ARCH & GARCH

- **Volatility**
- **ARCH**
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- **Features**
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Volatility in time series

- Volatility in time series is used to study the fluctuations present in the time series data. Some examples of frequently fluctuating time series can be in domains like stock prices, fuel prices, sales prices, and more. These fluctuations in the time series data have to be analyzed appropriately. Volatility in time series data can be a crucial factor that will be considered by various investors to analyze the possible profits/losses that can burst over a period of time.
- Volatility in the time series data can be used to determine certain strategies and shorter outcomes for a particular period of time. Volatility helps us to study the behavioral pattern of the data in the past day or over a period of time and deduce some interpretations of possible profits or losses that can be experienced.

Introduction to ARCH

An ARCH (autoregressive conditionally heteroscedastic) model is a model for the variance of a time series. ARCH models are used to describe a changing, possibly volatile variance. Although an ARCH model could possibly be used to describe a gradually increasing variance over time, most often it is used in situations in which there may be short periods of increased variation. (Gradually increasing variance connected to a gradually increasing mean level might be better handled by transforming the variable.)

ARCH models were created in the context of econometric and finance problems having to do with the amount that investments or stocks increase (or decrease) per time period, so there's a tendency to describe them as models for that type of variable. For that reason, the authors of our text suggest that the variable of interest in these problems might either be $y_t = (x_t - x_{t-1})/x_{t-1}$, the proportion gained or lost since the last time, or $log(x_t/x_{t-1}) = log(x_t) - log(x_{t-1})$, the logarithm of the ratio of this time's value to last time's value. It's not necessary that one of these be the primary variable of interest. An ARCH model could be used for any series that has periods of increased or decreased variance. This might, for example, be a property of residuals after an ARIMA model has been fit to the data.

The ARCH(1) Variance Model

Suppose that we are modeling the variance of a series y_t . The ARCH(1) model for the variance of model y_t is that conditional on y_{t-1} , the variance at time t is

$$(1)\ \mathrm{Var}(y_t|y_{t-1}) = \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2$$

We impose the constraints $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ to avoid negative variance.

Note!
The variance at time t is connected to the value of the series at time t – *1*. A relatively large value of y_{t-1}^2 gives a relatively large value of the variance at time t. This means that the value of y_t is less predictable at time t *−1* than at times after a relatively small value of y_{t-1}^2.

If we assume that the series has mean = 0 (this can always be done by centering), the ARCH model could be written as

$$(2)\ y_t = \sigma_t \epsilon_t,$$

$$\text{with } \sigma_t = \sqrt{\alpha_0 + \alpha_1 y_{t-1}^2},$$

$$\text{and } \epsilon_t \overset{iid}{\sim} (\mu = 0, \sigma^2 = 1)$$

For inference (and maximum likelihood estimation) we would also assume that the ϵ_t are normally distributed.

Possibly Useful Results

Two potentially useful properties of the useful theoretical property of the ARCH(1) model as written in equation line (2) above are the following:

- y_t^2 has the AR(1) model $y_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 +$ error.
  This model will be causal, meaning it can be converted to a legitimate infinite order MA only when $\alpha_1^2 < \frac{1}{3}$
- y_t is white noise when $0 \leq \alpha_1 \leq 1$.

Generalizations

An ARCH(m) process is one for which the variance at time t is conditional on observations at the previous m times, and the relationship is

$$\mathrm{Var}(y_t|y_{t-1},\dots,y_{t-m}) = \sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \cdots + \alpha_m y_{t-m}^2.$$

With certain constraints imposed on the coefficients, the y_t series squared will theoretically be AR(m).

A GARCH (generalized autoregressive conditionally heteroscedastic) model uses values of the past squared observations and past variances to model the variance at time t. As an example, a GARCH(1,1) is

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

In the GARCH notation, the first subscript refers to the order of the y^2 terms on the right side, and the second subscript refers to the order of the σ^2 terms.

Introduction to GARCH

- Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models are generally used for handling time series data which are highly volatile. Some of the domains where volatility in time series can be expected include e-commerce, finance, trading, and more. Time series data with signs of volatility are generally not stationary and can be expected to have a higher deviation from the expected mean value.

- The ability of the GARCH model to handle the moving average component facilitates the GARCH model to handle the dispersion of the time series data with respect to the error term and the time-dependent dispersion over a certain period.

- GARCH models are generally represented as GARCH(p,q). The p parameter in GARCH basically represents the number of lag variances, and the q parameter in the GARCH model is used to represent the number of lag residual errors. The optimal values of p and q can be obtained by taking up certain through certain searching techniques. The easiest way to obtain the optimal values of p and q would be by visualizing the ACF and PACF plots. The lag values in the time series data are very much essential to understand the shifts in the time series data and understanding the effect of autocorrelation among the various parameters.

Features of the GARCH model

As mentioned earlier GARCH models are generally used to handle the moving average and the autoregressive component in the time series data. So let us list down some of the features of the GARCH model.

- GARCH models are generally statistical models used to handle time series data with higher degrees of volatility.
- GARCH models are best suitable for time series data where the variance of the error term is auto-correlated.
- GARCH models are extremely useful to study the volatility of time series data and deduce the possible risks or the period of time to gain maximum profit.

These are some of the features of the GARCH model that makes forecasting time series data with higher degrees of volatility easier.

The necessity of GARCH models

- GARCH models are generally used over the ARCH models, where along with the autoregressive component the model will also have the ability to handle the fluctuating mean in the data. The fluctuating mean in time series data can be interpreted as the moving average, and GARCH models on a whole are used to handle the moving average component along with the autoregressive component in the time series data. The lag variance terms in the time series data are generally known as white noise and GARCH models are very much suitable to handle the noises present in the time series data due to the lag induced by variance. GARCH models are best suitable for data where the error terms possess higher autocorrelation. GARCH models assume that the variance of the error terms follows an autoregressive moving average process.
- So this is why GARCH models are very much essential to capture the dispersion pattern of the time series data from the mean. GARCH models are extremely useful to handle heteroskedastic error parameters. Heteroscedasticity basically describes the irregular pattern of variation of an error term in the time series data. Heteroskedastic error parameters can be interpreted as volatility error terms, and to handle such varying error parameters in time series data GARCH models are very much essential.

References

To understand more about ARCH and GARCH models one can refer to following links:

- https://medium.com/@ranjithkumar.rocking/time-series-model-s-arch-and-garch-2781a982b448
- https://online.stat.psu.edu/stat510/lesson/11/11.1
- https://kevinkotze.github.io/ts-12-volatility/
- https://machinehack.com/story/handling-volatility-in-time-series-by-garch-model