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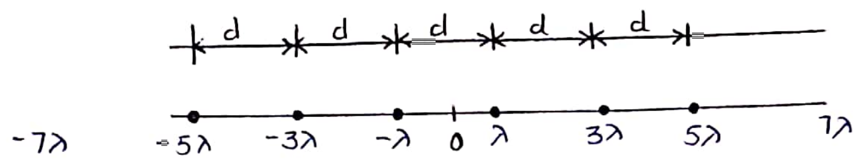
Problem 1 : Determine the average energy of a set of M-ary ASK signals of the form

$$S_m(t) = S_m \psi(t) \quad , \; m = 1, 2, \ldots\ldots M$$

where $S_m = \sqrt{\mathcal{E}_g}\, A_m$ , $m = 1, 2, \ldots M$ and $\psi(t)$ denotes unit energy basis signal. The signals are equally probable with amplitudes that are symmetric about zero and are uniformly spaced with distance 'd' between adjacent amplitudes as shown in figure.




let $d = 2\lambda$

$$\mathcal{E}_m = \|S_m\|^2 = \mathcal{E}_g A_m^2$$

as, the signals are equiprobable

So, $P_m = \frac{1}{M}$ , $m = 1, 2 \ldots\ldots M$

$$\mathcal{E}_{avg} = \sum_{m=1}^{M} P_m \mathcal{E}_m$$

$$= \frac{\mathcal{E}_g}{M} \sum_{m=1}^{M} A_m^2$$

$$A_m = (2m - M - 1)\lambda$$

$$\mathcal{E}_{avg} = \frac{\mathcal{E}_g}{M} \sum_{m=1}^{M} (2m-M-1)^2 \lambda^2 = \frac{\mathcal{E}_g}{M} \sum_{m=1}^{M} \left(4m^2 + (M+1)^2 - 4m(M-1)\right) \lambda^2$$

$$= \frac{\mathcal{E}_g}{M} \left[ 4(1^2 + 2^2 + \ldots M^2) + M(M+1)^2 - 4(1+2+\ldots M)(M-1) \right] \lambda^2$$

$$= \frac{\mathcal{E}_g}{M} \left[ 4\left(\frac{M(M+1)(2M+1)}{6}\right) + M(M^2+1+2M) - 4\frac{(M(M+1))}{2}(M-1) \right] \lambda^2$$

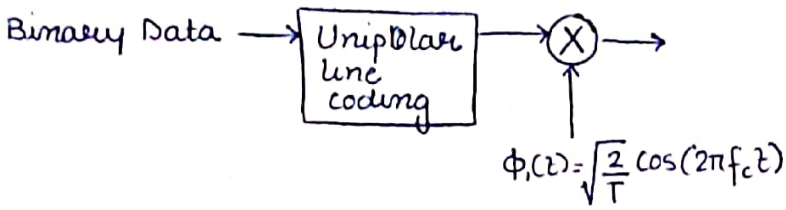
$$= \mathcal{E}_g \left[ \frac{2}{3}(2M^2+3M+1) + M^2+1+2M - 2M^2+2 \right] \lambda^2$$

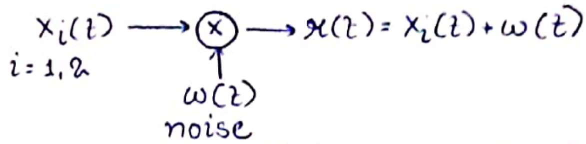
$$\mathcal{E}_{avg} = \mathcal{E}_g \left[ \frac{M^2}{3} + 4M + \frac{11}{3} \right] \lambda^2 \qquad \lambda = \frac{d}{2}$$

$$\mathcal{E}_{avg} = \frac{\mathcal{E}_g d^2 (M^2 + 12M + 11)}{6}$$

**Problem 2:** Derive the expression for the average probability error of the binary ASK modulation scheme corrupted by additive white Gaussian noise with variance $\sigma^2$. Assume that transmission of symbols '1' and '0' are equally likely.

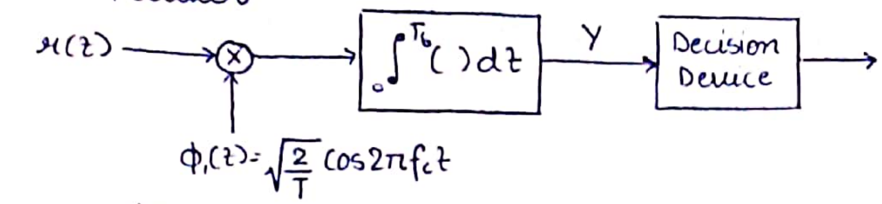
BASK Transmitter




$x_1(t) = \sqrt{E_b}\, \phi_1(t) \rightarrow$ bit '1'
$x_2(t) = \sqrt{E_b}\, \phi_2(t) \rightarrow$ bit '0'

BASK Receiver




$Y > T = \sqrt{E_b}/2 \rightarrow 1$
$Y < T = \sqrt{E_b}/2 \rightarrow 0$
threshold. Detected bit

probability of error analysis.

$$r(t) = \begin{cases} x_1(t) + \omega(t) & \leftarrow \text{bit } 1 \\ 0 + \omega(t) & \leftarrow \text{bit } 2 \quad [\because x_2(t) = 0] \end{cases}$$

$$Y = r(t) = \int_0^{T_b} r(t)\, \phi_1(t)\, dt$$

for bit 1

$$Y = \int_0^{T_b} x_1(t)\phi_1(t)dt + \int_0^{T_b} \omega(t)\, \phi_1(t)\, dt$$

$$= \int_0^{T_b} \sqrt{E_b}\, \phi_1(t)\phi_1(t)\, dt + \int_0^{T_b} \omega(t)\, \phi_1(t) dt$$

$$= \sqrt{E_b} \underbrace{\int_0^{T_b} \phi_1^2(t) dt}_{1} + W \qquad = \sqrt{E_b} + W$$

for bit 0

$$Y = \int_0^{T_b} \omega(t)\, \phi_1(t)\, dt = W$$

observation : $\quad$ $$Y = \begin{cases} \sqrt{E_b} + W & \leftarrow \text{bit } 1 \\ W & \leftarrow \text{bit } 0 \end{cases}$$

to represent guassian we need mean & variance.

mean of observation : $$E[Y] = \begin{cases} E(\sqrt{E_b}) + \cancel{E(W)}^{\,0} = \sqrt{E_b} & \leftarrow \text{bit } 1 \\ E[W] = 0 & \leftarrow \text{bit } 0 \end{cases}$$

Variance of observation

bit-1 $\rightarrow$ $E\{(Y-\bar{Y})^2\} = E\{(\sqrt{E_b} + N - \sqrt{E_b})^2\} = E\{W^2\} \rightarrow$ variance $= \sigma^2$

bit-0 $\rightarrow$ $E\{(Y-\bar{Y})^2\} = E\{(W-0)^2\} = E\{W^2\} = \sigma^2$

when bit 0 is transmitted the likelihood function is

$$f_Y(Y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(Y-\mu_Y)^2}{2\sigma^2}}$$

$$f_Y(Y/0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-Y^2/2\sigma^2}$$

when bit '1' is transmitted the likelihood function is

$$f_Y(Y/1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(Y-\sqrt{E_b})^2/2\sigma^2}$$

The probability of error when symbol 0 is sent and detected as 1.

$$P(1/0) = \int_{\sqrt{E_b}/2}^{\infty} f_Y(Y/0)\, dy = \int_{\sqrt{E_b}/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(Y)^2/2\sigma^2} dy = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\sqrt{E_b}/2}^{\infty} e^{-Y^2/2\sigma^2} dy$$

let $z = \frac{Y}{\sqrt{2}\sigma}$ $\quad \sqrt{2}\sigma\, dz = dy$

$Y = \infty$ , $z = \infty$

$Y = \sqrt{E_b}/2$ , $z = \frac{\sqrt{E_b}}{2\sqrt{2}\sigma}$

$$P(1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\sqrt{E_b}/2\sqrt{2}\sigma}^{\infty} e^{-z^2} \sqrt{2}\sigma\, dz = \left(\frac{1}{\sqrt{\pi}} \int_{\frac{\sqrt{E_b}}{2\sqrt{2}\sigma}}^{\infty} e^{-z^2} dz\right) \times 2 \times \frac{1}{2}$$

$$\left( erf_c(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-z^2} dz \right)$$

$$P(1|0) = \frac{1}{2} erf_c\left(\frac{\sqrt{E_b}}{2\sqrt{2}\sigma}\right)$$

Similarly,

The probability of error when symbol 1 is sent and detected as 0.

$$P(0|1) = \frac{1}{2} erf_c\left(\frac{1}{2}\sqrt{\frac{E_b}{2\sigma^2}}\right)$$

The average of probability error

$$P_e = \frac{P_e(0) + P_e(1)}{2}$$

$$= \frac{1}{2} erf_c\left(\frac{1}{2}\sqrt{\frac{E_b}{2\sigma^2}}\right).$$

Problem 3: Discuss the merits and limitation of M-ary ASK, M-ary PSK and M-ary QAM modulation techniques in terms of power, spectral efficiency and Error performance.

1) merits of M-ary ASK

- it offers high bandwidth efficiency
- it can be used to transmit digital data over optical fiber.

limitations of M-ary ASK

- it offers lower power efficiency
- ASK modulation is very susceptible to noise interference.

2) merits of M-ary PSK

- it is less susceptible to errors compare to ASK modulation & occupies same bandwidth as ASK.

limitations of M-ary PSK

- it has lower bandwidth efficiency

3) merits of M-ary QAM

- QAM increase the efficiency of transmission by utilizing both amplitude & phase variations.

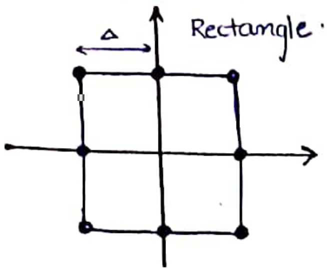
limitations of M-ary QAM

- more susceptible to noise.

more susceptible to noise

Problem 4: Find the average symbol energy $E_s$ for the constellations shown. Consider the same minimum distance of all the constellations as $\Delta$. With the same minimum distance of all the constellations, a more efficient signal constellation is one that has smaller average transmitted energy. Also find the most efficient signal constellation.

Constellation 1

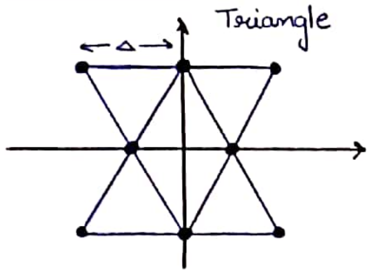



average energy $E_s = \frac{1}{8}\left[4(\Delta)^2 + 4(\sqrt{2}\Delta)^2\right]$

$= \frac{1}{8}\left[12\Delta^2\right] = \frac{3}{2}\Delta^2$

$= 1.5\Delta^2$

Constellation 2

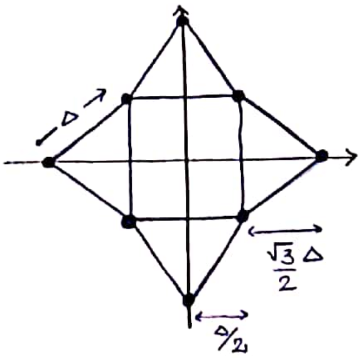



average energy $E_s = \frac{1}{8}\left[2\left(\frac{\Delta}{2}\right)^2 + 2(\Delta)^2 + 4(\sqrt{2}\Delta)^2\right]$

$= \frac{1}{8}\left[\frac{\Delta^2}{2} + 2\Delta^2 + 8\Delta^2\right]$

$= \frac{21}{16}\Delta^2 = 1.3125\Delta^2$

Constellation 3



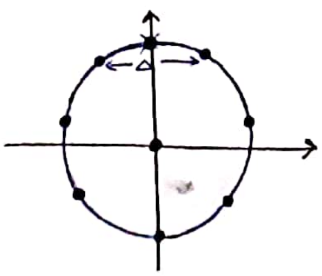

Average energy $E_s = \frac{1}{8}\left[\Delta^2\left(\frac{1+\sqrt{3}}{2}\right)^2 \times 4 + 4\left(\frac{\sqrt{2}\Delta}{2}\right)^2\right]$

$= \frac{1}{8}\left[(1+3+2\sqrt{3})\Delta^2 + 2\Delta^2\right]$

$= \frac{1}{8}\left[6\Delta^2 + 2\sqrt{3}\Delta^2\right]$

$= \frac{3\Delta^2}{4} + \frac{\sqrt{3}\Delta^2}{4} = 1.1830\,\Delta^2$

Constellation 4




Average energy $E_s = \frac{1}{8}\left[7\Delta^2\right] = \frac{7}{8}\Delta^2$

$= 0.875\Delta^2$

as a more efficient signal constellation is one that has a smaller average transmitted energy.

So, Constellation 4 is the more efficient signal as compared to others.

Efficient signal → Constellation 4 > Constellation 3 > Constellation 2 > Constellation 1

Problem 5: For Binary FSK calculate the minimum frequency seperation between two transmit pulses corresponding to transmitted bit '1' and '0' and data rate R?

Two carrier frequencies are used for binary frequency shift keying modulation. $f_1$ & $f_2$.

input is either '0' or '1' in one signaling duration.

$s_1(t) = A\cos 2\pi f_1 t$ , for transmitted bit '1'

$s_2(t) = A\cos 2\pi f_2 t$ , for transmitted bit '0'

frequencies are separated by the minimum amount in the case of orthogonality.

for two orthogonal basis function (inner product is 0).

$\langle S_1, S_2 \rangle = 0$ $\quad \int_0^{T_b} S_1(t)\cdot S_2(t) = 0$

$$\int_0^{T_b}\left[(A\cos 2\pi f_1 t)\cdot(A\cos 2\pi f_2 t)\right] = 0 \quad = \int_0^{T_b} \cos 2\pi f_1 t \cdot \cos 2\pi f_2 t = 0$$

$$\int_0^{T_b}\left(\cos 2\pi(f_1-f_2)t\right) - \cos\left(2\pi(f_1+f_2)t\right)dt = 0$$

$$\left[\frac{\sin 2\pi(f_1-f_2)t}{2\pi(f_1-f_2)} - \frac{\sin 2\pi(f_1+f_2)t}{2\pi(f_1+f_2)}\right]_0^{T_b} = 0$$

$$(f_1+f_2)\sin 2\pi T_b(f_1-f_2) - (f_1-f_2)\sin 2\pi T_b(f_1+f_2) = 0$$

$$f_1\left[\sin 2\pi T_b(f_1-f_2) - \sin 2\pi T_b(f_1+f_2)\right] + f_2\left[\sin 2\pi T_b(f_1-f_2) + \sin 2\pi T_b(f_1+f_2)\right] = 0$$

So, $\sin 2\pi T_b(f_1-f_2) - \sin 2\pi T_b(f_1+f_2) = 0$ .... (i)

and

$\sin 2\pi T_b(f_1-f_2) + \sin 2\pi T_b(f_1+f_2) = 0$ ..... (ii)

Solving equation (i) and (ii)

$\sin 2\pi T_b f_1 = 0$ OR $\cos 2\pi f_2 T_b = 0$ and $\cos 2\pi T_b f_1 = 0$ OR $\sin 2\pi T_b f_2 = 0$

$2\pi T_b f_1 = n\pi$ $\qquad$ $2\pi T_b f_2 = m\pi$

$f_1 = \dfrac{n}{2T_b}$ $\qquad$ $f_2 = \dfrac{m}{2T_b}$

$$f_1 - f_2 = (n-m)\times\frac{1}{2T_b}$$

for two carrier to be orthogonal $\quad aT_1 = bT_2 = T_b$

$R = \dfrac{1}{T_b}$

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