

# On the Validity of Analytic Potential Approximations for Galaxy Disks

ASTR 513, Statistics & Computation, Term Project

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# Outline

- 1 Introduction
- 2 Disk Models
- 3 Motivating the Problem
- 4 Data Description
- 5 Fitting Methodology
- 6 Results
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# Why Analytic Potentials ?

Galaxy dynamics is directly impacted by its gravitational potential.  
Advantages of an analytic potential:

- Intuitive understanding of internal orbits and resonances
- Tracer particle simulations & precision control
- Satellite dynamics
- Harmonic and an-harmonic expansions
- Understanding of phase space behaviour
- Non-equilibrium processes and perturbations

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# Coordinate System

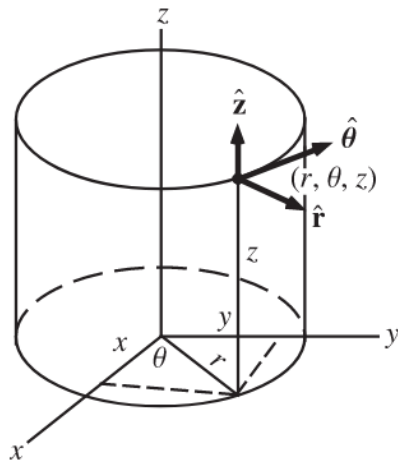


Figure: Cylindrical coordinate system. Source: Wolfram

# Potential - Density Pairs

Potential-Density relation:

$$\nabla^2 \Phi = 4\pi G \rho \quad (1)$$

From observations of the brightness profile of disks,

$$\rho(r, z) = \frac{M}{4\pi a^2 b} \exp\left(-\frac{r}{a}\right) \operatorname{sech}^2\left(\frac{z}{b}\right), \quad (2)$$

$M$  is a normalization mass,  $a$  is the radial scale length,  $b$  is the vertical scale length.

Potential corresponding to this mass distribution is not analytical. Therefore, analytic approximations need to be employed.

# Miyamoto-Nagai Approximation

The Miyamoto-Nagai (1975) disk is the most popular analytical approximation to the exponential density profile:

$$\Phi(r, z) = \frac{GM}{\sqrt{r^2 + \left[a + \sqrt{z^2 + b^2}\right]^2}}, \quad (3)$$

The corresponding density distribution is:

$$\rho(r, z) = \frac{b^2 M}{4\pi} \left[ \frac{ar^2 + \left(a + 3\sqrt{z^2 + b^2}\right) \left(a + \sqrt{z^2 + b^2}\right)^2}{\left[r^2 + \left(a + \sqrt{z^2 + b^2}\right)^2\right]^{\frac{5}{2}} (z^2 + b^2)^{\frac{3}{2}}} \right] \quad (4)$$



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# Validity of Miyamoto - Nagai Approximation ?

The validity of the Miyamoto - Nagai approximation has not been investigated quantitatively in depth. Questions I ask:

- How well does the Miyamoto - Nagai profile fit the exponential disk ?
- Can the same parameters describing the exponential disk be used for the Miyamoto - Nagai approximation ?
- Where does the Miyamoto - Nagai approximation break down ?
- What can be the physical consequences if the Miyamoto-Nagai approximation is not valid ?

I utilize existing simulation data of the equilibrium disk of the Large Magellanic Cloud for this work

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# LMC Equilibrium Disk

I extract the phase space information of stars ( $x, y, z, v_x, v_y, v_z$ ) from Besla et al. 2012 simulations. These simulations contain  $10^6$  star particles ( $M_* = 2500$ ) in an exponential disk, and  $10^5$  CDM particles in a Hernquist halo.

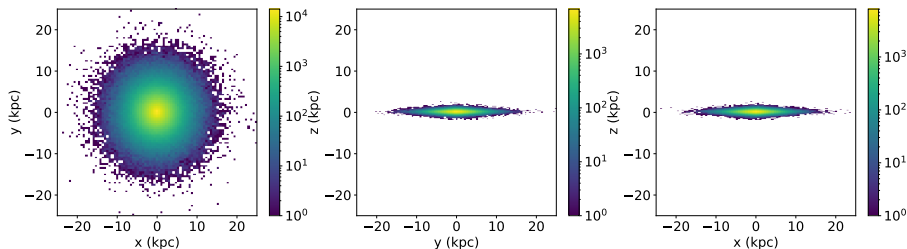


Figure: Stellar density map of the equilibrium LMC disk.

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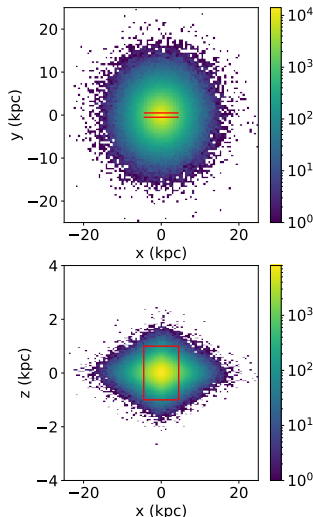
# Obtaining Radial & Vertical Densities

- Extract stars in a cuboidal volume  $x \in (-4.5, 4.5)$  kpc,  $y \in (-0.5, 0.5)$  kpc,  $z \in (-1, 1)$  kpc, and binned them.
- Density:

$$\rho(x, z) = \frac{N(x, z)M_*}{\Delta V}$$

- Error on density:

$$\delta\rho(x, z) = \frac{\sqrt{N(x, z)M_*}}{\Delta V}$$



# Sampling Setup

I use parallel MCMC to sample the posterior distribution of the model parameters. The  $\chi^2$  is defined as:

$$\chi^2 = \sum_{ij} \left( \frac{\rho_{data}(r_i, z_j) - \rho_{model}(r_i, z_j, a, b, M)}{\sigma_{ij}} \right)^2, \quad (5)$$

where  $model \in \{exponential, Miyamoto - Nagai\}$ .

The likelihood is defined as:

$$\mathcal{L} = e^{-\frac{\chi^2}{2}} \quad (6)$$

The parameters  $a, b, \log(M/M_\odot)$  are sampled using uniform priors.

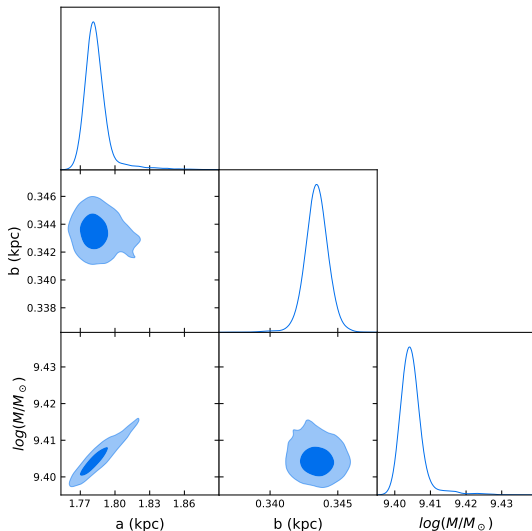
# Fitting of Exponential Profile

Priors:

- $a \in (0.1, 5)$  kpc
- $b \in (0.1, 0.5)$  kpc
- $\log(M/M_{\odot}) \in (9, 10)$

Chain length = 200

No. of // chains = 200





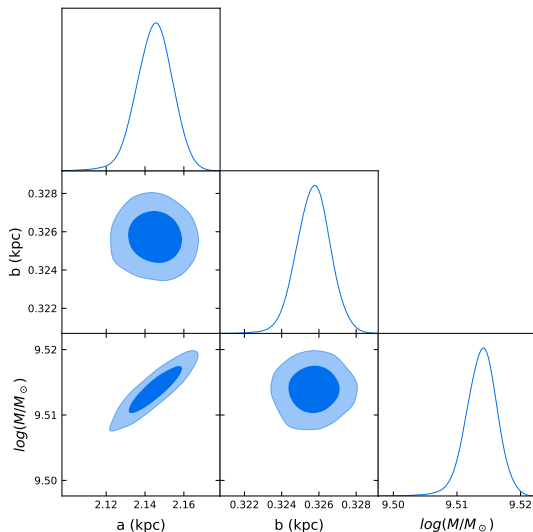
# Fitting of Miyamoto-Nagai Profile

Priors:

- $a \in (0.1, 5)$  kpc
- $b \in (0.1, 0.5)$  kpc
- $\log(M/M_{\odot}) \in (9, 10)$

Chain length = 200

No. of // chains = 200



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# Exponential Profile Fit

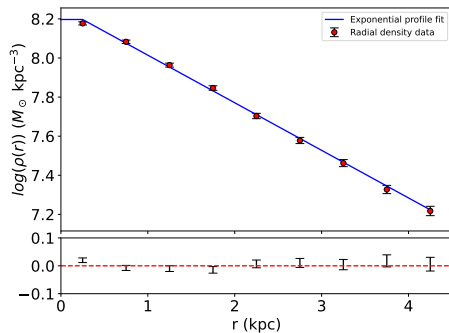


Figure: Radial density profile

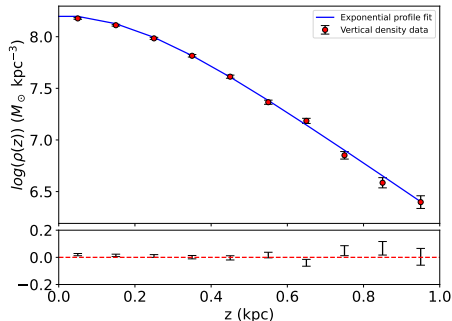


Figure: Vertical density profile

# Miyamoto-Nagai Profile Fit

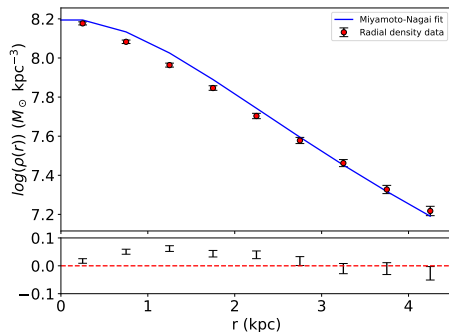


Figure: Radial density profile

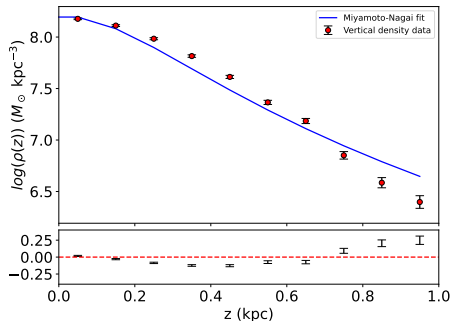


Figure: Vertical density profile

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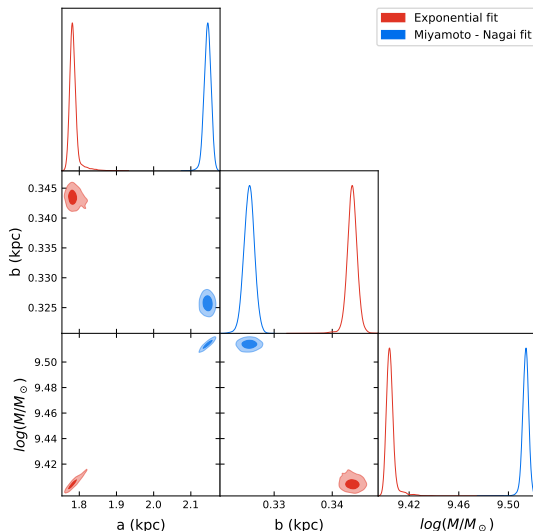
# Comparison of Exponential & Miyamoto-Nagai Fits

Exponential fit:

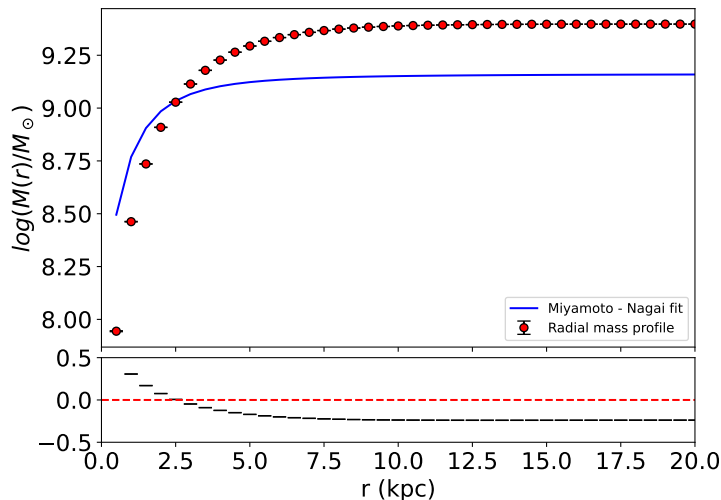
- $a = 1.78 \pm 0.01$  kpc
- $b = 0.343 \pm 0.001$  kpc
- $\log(\frac{M}{M_{\odot}}) = 9.405 \pm 0.004$
- $\chi^2_{red,dof} = 1.03$

Miyamoto-Nagai fit:

- $a = 2.14 \pm 0.01$  kpc
- $b = 0.326 \pm 0.001$  kpc
- $\log(\frac{M}{M_{\odot}}) = 9.514 \pm 0.003$
- $\chi^2_{red,dof} = 13.42$



# Mass Profile Comparison



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I explored quantitatively how well does the Miyamoto-Nagai analytical approximation describe an exponential galaxy disk:

- Miyamoto-Nagai is a considerably worse fit to the exponential disk, particularly in the vertical direction and disk outskirts
- The parameters of the exponential disk cannot be used to describe the Miyamoto-Nagai approximation
- Miyamoto-Nagai approximation can lead to underestimation of forces by a factor of  $\sim 2$  and orbital speeds by a factor of  $\sim 1.5$  in the disk outskirts

Multiple component Miyamoto-Nagai potential, or a combination of Miyamoto-Nagai and Hernquist potential might be the way to go forward.