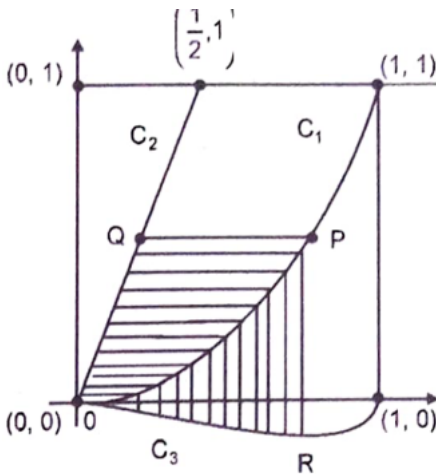


18/A/E/36-49

AI24BTECH11011 - HIMANI GOURISHETTY

- 1) Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x$, $0 \leq x \leq 1$ respectively. Let C_3 be graph of a function $y = f(x)$, $0 \leq x \leq 1$, $f(0) = 0$. For a point P on C_1 , let the lines through P, parallel to the axes, meet C_2 and C_3 at Q and R respectively (see figure). If for every position of P (on C_1), the areas of the shaded regions OPQ and ORP are equal, determine the function of $f(x)$. (1998-8 Marks)



- 2) Integrate $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$ (1999-2 Marks)

- 3) Let $f(x)$ be a continuous function given by $f(x) = \begin{cases} 2x, & |x| \leq 1 \\ x^2 + ax + b, & |x| > 1 \end{cases}$
- 4) Find the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$. (1999-10 marks)
- 5) For $x > 0$, let $f(x) = \int_e^x \frac{\ln t}{1+t} dt$. Find the function $f(x) + f\left(\frac{1}{x}\right)$ and show that $f(e) + f\left(\frac{1}{e}\right) = \frac{1}{2}$. Here, $\ln = \log e$. (2000-5 Marks)
- 6) Let $b \neq 0$ and for $j = 0, 1, 2, \dots, n$, S_j be the area of the region bounded by the y-axis and the curve $xe^{ay} = \sin by$ by $\frac{j\pi}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \dots, S_n$ are in geometric progression. Also, find their sum for $a = -1$ and $b = \pi$. (2001-5 Marks)

- 7) Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x|$ and $y = 2$, which lies to the right of the line $x = 1$. (2002-5 Marks)
- 8) If f is an even function then prove that $\int_0^{\frac{\pi}{2}} f(\cos 2x) \cos x dx = \sqrt{2} \int_0^{\frac{\pi}{4}} f(\sin 2x) \cos x dx$ (2003-2 Marks)
- 9) If $y(x) = \int_{\frac{\pi}{16}}^{x^2} \frac{\cos x \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta$, then find $\frac{dy}{dx}$ at $x = \pi$ (2004-2 Marks)
- 10) Find the value of $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{\pi + 4x^3}{2 - \cos(|x| + \frac{\pi}{3})} dx$ (2004-4 Marks)
- 11) Evaluate $\int_0^\pi e^{\cos x} (2 \sin(\frac{1}{2} \cos x) + 3 \cos(\frac{1}{2} \cos x)) \sin x dx$ (2005-2 Marks)
- 12) Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$ (2005-4 Marks)
- 13) $f(x)$ is a differentiable function and $g(x)$ is double differentiable function such that $f(x) \leq 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 9$. Prove that there exist some $c \in (-3, 3)$ such that $g(c) \cdot g''(c) < 0$. (2005-6 Marks)

14)

$$\begin{pmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{pmatrix} \begin{pmatrix} f(-1) \\ f(1) \\ f(2) \end{pmatrix} = \begin{pmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{pmatrix}$$

$f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB. (2005-6 Marks)

- 15) The value of $5050 \frac{\int_0^1 (1-x^5)^{100} dx}{\int_0^1 (1-x^5)^{101} dx}$ (2006-6M)