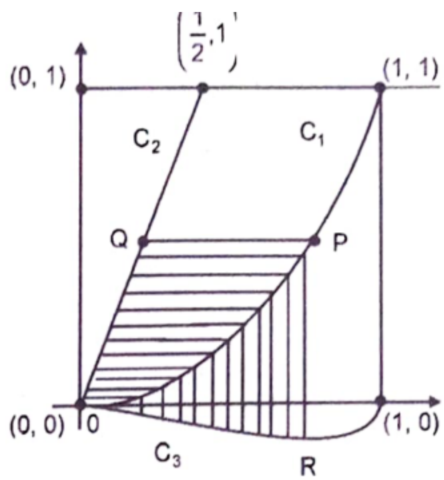
18/A/E/36-49

AI24BTECH11011 - HIMANI GOURISHETTY

1) Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x, 0 \leq x \leq 1$ respectively.Let C_3 be graph of a function y=f(x),$0 \leq x \leq 1$, f(0)=0.For a point P on C_1,let the lines through P,parallel to the axes, meet $C_2$and C_3 at Q and R respectively(see figure).If for every position of P(on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function of f(x). (1998-8 Marks)



2) Integrate $\int_0^{\pi} \frac{e^{cosx}}{e^{cosx}+e^{-cosx}}$ (1999-2 Marks)

3) Let f(x) be a continuos function given by

f(x)=
$$\begin{cases} 2x, & |x| \leq 1 \\ x^2 + ax + b, & |x| > 1 \end{cases}$$

4) Find the area of the region in the third quadrant bounded by the curves x=-2y^2 and y=f(x) lying on the left of the line $8x + 1$=0. (1999-10marks)

5) For $x > 0, let f(x) = \int_e^x \frac{lnt}{1+t}$.Find the function$f(x)+f(\frac{1}{x})$ and show that $f(e)+f(\frac{1}{e}) = \frac{1}{2}$ .Here ,lnt=log t. (2000-5Marks)

6) Let $b \neq 0$ and for j=0,1,2,...,n, S_j be the area of the region bounded by the y-axis and the curve xe^{ay} = sin by $\frac{jr}{b} \leq y \leq \frac{(j+1)\pi}{b}$ .Show that $S_0, S_1, S_2, ..., S_n$ are in geometric progression . Also , find their sum for a=-1and b=pi. . (2001-5Marks)

7) Find the area of the region bounded by the curves $y = x^2$,$y = |2 - x|$ and $y = 2$, which lies to the right of the line $x = 1$. (2002-5 Marks)

8) If f is an even function then prove that $\int_0^{\frac{\pi}{2}} f(cos2x)cox = \sqrt{2}\int_0^{\frac{\pi}{4}} f(sin2x)cosx$ (2003-2Marks)

9) If $y(x) = \int_{\frac{\pi^2}{16}}^{x^2} \frac{cosxcos\sqrt{\theta}}{1+sin^2\sqrt{\theta}}$, then find $\frac{dy}{dx}$ at x=pi . (2004-2Marks)

10) Find the value of $\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}} \frac{\pi+4x^3}{2-cos(|x|+\frac{\pi}{3})}$. (2004-4Marks)

11) Evaluate $\int_0^{\pi} e^{cosx}(2sin(\frac{1}{2}cosx) + 3cos(\frac{1}{2}cosx))sinx$. (2005-2Marks)

12) Find the area bounded by the curves $x^2 = y, x^2 = -y$ and $y^2 = 4x - 3$ (2005-4Marks)

13) f(x) is a differentiable function and g(x) is double differentiable function such that $f(x)| \leq 1$ and f'(x)=g(x). if $f^2(0)$ + $g^2(0)$=9. Prove that there exist some $c \in (-3, 3)$ such that $g(c).g''(c) < 0$. . (2005-6Marks)

14)
$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$$

f(x) is a quadratic function and its maximum value occurs at a point V.A is a point of intersection of y=f(x) with x axis and point B is such that chord AB subtends a right angle at V . Find the area enclosed by f(x) and chord AB. (2005-6Marks)

15) The value of 5050 $\frac{\int_0^1(1-x^50)^100}{\int 0^1(1-x^50)^101}$ (2006-6M)