

Assignment 4 - Theory- Himanshu Aggarwal
MT17015

Q1. (a) $V^1(s_0) = 1$
 $V^1(s_1) = 2$
 $V^1(s_2) = 3$
 $V^1(s_3) = 10$

$$V^2(s_0) = 1 + 0.9(0.5 \times 2 + 0.5 \times 3)$$

$$= 3.25$$

$$V^2(s_1) = \max(2 + 0.9(0.5 \times 2 + 0.5 \times 10),$$

$$2 + 0.9 \times 3)$$

$$= 7.4$$

$$V^2(s_2) = 3 + 0.9 \times 1 = 3.9$$

$$V^2(s_3) = 10 + 0.9 \times 10 = 19$$

$$V^3(s_0) = 1 + 0.9(0.5 \times 7.4 + 0.5 \times 3.9)$$

$$= 5.65$$

$$V^3(s_1) = \max(2 + 0.9(0.5 \times 7.4 + 0.5 \times 19),$$

$$2 + 0.9 \times 3.9)$$

$$= 13.88$$

$$V^3(s_2) = 3 + 0.9 \times 3.25$$

$$= 5.925$$

$$V^3(s_3) = 10 + 0.9 \times 19$$

$$= 27.1$$

(b) Optimal policy of state S_1 is to take action and go to state S_1 again.

(c) (i) False, this can't be determined as there can be multiple actions associated with a state and because of which each state will not converge. In just one value iteration, also, there can be cycles.

(ii) False, $\gamma=1$ can make the future rewards or the rewards that will be gathered later, more desirable, and will not let the state converge after 1 iteration.

(iii) True, all the converged values will be just immediate rewards.

(iv) True, since every state will converge within one iteration each, i.e. after every iteration ~~conver~~ atleast one state will reach optimality.

(v) False, without absorbing state, the value iteration will keep on changing the values of states, making it difficult for states to ~~cat~~ achieve convergence in N iterations.

Q2.

$$\begin{aligned}\text{No. of bits to transmit image} &= 3 \times 8 \times N^2 \\ &= \underline{24N^2}\end{aligned}$$

Now, using k-means we want to quantize the no. of colors ~~is~~ used in the image.

Originally we ~~are~~ have 2^{24} colors available for an image, i.e. we are storing this much types of different colors.

Now using k-means with optimal 'k' we are quantizing the colors to ~~24~~ k colors.

To store values for these k colors, we just need $(\log_2 k)$ bits.

$$\therefore \text{Total bits required to transmit image} = (\log_2 k) \times N^2$$

$$\begin{aligned}\text{Compression Ratio} &= \frac{24 \times N^2}{(\log_2 k) \times N^2} \\ &= \left(\frac{24}{\log_2 k} \right)\end{aligned}$$