $\frac{1}{4} \tan^{-1}(4)$
4) Let ABC be a triangle with $\mathbf{A}(-3, 1)$ and $\langle ACB = \theta, 0 < \theta < \frac{\pi}{2}$. If the equation of the median through $\mathbf{B}$ is $2x + y - 3 = 0$ and the equation of the angle bisector of $\mathbf{C}$ is $7x - 4y - 1 = 0$, then $\tan\theta$ is equal to:
   a) $\frac{1}{2}$
   b) $\frac{3}{4}$
   c) $\frac{4}{3}$
   d) 2
5) If the truth value of the Boolean expression $((p \vee q) \wedge (q \to r) \wedge (\neg r)) \to (p \wedge q)$ is false, then truth values of the statements p, q, r respectively can be:
   a) T F T
   b) F F T
   c) T F F
   d) F T F

I. SECTION-B

1) Let $z = \frac{1-i\sqrt{3}}{2}, i = \sqrt{-1}$. Then the value of $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \cdots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$ is _____ .

2) The sum of all integral values of $k\,(k \neq 0)$ for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is _____ .
3) Let the line L be the projection of the line $\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$ in the plane $x - 2y - z = 3$. If d is the distance of the point $(0, 0, 6)$ from L, then d^2 is equal to _____ .
4) If ${}^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + \cdots + 15 \cdot {}^{15}P_{15} = {}^qP_r = {}^qP_{r-s}, 0 \leq s \leq 1$, the ${}^{q+s}C_{r-s}$ is equal to _____ .
5) A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is $k(meter)$, then $\left(\frac{4}{\pi} + 1\right)k$ is equal to _____ .
6) The area of the region $S = \{(x, y) : 3x^2 \leq 4y \leq 6x + 24\}$ is _____
7) The locus of a point, which moves such that the sum of squares of its distances from the points $(0, 0), (1, 0), (0, 1)(1, 1)$ is 18 units, is a circle of diameter d. Then d^2 is equal to _____ .
8) If $y = y(x)$ is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to _____ .
9) The number of three-digit even numbers, formed by the digits $0, 1, 3, 4, 6, 7$ if the repetition of digits is not allowed, is _____ .
10) Let $a, b \in \mathbb{R}, b \neq 0$. Define a function

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x - 1), & \text{for } x \leq 0 \\ \frac{\tan 2x - \sin 2x}{bx^3}, & \text{for } x > 0 \end{cases}$$

If f is continous at $x = 0$, then $10 - ab$ is equal to _____ .