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Homework Assignment 2
CS 385-B Algorithms

"I pledge my honor that I have abided by the Stevens Honor System" - Himanshu Rana(hrana2)

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4. a) This algorithm computes the sum of the first n squares
   b) multiplication
   c) number of execution = n
   d) $\theta(n)$
   e) A better algorithm that will produce the same result would be $\frac{n(n+1)(2n+1)}{6}$ because that is the same thing as $\sum_{i=1}^{n} i^2$

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1. a) $x(n) = x(n-1) + 5, n > 1, x(1) = 0$
   $x(n-1) = x(n-2) + 5$
   I) $x(n) = x(n-2) + 10$
   $x(n-2) = x(n-3) + 5 + 10$
   II) $x(n) = x(n-3) + 15$
   III) $x(n) = x(n-i) + 5i$
   IV) $n - i = 1 \rightarrow i = n - 1$
   V) $x(n) = x(n-(n-1)) + 5(n-1)$
   $x(n) = x(1) + 5(n-1$
   $x(n) = 5(n-1)$

   b) $x(n) = 3x(n-1), n > 1, x(1) = 4$
   $x(n-1) = 3x(n-2)$
   I) $x(n) = 3(3x(n-2) \rightarrow x(n) = 9x(n-2)$
   $x(n-2) = 3x(n-3)$
   II) $x(n) = 9(3x(n-3) \rightarrow x(n) = 27x(n-3)$
   III) $x(n) = 3^k x(n-k)$
   IV) $n - k = 1 \rightarrow k = n - 1$
   V) $x(n) = 3^{n-1}x\big(n-(n-1)\big) \rightarrow x(n) = 3^{n-1}(4)$

   c) $x(n) = x(n-1) + n, n > 0, x(0) = 0$
   $x(n-1) = x(n-2) + (n-1)$
   I) $x(n) = x(n-2) + (n-1) + n$
   $x(n-2) = x(n-3) + (n-2)$
   II) $x(n) = x(n-3) + (n-2) + (n-1) + n$
   III) $x(n) = x(n-i) + (n-i+1) + (n-i+2) + (n-i+3)$
   IV) $n - i = 0 \rightarrow n = i$
   V) $x(n) = x(n-n) + (n-n+1) + (n-n+2) + (n-n+3)$
   $x(n) = 0 + 1 + 2 + 3 + \cdots \rightarrow \frac{n(n+1)}{2}$

d) $x(n) = x(n/2) + n, n > 1, x(1) = 1$

I) $x(n) = x(2^{k-1}) + 2^k$

II) $x(n) = x(2^{k-2}) + 2^{k-1} + 2^k$

III) $x(n) = x(2^{k-i}) + 2^{k-i+1} + \cdots + 2^k$

IV) $k - i = 1 \to k = i$

V) $x(n) = x(2^{k-k}) + 2^{k-k+1} + 2^{k-k+2} + \cdots + 2^k$

$x(n) = x(1) + 2^1 + 2^2 + \cdots + 2^k$

$1 + 2 + 2^2 + \cdots + 2^k$

$2 * 2^k - 1 \to 2n - 1$

e) $x(n) = x\left(\frac{n}{3}\right) + 1, n > 1, x(1) = 1$

I) $x(n) = 3^{k-1} + 1$

II) $x(n) = 3^{k-2} + 2$

III) $x(n) = 3^{k-i} + i$

IV) $k - i = 1 \to k = i$

V) $x(n) = 3^{k-k} + k$

$n = 3^k \to k = \log_3 n$

$x(n) = 1 + \log_3 n$

3. a) this is the number of times the basic operation (multiplication) is executed:

$x(n) = x(n-1) + 2, x(1) = 0$

I) $x(n) = x(n-2) + 2 + 2$

II) $x(n) = x(n-3) + 2 + 2 + 2$

III) $x(n) = x(n-i) + 2i$

IV) $n - i = 1 \to i = n - 1$

V) $x(n) = x(n - (n-1) + 2(n-1) \to 2n - 2$

b) The non-recursive algorithm does $\sum_{i=2}^{n} 2$ number of multiplications. This simplifies down to 2(n – 1), which is the same as the recursive. However, this new non-recursive algorithm does not take the same space in memory as the recursive algorithm uses a stack.