(i) $G(s) = \sum_{k=0}^{\infty} a_k s^k = P(X \leq 0)s^0 + P(X \leq 1)s^1 + P(X \leq 2)s^2 + \cdots$

$\quad = p_0 + (p_0 + p_1)s + (p_0 + p_1 + p_2)s^2 + \cdots$

$\quad = p_0(1 + s + s^2 + \cdots) + p_1(s + s^2 + \cdots) + \cdots$

$\quad = (p_0 + p_1 s + p_2 s^2 + \cdots)(1-s)^{-1} = (1-s)^{-1} P(s).$

(b) $G(s) = \sum_{k=0}^{\infty} a_k s^k = P(X < 0)s^0 + P(X < 1)s^1 + P(X < 2)s^2 + \cdots$

$\quad = p_0 s + (p_0 + p_1)s^2 + \cdots = p_0(s + s^2 + \cdots) + p_1(s^2 + s^3 + \cdots) + \cdots$

$\quad = \frac{1}{1-s}\left[p_0 s + p_1 s^2 + \cdots \right] = \frac{s}{1-s} P(s).$

(c) $G(s) = \sum_{k=0}^{\infty} a_k s^k = P(X \geq 0)s^0 + P(X \geq 1)s + P(X > 2)s^2 + \cdots$

$\quad = (1 - P(X < 0))s^0 + (1 - P(X < 1))s + (1 - P(X < 2))s^2 + \cdots$

$\quad = 1 + (1 - p_0)s + (1 - p_0 - p_1)s^2 + \cdots$

$\quad = \frac{1}{1-s} - \frac{s}{1-s} P(s) = \frac{s(1 - P(s))}{1-s}$

(d) $G(s) = \sum_{k=0}^{\infty} a_k s^k = P(X > 1)s^0 + P(X > 2)s^1 + P(X > 3)s^2 + \cdots$

$\quad = (1 - P(X \leq 1))s^0 + (1 - P(X \leq 2))s + (1 - P(X \leq 3))s^2 + \cdots$

$\quad = (1 - p_0 - p_1)s^0 + (1 - p_0 - p_1 - p_2)s + (1 - p_0 - p_1 - p_2 - p_3)s^2 + \cdots$

$\quad = \frac{1}{1-s} - p_0(1 + s + s^2 + \cdots) - p_1(1 + s + s^2 + \cdots) - \cdots$

$\quad = \frac{1}{1-s} - \frac{p_0}{1-s} - \left\{ \frac{p_1}{1-s} + \frac{p_2 s}{1-s} + \cdots \right\}$

$\quad = \frac{1}{1-s} - \frac{p_0}{1-s} - \frac{1}{s(1-s)}\sum_{k=1}^{\infty} p_k s^k$

$\quad = \frac{1}{1-s} - \frac{p_0}{1-s} - \frac{1}{s(1-s)}\left\{ P(s) - p_0 \right\}$

$\quad = \frac{1}{1-s} - \frac{P(s)}{s(1-s)} + \frac{p_0}{s(1-s)} - \frac{p_0}{1-s}$

$\quad = \frac{s - P(s)}{s(1-s)} + \frac{p_0}{s} \Rightarrow G(s) = \frac{1 - P(s)s}{1-s} + s^{-1} p_0.$

(e) $G(s) = \sum_{k=0}^{\infty} a_k s^k = P(X=0)s^0 + P(X=2)s^1 + P(X=4)s^2 + \cdots$

$\quad = p_0 + p_2 s + p_4 s^2 + \cdots \qquad\qquad = \frac{1}{2}\left(P(\sqrt{s}) + P(-\sqrt{s}) \right)$

$\quad = \left(\frac{p_0}{2} + \frac{p_1}{2}(\sqrt{s}) + \frac{p_2}{2}(\sqrt{s})^2 + \cdots \right) + \left(\frac{p_0}{2}(-\sqrt{s})^0 + \frac{p_1}{2}(-\sqrt{s}) + \frac{p_2}{2}(-\sqrt{s})^2 + \cdots \right)$